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Human Reasoning and the Weak Completion Semantics

TECHNISCHE UNIVERSITÄT DRESDEN

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The Weak Completion Semantics – Introduction

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- Human Reasoning and Deduction
- The Goal
- The Suppression Task
- Counterexamples
- Further Remarks
- Suggested Readings

"Logic is everywhere ..."

Human Reasoning and Deduction

- Johnson-Laird, Byrne: Deduction 1991

  You need to make deductions to formulate plans and to evaluate actions;
  to determine the consequences of assumptions and hypotheses;
  to interpret and to formulate instructions, rules, and general principles;
  to pursue arguments and negotiations;
  to weigh evidence and to assess data;
  to decide between competing theories;
  and to solve problems.
  A world without deduction would be a world without science, technology, laws, social conventions, and culture.

- Johnson-Laird: Models of Deduction 1984

  Are there any general ways of thinking that humans follow when they make deductions?

The Goal

- The development of a cognitive theory for adequately modelling human reasoning tasks
  - computational
  - comprehensive
  - a connectionist realization
- Background
  - logic programming
  - logic-based knowledge representation and reasoning

The Suppression Task

- **12 experiments carried out by Ruth Byrne in the 1980s**
- **Repeated several times leading to similar results**
- **Showing that humans suppress previously drawn inferences**
  - **valid inferences**
  - **invalid inferences**
  - **with respect to classical two-valued logic**

  Byrne: Suppressing Valid Inferences with Conditionals 1989

Affirmation of the Antecedent

- *She has an essay to write*
  If she has an essay to write, then she will study late in the library
  Will she study late in the library?
  - 96% yes
- *She has an essay to write*
  If she has an essay to write, then she will study late in the library
  If she has textbooks to read, then she will study late in the library
  Will she study late in the library?
  - 96% yes
- *She has an essay to write*
  If she has an essay to write, then she will study late in the library
  If the library stays open, then she will study late in the library
  Will she study late in the library?
  - 38% yes

Naive Two-Valued Classical Logic

- $\{e,\ e \rightarrow \ell\} \models \ell$
  - ok 96%
  - Modus ponens
- $\{e,\ e \rightarrow \ell,\ t \rightarrow \ell\} \models \ell$
  - ok 96%
  - Two-valued classical logic is monotonic
- $\{e,\ e \rightarrow \ell,\ o \rightarrow \ell\} \models \ell$
  - Upps only 38% of the participants were doing this
  - Human reasoning appears to be nonmonotonic

Adequateness

- **Two-valued classical logic is universal**
  - **If human reasoning can be computed, then we should be able to model the three experiments in two-valued classical logic** **How?**
- Bibel: Perspectives on Automated Deduction 1991

  There is an adequate general proof method that can automatically discover any proof done by humans provided the problem (including all required knowledge) is stated in appropriately formalized terms

  Adequateness is understood as the property of a theorem proving method that

  for any given knowledge base, the method solves simpler problems faster than more difficult ones

  Simplicity is measured under consideration of all (general) formalisms available to capture the problem and intrinsic in this assumption is a belief in the existence of an algorithm that is feasible (from a complexity point of view) for the set of problems humans can solve

Towards a Simple Formalism to Capture the Suppression Task

- **We need to answer the following questions**
  - **If the participants in the third experiment did not use two-valued classical logic what else did they use?**
  - **How did they come up with their answers?**
  - **Can we formally specify a system in which the three experiments can be uniformly modeled such that the answers given by the majority of the participants can be computed?**
- **My proposal**
  - **Take a nonmonotonic and multi-valued logic**
  - **Take the Weak Completion Semantics**

The Weak Completion Semantics in a Nutshell

- **Inspired by**
  Stenning, van Lambalgen: Human Reasoning and Cognitive Science 2008
- **The six stages of reasoning according to the Weak Completion Semantics**
  - **Reasoning towards a (logic) program**
  - **Weakly completing the program**
  - **Computing its least model**
  - **Reasoning with respect to the least model**
  - **If necessary, applying skeptical abduction**
  - **If possible, searching for counterexamples**

Affirmation of the Antecedent

- ***She has an essay to write***
  If she has an essay to write, then she will go to the library

- **Program $\mathcal{P}$**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_e$	rule	definition of ℓ
$ab_e \leftarrow \bot$	assumption	ab_e is assumed to be false

- **Weakly completed program & Generation of least model**

$$\begin{array}{rcl} e & \leftrightarrow & \top \\ \ell & \leftrightarrow & e \wedge \neg ab_e \\ ab_e & \leftrightarrow & \bot \end{array}$$

true	*false*	$\Phi_{\mathcal{P}}$
e	ab_e	1
ℓ		2

- **Computing logical consequences with respect to the least model**
  - **She will go to the library**

Łukasiewicz Three-Valued Logic

- Łukasiewicz: O logice trójwartościowey 1920

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Affirmation of the Antecedent and Alternative Arguments

- *She has an essay to write*
  If she has an essay to write, then she will go to the library
  If she has textbooks to read, then she will go to the library

- **Program $\mathcal{P}$**

e	$\leftarrow$	$\top$	fact	definition of e
ℓ	$\leftarrow$	$e \wedge \neg ab_e$	rule	definition of ℓ
ab_e	$\leftarrow$	$\bot$	assumption	ab_e is assumed to be false
ℓ	$\leftarrow$	$t \wedge \neg ab_t$	rule	definition of ℓ
ab_t	$\leftarrow$	$\bot$	assumption	ab_t is assumed to be false

- **Weakly completed program & Generation of least model**

$$
\begin{array}{rcl}
e & \leftrightarrow & \top \\
\ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (t \wedge \neg ab_t) \\
ab_e & \leftrightarrow & \bot \\
ab_t & \leftrightarrow & \bot
\end{array}
$$

true	false	$\Phi_{\mathcal{P}}$
e	ab_e	1
	ab_t	
ℓ		2

- **Computing logical consequences with respect to the least model**
  - She will go to the library

Reasoning Towards an Appropriate Logical Form

- ***If she has an essay to write, then she will go to the library***
  If the library stays open, then she will go to the library
- Kowalski: Computational Logic and Human Thinking 2011
- **Context independent rules**
  - ***If she has an essay to write and the library stays open,***
    then she will study late in the library
    If the library stays open and she has a reason for studying in the library,
    then she will study late in the library
- **Context dependent rule plus exception**
  - ***If she has an essay to write, then she will study late in the library***
    However, if the library does not stay open, then she will not study late in the library
  - **The last statement is the contrapositive of the converse of the original sentence!**

Affirmation of the Antecedent and Additional Arguments

- *She has an essay to write*
  If she has an essay to write, then she will go to the library
  If the library stays open, then she will go to the library

- **Programs $\mathcal{P}$**

e	$\leftarrow$	$\top$	fact	definition of e
ℓ	$\leftarrow$	$e \wedge \neg ab_e$	rule	definition of ℓ
ab_e	$\leftarrow$	$\bot$	assumption	ab_e is assumed to be false
ℓ	$\leftarrow$	$o \wedge \neg ab_o$	rule	definition of ℓ
ab_o	$\leftarrow$	$\bot$	assumption	ab_o is assumed to be false
ab_e	$\leftarrow$	$\neg o$	rule	definition of ab_e
ab_o	$\leftarrow$	$\neg e$	rule	definition of ab_o

- **Weakly completed program & Generation of least model**

$$\begin{array}{rcl} e & \leftrightarrow & \top \\ \ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (o \wedge \neg ab_o) \\ ab_e & \leftrightarrow & \bot \vee \neg o \\ ab_o & \leftrightarrow & \bot \vee \neg e \end{array}$$

true	*false*	$\Phi_{\mathcal{P}}$
e		1
	ab_o	2

- **Computing logical consequences with respect to the least model**
  - **We can neither conclude that she will go nor that she will not go to the library**

Denial of the Antecedent

- ***She does not have an essay to write***
  If she has an essay to write, then she will study late in the library
  Will she not study late in the library?
  - **46% yes**
- ***She does not have an essay to write***
  If she has an essay to write, then she will study late in the library
  If she has textbooks to read, then she will study late in the library
  Will she not study late in the library?
  - **4% yes**
- ***She does not have an essay to write***
  If she has an essay to write, then she will study late in the library
  If the library stays open, then she will study late in the library
  Will she not study late in the library?
  - **63% yes**

Denial of the Antecedent

- *She does not have an essay to write*
  If she has an essay to write, then she will go to the library

- **Program $\mathcal{P}$**

$$\begin{array}{rcll} e & \leftarrow & \bot & \text{assumption} \\ \ell & \leftarrow & e \wedge \neg ab_e & \text{rule} \\ ab_e & \leftarrow & \bot & \text{assumption} \end{array}$$

- **Weakly completed program & Generation of least model**

$$\begin{array}{rcl} e & \leftrightarrow & \bot \\ \ell & \leftrightarrow & e \wedge \neg ab_e \\ ab_e & \leftrightarrow & \bot \end{array}$$

true	*false*	$\Phi_{\mathcal{P}}$
	e	1
	ab_e	
	ℓ	2

- **Computing logical consequences with respect to the least model**
  - She will not go to the library

Denial of the Antecedent and Alternative Arguments

- *She does not have an essay to write*
  If she has an essay to write, then she will go to the library
  If she has textbooks to read, then she will go to the library

- **Program $\mathcal{P}$**

$$
\begin{array}{rcll}
e & \leftarrow & \bot & \text{assumption} \\
\ell & \leftarrow & e \wedge \neg ab_e & \text{rule} \\
ab_e & \leftarrow & \bot & \text{assumption} \\
\ell & \leftarrow & t \wedge \neg ab_t & \text{rule} \\
ab_t & \leftarrow & \bot & \text{assumption}
\end{array}
$$

- **Weakly completed program & Generation of least model**

$$
\begin{array}{rcl}
e & \leftrightarrow & \bot \\
\ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (t \wedge \neg ab_t) \\
ab_e & \leftrightarrow & \bot \\
ab_t & \leftrightarrow & \bot
\end{array}
$$

true	false	$\Phi_{\mathcal{P}}$
	e	1
	ab_e	
	ab_t	

- **Computing logical consequences with respect to the least model**
  - We can neither conclude that she will go nor that she will not go to the library

Denial of the Antecedent and Additional Arguments

- *She does not have an essay to write*
  If she has an essay to write, then she will go to the library
  If the library stays open, then she will go to the library

- Programs $\mathcal{P}$

$$\begin{array}{rcll} e & \leftarrow & \bot & \text{assumption} \\ \ell & \leftarrow & e \wedge \neg ab_e & \text{rule} \\ ab_e & \leftarrow & \bot & \text{assumption} \\ \ell & \leftarrow & o \wedge \neg ab_o & \text{rule} \\ ab_o & \leftarrow & \bot & \text{assumption} \\ ab_e & \leftarrow & \neg o & \text{rule} \\ ab_o & \leftarrow & \neg e & \text{rule} \end{array}$$

- Weakly completed program & Generation of least model

$$\begin{array}{rcl} e & \leftrightarrow & \bot \\ \ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (o \wedge \neg ab_o) \\ ab_e & \leftrightarrow & \bot \vee \neg o \\ ab_o & \leftrightarrow & \bot \vee \neg e \end{array}$$

true	*false*	$\Phi_{\mathcal{P}}$
	e	1
ab_o		2
	ℓ	3

- Computing logical consequences with respect to the least model
  - **She will not go to the library**

Affirmation of the Consequent

- *She will study late in the library*
  If she has an essay to write, then she will study late in the library
  Has she an essay to write?
  - 71% yes
- *She will study late in the library*
  If she has an essay to write, then she will study late in the library
  If she has textbooks to read, then she will study late in the library
  Has she an essay to write?
  - 13% yes
- *She will study late in the library*
  If she has an essay to write, then she will study late in the library
  If the library stays open, then she will study late in the library
  Has she an essay to write?
  - 54% yes

Affirmation of the Consequent

- *She will go to the library*
  If she has an essay to write, then she will go to the library

- **Program**

$$\begin{array}{rcl} \ell & \leftarrow & \top \\ \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \end{array}$$

- **Weakly completed program & Generation of least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & \top \vee (e \wedge \neg ab_e) \\ ab_e & \leftrightarrow & \bot \end{array}$$

true	*false*
ℓ	ab_e

- **Computing logical consequences with respect to the least model**
  - **We cannot conclude that she has an essay to write**
  - **But most humans conclude that she has**
  - **Don't consider ℓ as a fact in the presence of a rule for ℓ**
    - **Consider ℓ to be an observation that needs to be explained**

Abduction

- **Program & Observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \end{array} \qquad \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad e \leftarrow \bot$$

- **Weakly completed program plus explanation & Generation of least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & e \wedge \neg ab_e \\ ab_e & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \end{array}$$

true	*false*
e	ab_e
ℓ	

- **Computing logical consequences with respect to the least model**
  - She has an essay to write

Affirmation of the Consequent and Alternative Arguments

- **Program & Observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \\ \ell & \leftarrow & t \wedge \neg ab_t \\ ab_t & \leftarrow & \bot \end{array}$$

$$\ell$$

- **Abducibles**

$$e \leftarrow \top \qquad t \leftarrow \top \qquad e \leftarrow \bot \qquad t \leftarrow \bot$$

- **Weakly completed program plus explanations & Generation of least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (t \wedge \neg ab_t) \\ ab_e & \leftrightarrow & \bot \\ ab_t & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \quad \text{or} \quad t \leftrightarrow \top \end{array}$$

true	false
e	ab_e
	ab_t
ℓ	

true	false
t	ab_e
	ab_t
ℓ	

- **Computing skeptical consequences with respect to both models**
  - We cannot conclude that she has an essay to write
  - Reasoning creduluously we can but the participants did not do this

Affirmation of the Consequent and Additional Arguments

- **Program & Observation**

$$
\begin{array}{rcl}
\ell & \leftarrow & e \wedge \neg ab_e \\
ab_e & \leftarrow & \bot \\
\ell & \leftarrow & o \wedge \neg ab_o \\
ab_o & \leftarrow & \bot \\
ab_e & \leftarrow & \neg o \\
ab_o & \leftarrow & \neg e
\end{array}
\qquad\qquad \ell
$$

- **Abducibles**

$$
e \leftarrow \top \qquad o \leftarrow \top \qquad e \leftarrow \bot \qquad o \leftarrow \bot
$$

- **Weakly completed program plus explanations & Generation of least model**

$$
\begin{array}{rcl}
\ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (o \wedge \neg ab_o) \\
ab_e & \leftrightarrow & \bot \vee \neg o \\
ab_o & \leftrightarrow & \bot \vee \neg e \\
e & \leftrightarrow & \top \\
o & \leftrightarrow & \top
\end{array}
$$

true	false
e	
o	
	ab_e
	ab_o
ℓ	

- **Computing consequences with respect to the least model**
  - **She has an essay to write**

Denial of the Consequent

- *She will not study late in the library*
  If she has an essay to write, then she will study late in the library
  Does she not have an essay to write?
  - 92% yes
- *She will not study late in the library*
  If she has an essay to write, then she will study late in the library
  If she has textbooks to read, then she will study late in the library
  Does she not have essay to write?
  - 96% yes
- *She will not study late in the library*
  If she has an essay to write, then she will study late in the library
  If the library stays open then, she will study late in the library
  Does she not have an essay to write?
  - 33% yes

Denial of the Consequent

- **Program & Observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \end{array} \qquad\qquad \neg\ell$$

- **Abducibles**

$$e \leftarrow \top \qquad\qquad e \leftarrow \bot$$

- **Weakly completed program plus explanation & Generation of least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & e \wedge \neg ab_e \\ ab_e & \leftrightarrow & \bot \\ e & \leftrightarrow & \bot \end{array}$$

true	false
	ab_e
	e
	ℓ

- **Computing logical consequences with respect to the least model**
  - She does not have an essay to write

Denial of the Consequent and Alternative Arguments

- **Program & Observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \\ \ell & \leftarrow & t \wedge \neg ab_t \\ ab_t & \leftarrow & \bot \end{array} \qquad \neg \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad t \leftarrow \top \qquad e \leftarrow \bot \qquad t \leftarrow \bot$$

- **Weakly completed program plus explanations & Generation of least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (t \wedge \neg ab_t) \\ ab_e & \leftrightarrow & \bot \\ ab_t & \leftrightarrow & \bot \\ e & \leftrightarrow & \bot \\ t & \leftrightarrow & \bot \end{array}$$

true	*false*
	e
	t
	ab_e
	ab_t
	ℓ

- **Computing consequences with respect to the least model**
  - **She does not have an essay to write**

Denial of the Consequent and Additional Arguments

- **Program & Observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_e \\ ab_e & \leftarrow & \bot \\ \ell & \leftarrow & o \wedge \neg ab_o \\ ab_o & \leftarrow & \bot \\ ab_e & \leftarrow & \neg o \\ ab_o & \leftarrow & \neg e \end{array} \qquad \neg \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad o \leftarrow \top \qquad e \leftarrow \bot \qquad o \leftarrow \bot$$

- **Weakly completed program plus explanations & Generation of least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_e) \vee (o \wedge \neg ab_o) \\ ab_e & \leftrightarrow & \bot \vee \neg o \\ ab_o & \leftrightarrow & \bot \vee \neg e \\ e & \leftrightarrow & \bot \quad \text{or} \quad o \leftrightarrow \bot \end{array}$$

true	false
	e
ab_o	
	ℓ

true	false
	o
ab_e	
	ℓ

- **Computing skeptical consequences with respect to both models**
  - **We cannot conclude that she does not have an essay to write**

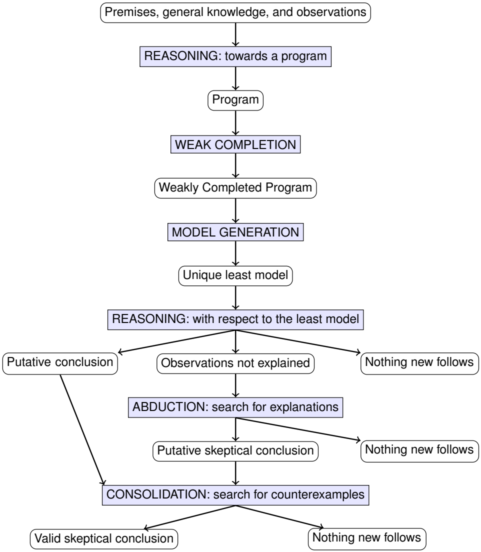
Summary (1)

Ex	atomic sentences				conditional sentences			queries				WCS
	e	$\neg e$	ℓ	$\neg \ell$	$e \Rightarrow \ell$	$t \Rightarrow \ell$	$o \Rightarrow \ell$	ℓ	$\neg \ell$	e	$\neg e$	
1	X				X			96%				$\top$
2	X				X	X		96%				$\top$
3	X				X		X	38%				**U**
4		X			X				46%			$\top$
5		X			X	X			4%			**U**
6		X			X		X		63%			$\top$
7			X		X					71%		$\top$
8			X		X	X				13%		**U**
9			X		X		X			54%		$\top$
10				X	X						92%	$\top$
11				X	X	X					96%	$\top$
12				X	X		X				33%	**U**

Summary (2)

- **The Weak Completion Semantics appears to be adequate**
  - **The suppression effect is modeled**
  - **The average reasoner is modeled**
- **Principles**
  - **Licenses for inference**
    - **Abnormalities**
    - **Modeling additional antecedents by context dependent rules**
  - **Abduction**
    - **If a fact corresponds to the consequent of a conditional then treat the fact as an observation which needs to be explained**
    - **Skeptical abduction is adequate**
    - **Credulous abduction is not**

The Six Stages of Reasoning



Necessary and Non-Necessary Antecedents

- Given a conditional sentence *if A then C*
  - *A* is necessary iff *C* cannot be true unless *A* is true
  - *A* is non-necessary iff *C* can be true irrespective of the truth of *A*
- Are the following antecedents necessary or non-necessary?
  - *If the library stays open, then she will study late in the library*
  - *If she has an essay to write, then she will study late in the library*
- The answer depends on experience, culture, etc

Non-Necessary Antecedents

- Suspect the antecedent of

  if she has an essay to write, then she will study late in the library

  was classified as non-necessary
- But then there are other (unknown) reasons for studying late in the library
  - This can be taken into consideration by the abducible $\ell \leftarrow \top$
- Recall Experiment 4 (denial of the antecedent)
  - The least model was $\langle \emptyset, \{e, \ell, ab_e\} \rangle$
  - 46% answered *she will not study late in the library*
  - What about the others?
  - Due to the abducible we can construct a counterexample $\langle \{\ell\}, \{e, ab_e\} \rangle$
  - Reasoning skeptically *she may or may not study late in the library*

Formal and/or Cognitive Theory

- Collins English Dictionary
  - A **formal theory** is an uninterpreted symbolic system whose syntax is precisely defined and on which a relation of deducibility is defined in purely syntactic terms
  - A **cognitive theory** is any theory of mind that focuses on mental activities, such as perceiving, attending, thinking, remembering, evaluating, planning, language, and creativity, especially a theory that suggests a model for the various processes involved
- The Weak Completion Semantics is a formal theory
- **But is it also a cognitive theory?**

Human Disjunctive Reasoning

- In classical two-valued logic $\{A \vee B, \neg A\} \models B$ holds
  - Can you prove it?
- What do you think about the following human reasoning episode?

  Eva's in Rio or she's in Brazil
  She's not in Brazil
  Therefore, she is in Rio

  - Johnson-Laird, Byrne: Conditionals 2002
    No sensible person other than a logician is likely to draw this conclusion as it is impossible for Eva to be in Rio and not in Brazil, because Rio is in Brazil
  - What should a computer scientist reply?

Expected and Suggested Readings

- **I expect students to read**
  - Byrne: Suppressing Valid Inferences with Conditionals 1989
  - Łuksiewicz: O logice trójwartościowey 1920
- **I suggest that students have a look at**
  - Stenning, van Lambalgen: Human Reasoning and Cognitive Science 2008
  - Kowalski: Computational Logic and Human Thinking 2011
- **Complete references are given in the manuscript**

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