MAI4CAREU

Master programmes in Artificial Intelligence 4 Careers in Europe

Human Reasoning and the Weak Completion Semantics

TECHNISCHE UNIVERSITÄT DRESDEN

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Applications and Extensions

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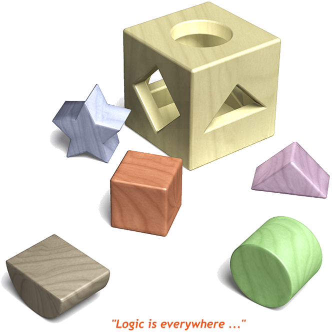
- Conditional Reasoning
- Syllogistic Reasoning
- Disjunctive Reasoning
- Contextual Reasoning
- Spatial Reasoning
- Ethical Decision Problems



Conditional Reasoning

- Conditionals
- The Semantics of Conditionals
- Reasoning with a Conditional
- Reasoning about a Conditional
- The Selection Task

Introduction – Conditionals

- **Conditionals** are statements of the form *if antecedent then consequence*
- Claim of membership in a class or category
  - If it is a dog then it is a mammal
  - If the city is Rio then it is in Brasil
- Declarative (indicative) statements of fact or assumed fact
  - If the serial number is less that 150000 then it was built before 1995
  - If it is raining then the roofs are wet
  - If the roofs are wet then it is raining
- Promise
  - If you clean your shoes
    then Santa Claus will fill them with nuts, fruits, and chocolate
- Threat
  - If you violate the terms of the contract then we will sue

More Conditionals

- **Advice**
  - If it will be cold then put your sweater on
  - If it is raining then take your umbrella
- **Tip**
  - If you want to make a good impression then wear a dress or a suit and tie
- **Legal rules**
  - If you want to drink alcohol in a restaurant
    then you must be older than 18 years of age
- **Command**
  - If you find termites then apply the pesticide
- **Request**
  - If it is convenient for you
    then please drop the package off on your way to work

Even More Conditionals

- **Counterfactual**
  - **If I had not taken this road today then I would have avoided the accident**
- **Prediction**
  - **If I take my umbrella then it will not rain in the afternoon**
  - **If there is a d on one side of a card then there is a 3 on the other side**
- **Question**
  - **If she graduates with 1**
    will she be promoted to the PhD program of her choice?
- **Warning**
  - **If you park there then your car will be towed**
- Nickerson: Conditional Reasoning: 2015

Conditionals in this Lecture

- In the sequel, let *if $\mathcal{A}$ then $\mathcal{C}$* be a conditional, where
  - antecedent $\mathcal{A}$ and consequence $\mathcal{C}$ are finite and consistent sets of ground literals
  - If $\mathcal{A}$ or $\mathcal{C}$ is a singleton set, then curly brackets are omitted
- Conditionals are evaluated wrt some background knowledge
  - a finite propositional or datalog program $\mathcal{P}$
  - an equational theory $\mathcal{E}$
  - a set of integrity constraints $\mathcal{IC}$
- Let $\mathcal{M}_{wc\mathcal{P}}$ be the least model of the weak completion of $\mathcal{P}$

The Semantics of Conditionals

- If it rains then the roofs are wet and she takes her umbrella
- Let $\mathcal{P}$ consist of
$$\begin{array}{rcl} \mathit{wet\_roofs} & \leftarrow & \mathit{rain} \wedge \neg \mathit{ab}_w \\ \mathit{ab}_w & \leftarrow & \bot \\ \mathit{umbrella} & \leftarrow & \mathit{rain} \wedge \neg \mathit{ab}_u \\ \mathit{ab}_u & \leftarrow & \bot \end{array}$$
- $\mathcal{M}_{wc\mathcal{P}} = \langle \emptyset, \{\mathit{ab}_w, \mathit{ab}_u\}\rangle \qquad \mathcal{A}_{\mathcal{P}} = \{\mathit{rain} \leftarrow \top,\ \mathit{rain} \leftarrow \bot\}$
- What follows if we additionally observe that
  - the roofs are wet?
  - she took her umbrella?
  - the roofs are not wet?
  - she did not take her umbrella?
- Are you happy with the formalization?

The Semantics of Conditionals – Obligation Conditionals

- **A conditional *if* $\mathcal{A}$ *then* $\mathcal{C}$ is said to be an obligation conditional iff its consequence $\mathcal{C}$ is obligatory given its antecedent $\mathcal{A}$**
- Byrne: The Rational Imagination: 2005
  - **We cannot easily imagine a case where the antecedent is true and the consequence is not**
  - **The possibility $\mathcal{A} \wedge \neg\mathcal{C}$ is forbidden or unlikely**
- **Can you name obligation conditionals?**
  - ***If a person is drinking beer then the person must be over 19 years of age***
  - ***If somebody is riding a motorbike then he/she must wear a helmet***
  - ***If a german tourist wants to enter Russia then he needs a visa***
  - ***If somebody's parents are elderly then he/she should look after them***
  - ***If there is no light then plants will not grow***
  - ***If an object is not supported it will drop to the floor***
  - ***If it is raining then the roofs are wet***

Obligation Conditionals 2

- Byrne: The Rational Imagination: 2005
- **For obligation conditionals there are two initial possibilities people think about**
  - **the conjunction of antecedent and consequent (permitted)**
    - ***it rains and the roofs are wet***
  - **the conjunction of antecedent and negation of consequent (forbidden/unlikely)**
    - ***it rains and the roofs are not wet***
- **Exceptions are possible but unlikely**

Factual Conditionals

- A conditional *if* $\mathcal{A}$ *then* $\mathcal{C}$ is said to be a factual conditional iff its consequent $\mathcal{C}$ is not obligatory given its antecedent $\mathcal{A}$
- There is no forbidden or unlikely possibility
- Can you name factual conditionals?
  - *If the letter d is on one side of a card then the number 3 is on the other side*
  - *If Nancy rides her motorbike she goes to the mountains*
  - *If Fred was in Paris then Joe was in Lisbon*
  - *If it raining then she is taking her umbrella*
  - *If the sun is shining then I will water my garden in the evening*

Obligation versus Factual Conditionals – Summary

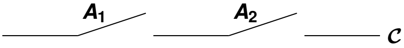
- Humans may classify conditionals as obligation or factual conditionals
- This is an informal and pragmatic classification
- It depends on
  - the background knowledge and experience of a human

  as well as on
  - the context in which a conditional is stated

Necessary Antecedents

- The antecedent $\mathcal{A}$ of a conditional *if* $\mathcal{A}$ *then* $\mathcal{C}$ is said to be necessary iff its consequent $\mathcal{C}$ cannot be true unless the antecedent is true
  - But the antecedent $\mathcal{A}$ may be true while the consequence $\mathcal{C}$ is not

series circuit



- Can you name conditionals with necessary antecedent?
  - *If the kid is tall enough then it can ride the roller coaster*
  - *If it is raining then the roofs are wet*
  - *If there is gas in the gas tank then the engine will start*
  - *If the switch is toggled then the light will be turned on*

Non-Necessary Antecedents

- The antecedent $\mathcal{A}$ of a conditional *if* $\mathcal{A}$ *then* $\mathcal{C}$ is said to be non-necessary iff $\mathcal{A}$ is not necessary
- $\mathcal{C}$ may be true without $\mathcal{A}$ being true
- Can you name conditionals with non-necessary antecedent?
  - *If Polly is a parrot then Polly is a bird*
  - *If the number ends with 3 then it is an odd number*
  - *If the car has no gas then it will not run*
  - *If it is raining then she is taking her umbrella*
  - *If a person is drinking beer then the person must be over 19 years of age*
  - *If the sun is shining then she is going to the swimming pool*
  - *If I want to meet friends then I will go to my favorite pub*
  - *If Nancy rides her motorbike she goes to the mountains*

Necessary versus Non-Necessary Antecedents – Summary

- **Humans may classify antecedents as necessary or non-necessary**
- **The classification is informal and pragmatic**
- **It depends on**
  - **the background knowledge and experience of a human**

  as well as on
  - **the context in which a conditional is stated**

Representing the Semantics of Conditionals

- Conditional *if A then C*
- Represented by

$$\begin{array}{lcl} C & \leftarrow & A \wedge \neg ab \\ ab & \leftarrow & \bot \end{array}$$

- Abducibles are

$$\mathcal{A}_\mathcal{P} = \{A \leftarrow \top,\ A \leftarrow \bot\}$$

- We extend the set of abducibles

$$\mathcal{A}^e_\mathcal{P} = \mathcal{A}_\mathcal{P} \cup \mathcal{A}^{nn}_\mathcal{P} \cup \mathcal{A}^f_\mathcal{P}$$

where

$$\begin{array}{lcll} \mathcal{A}^{nn}_\mathcal{P} & = & \{C \leftarrow \top \mid & C \text{ is head of a rule in } \mathcal{P} \text{ representing} \\ & & & \text{a conditional with non-necessary antecedent}\} \\ \mathcal{A}^{f}_\mathcal{P} & = & \{ab \leftarrow \top \mid & ab \text{ occurs in the body of a rule in } \mathcal{P} \\ & & & \text{representing a factual conditional}\} \end{array}$$

Returning to the Initial Example

$C \leftarrow A \wedge \neg ab$	A non-necessary	A necessary
Factual conditional	$ab \leftarrow \top,\ C \leftarrow \top$	$ab \leftarrow \top$
Obligation conditional	$C \leftarrow \top$	

- ***If it rains then the roofs are wet***
  - **Obligation conditional with necessary antecedent**
  - $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot\} = \mathcal{A}^e_{\mathcal{P}}$
- ***If it rains then she takes her umbrella***
  - **Factual conditional with non-necessary antecedent**
  - $\mathcal{A}^e_{\mathcal{P}} = \{rain \leftarrow \top,\ rain \leftarrow \bot,\ umbrella \leftarrow \top,\ ab_u \leftarrow \top\}$
- **Are you happier now?**

Reasoning with a Conditional

- First premise: conditional sentence *if A then C*
- Second premise: (possibly negated) atomic sentence
  - affirmation of the antecedent (AA)
  - denial of the antecedent (DA)
  - affirmation of the consequent (AC)
  - denial of the consequent (DC)
- What follows?

Reasoning with a Conditional – Examples

- If it rains then the roofs must be wet
  It rains **(AA)**
- If Pauls rides a motorbike then Paul must wear a helmet
  Paul does not ride a motorbike **(DA)**
- If the library is open then Elisa is studying late in the library
  Elisa is studying late in the library **(AC)**
- If Nancy rides her motorbike then Nancy goes to the mountains
  Nancy does not go to the mountains **(DC)**
- **What follows?**

Facts, Assumptions, or Observations

- **First premise**

$$\begin{array}{rcl} C & \leftarrow & A \wedge \neg ab \\ ab & \leftarrow & \bot \end{array}$$

  with set of abducibles

$$\mathcal{A} = \{A \leftarrow \top,\ A \leftarrow \bot\}$$

- **Shall the second premise be represented as fact, assumption, or observation?**
  - **So far, if atom undefined then fact or assumption else observation**
  - **In this section, always observation**

An Experiment

- 56 logically naive participants from mid-Europe including UK
- Proficient speakers in English
- They were given a short story and thereafter
  - a conditional sentence and a (possibly negated) atomic sentence
- What follows?
- 48 problems consisting of 12 conditionals classified by the authors
- Solved all four inference types (AA, DA, AC, DC)
- Participants could answer
  - corresponding atomic sentence which was not presented as second premise
  - corresponding negated atomic sentence
  - nothing (new) follows (nf)
- Participants acted as their own controls

Conditionals used in the Experiment

- **Obligation Conditionals with Necessary Antecedent**

(1) If it rains then the roofs must be wet
(2) If water in the cooking pot is heated over $99^\circ C$ then the water starts boiling
(3) If the wind is strong enough then the sand is blowing over the dunes

- **Obligation Conditionals with Non-Necessary Antecedent**

(4) If Paul rides a motorbike then Paul must wear a helmet
(5) If Maria is drinking alcoholic beverages in a pub then Maria must be over 19 years of age
(6) If it rains then the lawn must be wet

- **Factual Conditionals with Necessary Antecedent**

(7) If the library is open then Sabrina is studying late in the library
(8) If the plants get water then they will grow
(9) If my car's start button is pushed then the engine will start running

- **Factual Conditionals with Non-Necessary Antecedent**

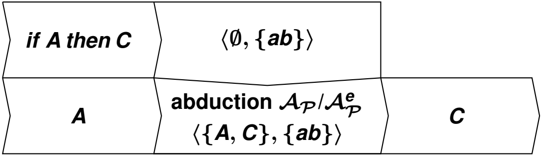
(10) If Nancy rides her motorbike then Nancy goes to the mountains
(11) If Lisa plays on the beach then Lisa will get sunburned
(12) If Ron scores a goal then Ron is happy

Affirmation of the Antecedent (AA)

Class	*C*	¬*C*	*nf*	Sum	*Mdn C*	*Mdn nf*
(1)	**55**	**1**	**0**	**56**	**3343**	***na***
(2)	**55**	**1**	**0**	**56**	**3487**	***na***
(3)	**53**	**3**	**0**	**56**	**3516**	***na***
Obligation+necessary	**163 (.97)**	**5 (.03)**	**0**	**168**	**3408**	***na***
(4)	**53**	**1**	**2**	**56**	**3403**	**3472**
(5)	**53**	**2**	**1**	**56**	**3903**	**3572**
(6)	**54**	**1**	**1**	**56**	**3088**	**6959**
Obligation+non-necessary	**160 (.95)**	**4 (.02)**	**4 (.02)**	**168**	**3543**	**4183**
(7)	**49**	**1**	**6**	**56**	**3885**	**7051**
(8)	**54**	**1**	**1**	**56**	**3559**	**7349**
(9)	**54**	**1**	**1**	**56**	**3710**	**3826**
Factual+necessary	**157 (.93)**	**3 (.02)**	**8 (.05)**	**168**	**3615**	**6926**
(10)	**51**	**2**	**3**	**56**	**3929**	**6647**
(11)	**54**	**1**	**1**	**56**	**3777**	**5073**
(12)	**55**	**1**	**0**	**56**	**2977**	***na***
Factual+non-necessary	**160 (.95)**	**4 (.02)**	**4 (.02)**	**168**	**3644**	**5860**
Obligation	**323**	**9**	**4**	**336**	**3516**	**4183**
Factual	**317**	**7**	**12**	**336**	**3640**	**6575**
Necessary	**320**	**8**	**8**	**336**	**3546**	**6926**
Non-necessary	**320**	**8**	**8**	**336**	**3588**	**4934**
Total	**640 (.95)**	**16 (.02)**	**16 (.02)**	**672**	**3570**	**5925**

AA – Details

- $\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\; ab \leftarrow \bot\}$ $\qquad\qquad \mathcal{A}_\mathcal{P} = \{A \leftarrow \top,\; A \leftarrow \bot\}$
- $\mathcal{O} = \{A\}$ is explained by $\{A \leftarrow \top\}$
- Neither $\{C \leftarrow \top\}$ nor $\{ab \leftarrow \top\}$ can explain $\mathcal{O}$



- **Please check an example for each class!**

Denial of the Antecedent (DA)

Class	*C*	$\neg C$	*nf*	Sum	*Mdn* $\neg C$	*Mdn nf*
(1)	**0**	**45**	**11**	**56**	**2863**	**4901**
(2)	**2**	**54**	**0**	**56**	**3367**	***na***
(3)	**2**	**51**	**3**	**56**	**3647**	**10477**
Obligation+necessary	**4 (0.2)**	**150 (.89)**	**14 (.08)**	**168**	**3356**	**5115**
(4)	**1**	**40**	**15**	**56**	**3722**	**7189**
(5)	**3**	**28**	**25**	**56**	**5735**	**7814**
(6)	**4**	**36**	**16**	**56**	**3602**	**6240**
Obligation+non-necessary	**8 (.05)**	**104 (.62)**	**56 (.33)**	**168**	**4064**	**7471**
(7)	**2**	**51**	**3**	**56**	**3928**	**7273**
(8)	**1**	**47**	**8**	**56**	**3296**	**5728**
(9)	**1**	**52**	**3**	**56**	**3549**	**8735**
Factual+necessary	**4 (.02)**	**150 (.89)**	**14 (.08)**	**168**	**3605**	**6582**
(10)	**1**	**39**	**16**	**56**	**3725**	**6874**
(11)	**0**	**41**	**15**	**56**	**3374**	**5887**
(12)	**1**	**41**	**14**	**56**	**3205**	**7002**
Factual+non-necessary	**2 (.01)**	**121 (.72)**	**45 (.28)**	**168**	**3374**	**6221**
Obligation	**12**	**254**	**70**	**336**	**3583**	**6613**
Factual	**6**	**271**	**59**	**336**	**3518**	**6221**
Necessary	**8 (.02)**	**300 (.89)**	**28 (.08)**	**336**	**3474**	**5808**
Non-necessary	**10 (.03)**	**225 (.67)**	**101 (.30)**	**336**	**3646**	**6700**
Total	**18 (.03)**	**525 (.78)**	**129 (.19)**	**672**	**3558**	**6450**

DA – Details

- $\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\}$ $\qquad \mathcal{A_P} = \{A \leftarrow \top,\ A \leftarrow \bot\}$
- $\mathcal{O} = \{\neg A\}$ is explained by
  - $\{A \leftarrow \bot\}$
  - $\{A \leftarrow \bot,\ C \leftarrow \top\}$ (in case of a non-necessary antecedent)

if A then C	$\langle \emptyset, \{ab\} \rangle$	
$\neg A$	abduction $\mathcal{A_P}$ $\langle \emptyset, \{A, C, ab\} \rangle$	$\neg C$
	abduction $\mathcal{A}^e_{\mathcal{P}}$	$\neg C$ / *nf*

- **Please check an example for each class!**

Affirmation of the Consequent (AC)

Class	*A*	¬*A*	*nf*	Sum	*Mdn A*	*Mdn nf*
(1)	**37**	**1**	**18**	**56**	**3952**	**7995**
(2)	**48**	**1**	**7**	**56**	**4003**	**4170**
(3)	**43**	**1**	**12**	**56**	**3458**	**9001**
Obligation+necessary	**128 (.76)**	**3 (.02)**	**37 (.22)**	**168**	**3797**	**8175**
(4)	**42**	**1**	**13**	**56**	**3659**	**8828**
(5)	**32**	**1**	**23**	**56**	**4704**	**6044**
(6)	**29**	**1**	**26**	**56**	**3593**	**4396**
Obligation+non-necessary	**103 (.61)**	**3 (.02)**	**62 (.37)**	**168**	**3968**	**5939**
(7)	**51**	**1**	**4**	**56**	**3767**	**4397**
(8)	**42**	**1**	**13**	**56**	**3798**	**4565**
(9)	**45**	**1**	**10**	**56**	**3492**	**4598**
Factual+necessary	**138 (.82)**	**3 (.02)**	**27 (.16)**	**168**	**3699**	**4565**
(10)	**34**	**2**	**20**	**56**	**5224**	**6289**
(11)	**29**	**2**	**25**	**56**	**3218**	**6205**
(12)	**33**	**1**	**22**	**56**	**3483**	**4992**
Factual+non-necessary	**96 (.57)**	**5 (.03)**	**67 (.40)**	**168**	**3885**	**6116**
Obligation	**231**	**6**	**99**	**336**	**3888**	**6044**
Factual	**234**	**8**	**94**	**336**	**3769**	**5650**
Necessary	**266 (.79)**	**6 (.02)**	**64 (.19)**	**336**	**3735**	**5450**
Non-necessary	**199 (.59)**	**8 (.02)**	**129 (.38)**	**336**	**3906**	**6039**
Total	**465 (.69)**	**14 (.02)**	**193 (.29)**	**672**	**3826**	**5802**

AC – Details

- $\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\}$ $\qquad\qquad \mathcal{A}_\mathcal{P} = \{A \leftarrow \top,\ A \leftarrow \bot\}$
- $\mathcal{O} = \{C\}$ is explained by
  - $\{A \leftarrow \top\}$
  - $\{C \leftarrow \top\}$ (in case of a non-necessary antecedent)

if A then C	$\langle \emptyset, \{ab\} \rangle$	
C	abduction $\mathcal{A}_\mathcal{P}$ $\langle \{A, C\}, \{ab\} \rangle$	A
	abduction $\mathcal{A}^e_\mathcal{P}$	A / *nf*

- **Please check an example for each class!**

Denial of the Consequent (DC)

Class	*A*	¬*A*	*nf*	Sum	*Mdn* ¬*A*	*Mdn nf*
(1)	**1**	**45**	**10**	**56**	**3449**	**4758**
(2)	**0**	**50**	**6**	**56**	**4058**	**7922**
(3)	**2**	**46**	**8**	**56**	**3796**	**4517**
Obligation+necessary	**3 (.02)**	**141 (.84)**	**24 (.14)**	**168**	**3767**	**5732**
(4)	**3**	**46**	**7**	**56**	**3872**	**4154**
(5)	**1**	**54**	**1**	**56**	**4946**	**8020**
(6)	**0**	**36**	**20**	**56**	**4062**	**5235**
Obligation+non-necessary	**4 (.02)**	**136 (.81)**	**28 (.17)**	**168**	**4293**	**5803**
(7)	**1**	**37**	**18**	**56**	**5974**	**4744**
(8)	**3**	**42**	**11**	**56**	**4367**	**5013**
(9)	**0**	**47**	**9**	**56**	**4208**	**3966**
Factual+necessary	**4 (0.2)**	**126 (.75)**	**38 (.23)**	**168**	**4849**	**4574**
(10)	**2**	**35**	**19**	**56**	**4879**	**4167**
(11)	**0**	**39**	**17**	**56**	**4411**	**5647**
(12)	**0**	**34**	**22**	**56**	**3726**	**3813**
Factual+non-necessary	**2 (.01)**	**108 (.64)**	**58 (.35)**	**168**	**4338**	**4542**
Obligation	**7 (.02)**	**277 (.82)**	**52 (.15)**	**336**	**4053**	**4790**
Factual	**6 (.02)**	**234 (.70)**	**96 (.28)**	**336**	**4459**	**4345**
Necessary	**7**	**267**	**62**	**336**	**4096**	**4758**
Non-necessary	**6**	**244**	**86**	**336**	**4325**	**4555**
Total	**13 (.02)**	**511(.76)**	**148 (.22)**	**672**	**4311**	**5162**

DC – Details

- $\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\}$ $\qquad$ $\mathcal{A}_\mathcal{P} = \{A \leftarrow \top,\ A \leftarrow \bot\}$
- $\mathcal{O} = \{\neg C\}$ is explained by
  - $\{A \leftarrow \bot\}$
  - $\{ab \leftarrow \top\}$ (in case of a factual conditional)

if A then C	$\langle \emptyset, \{ab\} \rangle$	
$\neg C$	abduction $\mathcal{A}_\mathcal{P}$ $\langle \emptyset, \{A, C, ab\} \rangle$	$\neg A$
	abduction $\mathcal{A}^e_\mathcal{P}$	$\neg A$ / *nf*

- **Please check an example for each class!**

Reasoning About a Conditional

- Revision
- Minimal Revision Followed by Abduction
- Pam is Well
- The Moon is Not Made out of Cheese
- The Suppression Task Revisited
- The Shooting of Kennedy
- The Firing Squad
- The Forest Fire
- Relevance
- The Selection Task

Experiment – The Firing Squad

- Pearl: Causality: Models, Reasoning, and Inference: 2000
- *If the court orders an execution, then the captain will give the signal upon which riflemen A and B will shoot the prisoner Consequently the prisoner will be dead*
- **We assume that**
  - *the court's decision is unknown*
  - *both riflemen are accurate, alert, and law-abiding*
  - *the rifles are operating as expected*
  - *the prisoner is unlikely to die from any other causes*
- **Evaluate the following conditionals (true, false, unknown)**
  - *If the prisoner is not dead then the captain did not signal*
  - *If rifleman A shot then rifleman B shot as well*
  - *If rifleman A did not shoot then the prisoner is not dead*
  - *If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution*

More on Conditionals

- In the sequel, let *if* $\mathcal{A}$ *then* $\mathcal{C}$ be a conditional, where
  - antecedent $\mathcal{A}$ and consequence $\mathcal{C}$
    are finite and consistent sets of ground literals
- Conditionals are evaluated wrt some background knowledge
  - a finite propositional or datalog program $\mathcal{P}$
  - an equational theory $\mathcal{E}$
  - a set of integrity constraints $\mathcal{IC}$ such that $\mathcal{M}_{wc\mathcal{P}}$ satisfies $\mathcal{IC}$
- We distinguish three cases wrt the value of the antecedent under $\mathcal{M}_{wc\mathcal{P}}$

Indicative Conditionals

- Let *if* $\mathcal{A}$ *then* $\mathcal{C}$ be a conditional such that $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = \top$
  - Such conditionals are often called indicative conditionals
  - Their consequent is asserted to be true if their antecedent is true
  - Check whether $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{C} = \top$ holds

Counterfactuals

- Let *if* $\mathcal{A}$ *then* $\mathcal{C}$ be a conditional such that $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = \bot$
  - Such conditionals are sometimes called counterfactuals
    - Their antecedent is false
    - Their consequent may or may not be true
    - But in the counterfactual circumstance of the antecedent being true the consequence is asserted to be true
  - *Counterfactuals are always true because the premise is false*
    Eco: The Name of the Rose: 1988
    - Humans do not consider counterfactuals this way
    - Counterfactuals are very important   Byrne: Counterfactuals in XAI: 2019
    - If the car had detected the pedestrian earlier and braked the passenger would not have been injured
    - If the car had not swerved and hit the wall the passenger would not have been injured
  - We need to revise the background knowledge

Revision

- Let $\mathcal{S}$ be a finite and consistent set of literals

$$rev(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus defs(\mathcal{P}, \mathcal{S})) \cup \mathcal{S}^{\uparrow}$$

  is called the revision of $\mathcal{P}$ with respect to $\mathcal{S}$

$$\begin{aligned} & rev(\{e \leftarrow \top,\ \ell \leftarrow e \wedge \neg ab_e,\ ab_e \leftarrow \bot\}, \{\neg \ell\}) \\ = \ & \{e \leftarrow \top,\ \ell \leftarrow \bot,\ ab_e \leftarrow \bot\} \end{aligned}$$

- **Proposition 31**
  Let $\mathcal{P}$ be a program, $\mathcal{E}$ an equational theory, and $\mathcal{S}$ a consistent set of literals
  - rev is nonmonotonic
  - If $\mathcal{M}_{wc\mathcal{P}}\, L = \mathsf{U}$ for all $L \in \mathcal{S}$ then rev is monotonic: $\mathcal{M}_{wc\mathcal{P}} \subseteq \mathcal{M}_{wc\, rev(\mathcal{P}, \mathcal{S})}$
  - $\mathcal{M}_{wc\, rev(\mathcal{P}, \mathcal{S})}\, \mathcal{S} = \top$
- Proof $\rightsquigarrow$ Exercise

Unknown Antecedents

- Let *if* $\mathcal{A}$ *then* $\mathcal{C}$ be a conditional such that $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = U$
  - To the best of my knowledge this case has not been considered so far
  - We believe that humans would like to assign true to the antecedent
    - Skeptical abduction
    - Revision
  - There are scenarios where abduction alone cannot solve the problem
  - We propose to
    - minimally revise the background knowledge
    - and to apply skeptical abduction
    - such that the antecedent becomes true
  - Do humans make an attempt to assign false to the antecedent?

Minimal Revision Followed by Abduction (MRFA)

- Given $\mathcal{P}$, $\mathcal{E}$, $\mathcal{IC}$, and the conditional sentence *if* $\mathcal{A}$ *then* $\mathcal{C}$
- If $\mathcal{M}_{wc\mathcal{P}}$ does not satisfy $\mathcal{IC}$, then
  - if $\mathcal{O} = \emptyset$ can be explained by $\mathcal{X} \subseteq \mathcal{A}_{\mathcal{P}}$
    then evaluate *if* $\mathcal{A}$ *then* $\mathcal{C}$ with respect to $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})}$
    else *nothing follows*
- If $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = \top$, then the value of *if* $\mathcal{A}$ *then* $\mathcal{C}$ is $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{C}$
- If $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = \bot$, then evaluate *if* $\mathcal{A}$ *then* $\mathcal{C}$ wrt $\mathcal{M}_{wc\ rev(\mathcal{P},\mathcal{S})}$, where
  - $\mathcal{S} = \{L \in \mathcal{A} \mid \mathcal{M}_{wc\mathcal{P}}\ L = \bot\}$
- If $\mathcal{M}_{wc\mathcal{P}}\ \mathcal{A} = U$, then evaluate *if* $\mathcal{A}$ *then* $\mathcal{C}$ wrt $\mathcal{M}_{wc\mathcal{P}'}$, where
  - $\mathcal{P}' = rev(\mathcal{P},\mathcal{S}) \cup \mathcal{X}$,
  - $\mathcal{S}$ is a minimal subset of $\mathcal{A}$,
  - $\mathcal{X} \subseteq \mathcal{A}_{rev(\mathcal{P},\mathcal{S})}$ is an explanation for $\mathcal{A} \setminus \mathcal{S}$
  - such that $\mathcal{P}' \models_{wcs} \mathcal{A}$ and $\mathcal{M}_{wc\mathcal{P}'}$ satisfies $\mathcal{IC}$
- Abduction has to be applied skeptically

Pam is well

- $\mathcal{P} = \{well \leftarrow \top\}$
- $\mathcal{M}_{wc\mathcal{P}} = \langle \{well\}, \emptyset \rangle$
- Evaluate *if Pam is not well, then she has the flu*
- $rev(\mathcal{P}, \neg well) = \{well \leftarrow \bot\}$
- $\mathcal{M}_{wc\, rev(\mathcal{P}, \neg well)} = \langle \emptyset, \{well\} \rangle$
- Hence, the value of the conditional is unknown
- The conditional is not treated as an implication

The Moon is Not Made out of Cheese

- $\mathcal{IC} = \{\bot \leftarrow \textit{cheese}\}$
- $\mathcal{P} = \emptyset$
- $\mathcal{M}_{wc\mathcal{P}} = \langle\emptyset, \emptyset\rangle$
- $\mathcal{X} = \{\textit{Cheese} \leftarrow \bot\}$ **explains** $\mathcal{O} = \emptyset$
- $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} = \langle\emptyset, \{\textit{cheese}\}\rangle$
- **Evaluate** *if the moon is made out of cheese, then life exists on other planets*
- $\textit{rev}(\mathcal{P} \cup \mathcal{X}, \textit{cheese}) = \{\textit{cheese} \leftarrow \top\}$
- $\mathcal{A}_{\{\textit{cheese}\leftarrow\top\}} = \emptyset$
- *nothing follows*

The Suppression Task Revisited Again – Background Knowledge

- **In the remainder of this section** $\mathcal{E} = \mathcal{IC} = \emptyset$
- **Group 1**
  - ***If she has an essay to write then she will study late in the library***
- **Group 2**
  - ***If she has an essay to write then she will study late in the library***
  - ***If she has some textbooks to read then she will study late in the library***
- **Group 3**
  - ***If she has an essay to write then she will study late in the library***
  - ***If the library stays open then she will study late in the library***

The Suppression Task Revisited Again – Conditionals

- **The groups are asked to evaluate the following conditionals**
  - *If she has an essay to write then she will study late in the library*
    - $\mathcal{S} = \emptyset,\ \mathcal{X} = \{e \leftarrow \top\}$
  - *If she does not have an essay to write then she will not study late in the library*
    - $\mathcal{S} = \emptyset,\ \mathcal{X} = \{e \leftarrow \bot\}$
  - *If she will study late in the library then she has an essay to write*
    - **Exercise**
  - *If she will not study late in the library then she does not have an essay to write*
    - **Exercise**
- **Applying MRFA yields the same results as before**
  - **Skeptical reasoning is required**
  - **It should be experimentally verified**

The Shooting of Kennedy

- Adams: Subjunctive and indicative conditionals: 1970
- **Background knowledge**
  - *If Oswald shot then the president was killed*
  - *If somebody else shot then the president was killed*
  - *Oswald shot*
- **Reasoning towards a program $\mathcal{P}$**

$$\begin{array}{lllllllll} k & \leftarrow & os \wedge \neg ab_{os} & ab_{os} & \leftarrow & \bot & os & \leftarrow & \top \\ k & \leftarrow & ses \wedge \neg ab_{ses} & ab_{ses} & \leftarrow & \bot & & & \end{array}$$

- **Weakly completing $\mathcal{P}$ and computing $\mathcal{M}_{wc\mathcal{P}}$**

$$\langle \{os, k\}, \{ab_{os}, ab_{ses}\} \rangle$$

- **Evaluate**
  - *If Oswald did not shoot Kennedy in Dallas then no one else would have*
  - *If Kennedy was killed in Dallas and Oswald did not shoot then no one else would have*

The Shooting of Kennedy – The Set of Abducibles

- **Recall**

$$\begin{array}{lllllllll} k & \leftarrow & os \wedge \neg ab_{os} & \quad ab_{os} & \leftarrow & \bot & \quad os & \leftarrow & \top \\ k & \leftarrow & ses \wedge \neg ab_{ses} & \quad ab_{ses} & \leftarrow & \bot & & & \end{array}$$

- **How would you classify the two conditionals of the background knowledge?**
  - **Factual conditionals with non-necessary antecedent**
- **Now consider**

  if Oswald shot or somebody else shot, then the president was killed

  - **Factual (generalized) conditional with necessary antecedent**
- **The set of abducibles**

$$\{ses \leftarrow \top,\ ses \leftarrow \bot,\ ab_{os} \leftarrow \top,\ ab_{ses} \leftarrow \top\}$$

  - $k \leftarrow \top$ **is not added**

The Shooting of Kennedy – First Conditional

- *If Oswald did not shoot Kennedy in Dallas then no one else would have*

$$\textit{if } \neg os \textit{ then } \neg ses$$

- $rev(\mathcal{P}, \{\neg os\})$

$$\begin{array}{lllllllll} k & \leftarrow & os \wedge \neg ab_{os} & ab_{os} & \leftarrow & \bot & os & \leftarrow & \bot \\ k & \leftarrow & ses \wedge \neg ab_{ses} & ab_{ses} & \leftarrow & \bot & & & \end{array}$$

- $\mathcal{M}_{wc\ rev(\mathcal{P}, \{\neg os\})}$

$$\langle \emptyset, \{os, ab_{os}, ab_{ses}\} \rangle$$

- **The counterfactual is unknown**

The Shooting of Kennedy – Second Conditional

- *If Kennedy was killed and Oswald did not shoot then no one else did*

$$\textit{if } \{k, \neg os\} \textit{ then } \neg ses$$

- $rev(\mathcal{P}, \{\neg os\})$

$$\begin{array}{lllllllll} k & \leftarrow & os \wedge \neg ab_{os} & ab_{os} & \leftarrow & \bot & os & \leftarrow & \bot \\ k & \leftarrow & ses \wedge \neg ab_{ses} & ab_{ses} & \leftarrow & \bot & & & \end{array}$$

- $\mathcal{M}_{wc\ rev(\mathcal{P},\{\neg os\})}$

$$\langle \emptyset, \{os, ab_{os}, ab_{ses}\} \rangle$$

- $\mathcal{A}^{e}_{rev(\mathcal{P},\{\neg os\})}$

$$\{ses \leftarrow \top,\ ses \leftarrow \bot, ab_{os} \leftarrow \top,\ ab_{ses} \leftarrow \top\}$$

- $\mathcal{M}_{wc(rev(\mathcal{P},\{\neg os\}) \cup \{ses \leftarrow \top\})}$

$$\langle \{ses, k\}, \{os, ab_{os}, ab_{ses}\} \rangle$$

- **The counterfactual is false**

Modeling the Firing Squad

- **Reasoning towards a program $\mathcal{P}$**

$$\begin{array}{rcl@{\qquad}rcl}
signal & \leftarrow & execution \wedge \neg ab_1 & ab_1 & \leftarrow & \bot \\
rifleman_A & \leftarrow & signal \wedge \neg ab_2 & ab_2 & \leftarrow & \bot \\
rifleman_B & \leftarrow & signal \wedge \neg ab_3 & ab_3 & \leftarrow & \bot \\
dead & \leftarrow & rifleman_A \wedge \neg ab_4 & ab_4 & \leftarrow & \bot \\
dead & \leftarrow & rifleman_B \wedge \neg ab_5 & ab_5 & \leftarrow & \bot \\
alive & \leftarrow & \neg dead \wedge \neg ab_6 & ab_6 & \leftarrow & \bot
\end{array}$$

- **Weakly completing the program and computing $\mathcal{M}_{wc\mathcal{P}}$**

$$\langle \emptyset, \{ab_1, ab_2, ab_3, ab_4, ab_5, ab_6\}\rangle$$

- **The set of abducibles $\mathcal{A}_{\mathcal{P}}$**

$$\{execution \leftarrow \top,\ execution \leftarrow \bot\}$$

  - $\mathcal{X}_\top = \{execution \leftarrow \top\}$
    explains $\{signal, rifleman_A, rifleman_B, dead, \neg alive\}$
  - $\mathcal{X}_\bot = \{execution \leftarrow \bot\}$
    explains $\{\neg signal, \neg rifleman_A, \neg rifleman_B, \neg dead, alive\}$
  - $\{\neg signal, rifleman_A\}$ cannot be explained

The Firing Squad – Conditionals

- **Recall**
  - $\mathcal{X}_\top = \{execution \leftarrow \top\}$ explains $\{signal, rifleman_A, rifleman_B, dead, \neg alive\}$
  - $\mathcal{X}_\bot = \{execution \leftarrow \bot\}$ explains $\{\neg signal, \neg rifleman_A, \neg rifleman_B, \neg dead, alive\}$
  - $\{\neg signal, rifleman_A\}$ cannot be explained
- *If the prisoner is alive then the captain did not signal*

$$\textit{if alive then} \neg signal : \mathcal{P} \mapsto \mathcal{P} \cup \mathcal{X}_\bot \mapsto \top$$

- *If rifleman A shot then rifleman B shot as well*

$$\textit{if } rifleman_A \textit{ then } rifleman_B : \mathcal{P} \mapsto \mathcal{P} \cup \mathcal{X}_\top \mapsto \top$$

- *If the captain gave no signal and rifleman A decides to shoot then the court did not order an execution*

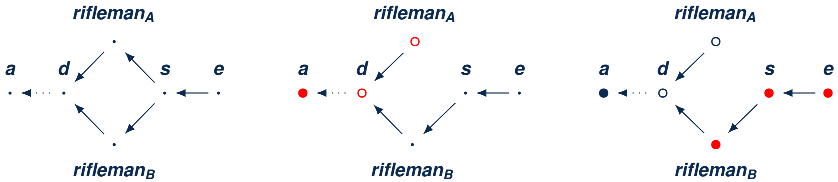
$$\textit{if } \{\neg signal, rifleman_A\} \textit{ then } \neg execution : \mathcal{P} \mapsto rev(\mathcal{P}, \{rifleman_A\}) \cup \mathcal{X}_\bot \mapsto \top$$

The Firing Squad – Last Conditional Revisited

- If the captain gave no signal and rifleman A decides to shoot then the court did not order an execution

$$\mathcal{P} \mapsto rev(\mathcal{P}, \{rifleman_A\}) \cup \mathcal{X}_\perp \mapsto \top$$

- Consider the dependency graphs (ignoring abnormalities)



· unknown ∘ true • false

The Forest Fire Example

- Byrne: The Rational Imagination: 2005
- **Suppose lightning hits a forest and a devastating forest fire breaks out**
  The forest was dry after a long hot summer and many acres were destroyed
- **Causal relationships** **lightning caused the forest fire**
- **Enabling relationships** **dry leaves made it possible for the fire to occur**
- **An enabler is usually not considered to be the cause for an event**
- **A missing enabler can prevent an event**

Encoding the Forest Fire Example

- ***Lightning may cause a forest fire   Lightning happened   Dry leaves are present***

$$\mathcal{P} = \{ \begin{array}{lll} ff & \leftarrow & lightning \wedge \neg ab_\ell, \\ ab_\ell & \leftarrow & \neg dryleaves, \end{array} \quad \begin{array}{lll} lightning & \leftarrow & \top, \\ dryleaves & \leftarrow & \top \} \end{array}$$

- ***If there had not been so many dry leaves on the forest floor then the forest fire would not have occurred***

	$\Phi_{\mathcal{P}}$	$\Phi_{rev(\mathcal{P},\{\neg dryleaves\})}$
$\uparrow 0$	$\langle\emptyset,\emptyset\rangle$	$\langle\emptyset,\emptyset\rangle$
$\uparrow 1$	$\langle\{dryleaves, lightning\},\emptyset\rangle$	$\langle\{lightning\},\{dryleaves\}\rangle$
$\uparrow 2$	$\langle\{dryleaves, lightning\},\{ab_\ell\}\rangle$	$\langle\{lightning, ab_\ell\},\{dryleaves\}\rangle$
$\uparrow 3$	$\langle\{dryleaves, lightning, ff\},\{ab_\ell\}\rangle$	$\langle\{lightning, ab_\ell\},\{dryleaves, ff\}\rangle$

$$rev(\mathcal{P},\{\neg dryleaves\}) = \{ \begin{array}{lll} ff & \leftarrow & lightning \wedge \neg ab_\ell, \\ ab_\ell & \leftarrow & \neg dryleaves, \end{array} \quad \begin{array}{lll} lightning & \leftarrow & \top, \\ dryleaves & \leftarrow & \bot \} \end{array}$$

- **The counterfactual is true**

The Extended Forest Fire Example 1

- Pereira, Dietz, H.: Contextual Abductive Reasoning with Side-Effects: 2014
- **Add to the previous example** ***Arson may cause a forest fire***
- ***If there had not been so many dry leaves on the forest floor then the forest fire would not have occurred***

$$
\begin{array}{rcl}
\mathcal{P} & = & \{ff \leftarrow lightning \wedge \neg ab_\ell,\; ff \leftarrow arson \wedge \neg ab_a, \\
& & lightning \leftarrow \top,\; ab_\ell \leftarrow \neg dryleaves, \\
& & dryleaves \leftarrow \top,\; ab_a \leftarrow \bot\} \\
\mathcal{M}_{wc\mathcal{P}} & = & \langle \{dryleaves, lightning, ff\}, \{ab_\ell, ab_a\} \rangle \\
rev(\mathcal{P}, \{\neg dryleaves\}) & = & \{ff \leftarrow lightning \wedge \neg ab_\ell,\; ff \leftarrow arson \wedge \neg ab_a, \\
& & lightning \leftarrow \top,\; ab_\ell \leftarrow \neg dryleaves, \\
& & dryleaves \leftarrow \bot,\; ab_a \leftarrow \bot\} \\
\mathcal{M}_{wc\; rev(\mathcal{P}, \{\neg dryleaves\})} & = & \langle \{lightning, ab_\ell\}, \{dryleaves, ab_a\} \rangle
\end{array}
$$

- **The counterfactual is unknown**

The Extended Forest Fire Example 2

- ***If there had not been so many dry leaves on the forest floor and there was no arson then the forest fire would not have occurred***
- **What will happen?**

The Selection Task – The Abstract Case

- Wason: Reasoning About a Rule: 1968
- ***If the letter d is on one side of a card then the number 3 is on the other side***

d	f	3	7

- **Which cards must be turned to show that the rule holds?**
- **Humans typically turn the cards showing *d* and *3***

An Analysis of the Abstract Case

- Stenning, van Lambalgen: Human Reasoning and Cognitive Science: 2008
  - **With respect to classical two-valued logic!**
- **Almost everyone (89%) correctly selects *d***
  - **Corresponds to modus ponens in classical logic**
- **Almost everyone (84%) correctly does not select *f***
  - ***Because the condition does not mention f***
- **Many (62%) incorrectly select 3**
  - ***If there is a 3 on one side then there is a d on the other side***
  - **Converse of the given conditional**
- **Only a small percentage of participants (25%) correctly selects 7**
  - ***If the number on one side is not 3 then the letter on the other side is not d***
  - **Contrapositive of the given conditional**

The Selection Task – The Social Case

- Griggs, Cox: The Elusive Thematic Materials Effect in the Wason Selection Task: 1982
- ***If a person is drinking beer then the person must be over 19 years of age***

beer	coke	22yrs	16yrs

- **Which cards must be turned to show that the rule holds?**
- **Humans typically turn the cards showing *beer* and *16yrs***

The Selection Task – Alternative Conditional 1

- ***If Nancy rides her motorbike she goes to the mountains***

rides	no ride	mountain	no mountain

- **Which cards must be turned to show that the rule holds?**

The Selection Task – Alternative Conditional 2

- *If it rains then the roofs are wet*

rain	no rain	wet roofs	dry roofs

- Which cards must be turned to show that the rule holds?

The Selection Task

- **The Abstract Case**
  - ***If there is the letter d on one side of the card then the number 3 is on the other side***
    - **Factual conditional with necessary antecedent**
- **The Social Case**
  - ***If a person is drinking beer then the person must be over 19 years of age***
    - **Obligational conditional with non-necessary antecedent**

$C \leftarrow A \wedge \neg ab$	non-necessary	necessary
factual	$ab \leftarrow \top,\ C \leftarrow \top$	$ab \leftarrow \top$
obligational	$C \leftarrow \top$	

The Abstract Case: Factual Conditional with Necessary Antecedent

- *If the letter d is on one side of a card then there is the letter 3 on the other side*
- **Reasoning towards a program yields** $\mathcal{P} = \{3 \leftarrow d \wedge \neg ab_a,\ ab_a \leftarrow \bot\}$
- **Its set of abducibles is** $\mathcal{A}^e_{\mathcal{P}} = \{d \leftarrow \top,\ d \leftarrow \bot,\ ab_a \leftarrow \top\}$
- **Observations, least models, and decisions**

$\boxed{d}$

true	false
d	ab_a
3	

turn
0.89

$\boxed{\neg d}$

true	false
	d
	ab_a
	3

no turn
0.16

$\boxed{3}$

true	false
d	ab_a
3	

turn
0.62

$\boxed{\neg 3}$

true	false
	d
	ab_a
	3

true	false
ab_a	
	3

no turn
0.25

The Social Case: Obligation with Non-Necessary Antecedent

- *If a person is drinking beer then the person must be over 19 years of age*
- **Reasoning towards a program yields** $\mathcal{P} = \{o \leftarrow b \wedge \neg ab_s,\ ab_s \leftarrow \bot\}$
- **Its set of abducibles is** $\mathcal{A}^e_{\mathcal{P}} = \{b \leftarrow \top,\ b \leftarrow \bot,\ o \leftarrow \top\}$
- **Observations, least models, and decisions**

Observation $\boxed{b}$:

true	false
b	ab_s
o	

turn
0.95

Observation $\boxed{\neg b}$:

true	false
	b
	ab_s
	o

no turn
0.025

Observation $\boxed{o}$:

true	false
o	ab_s

true	false
b	ab_s
o	

no turn
0.025

Observation $\boxed{\neg o}$:

true	false
	b
	ab_s
	o

turn
0.80

The Selection Task – Summary

- We obtain adequate answers if
  - the abstract case is interpreted as
    a factual conditional with necessary antecedent
  - the social case is interpreted as
    an obligational conditional with non-necessary antecedent
  - reasoning skeptically

Syllogisms

Introduction

- Consider the following inference

  In some cases when I go out, I am not in company
  Every time I am very happy I am in company
  Therefore, in some cases when I go out, I am not very happy

- It is valid
  - the conclusion is true in every case in which both premises are true
- Aristotle was the first to analyze syllogisms
- Syllogisms were central to logic until the second half of the 19th century
- Psychological studies of reasoning with determiners, such as *some* and *all*, have almost all concerned syllogistic reasoning

Reasoning

- **The ability to reason is at the core of human mentality**
- **Many contexts in daily life call for inferences**
  - **decisions about goals and actions**
  - **evaluation of conjectures and hypothesis**
  - **the pursuit of arguments and negotiations**
  - **the assessment of evidence and data**
  - **science, technology, and culture**
- **Examples**
  - ***Any experiment containing a confound is open to misinterpretation***
  - ***No current word processor spontaneously corrects a user's grammar***
  - ***Every chord containing three adjacent semitones is highly dissonant***

Common Sense Reasoning

- **In daily life, individuals reason in a variety of contexts, and often so rapidly that they are unaware of having made an inference**
- **Example**
  - ***Belinda: If you drop this cup it'll break***
  - ***Jeffrey: It looks pretty solid to me***
  - ***Belinda: Yes, but it's made from porcelain***

An Example

- **Try to determine, as quickly as you can, whether the following syllogism is valid**

  All roses are flowers
  Some flowers fade quickly
  Therefore, some roses fade quickly

- **Now, take your time and think about it again**

Another Example

- **What follows necessarily from the following premises?**

some a are b
no b are c

Aac	*all a are c*	
Iac	*some a are c*	
Eac	*no a are c*	**answer by humans**
Oac	*some a are not c*	**answer by humans the only correct answer wrt FOL**
Aca	*all c are a*	
Ica	*some c are a*	
Eca	*no c are a*	
Oca	*some c are not a*	
NVC	*no valid conclusion*	

Syllogisms

- **4 moods**

mood (*AFFIRMO NEGO*)	natural language	FOL	short
affirmative universal (A)	***all a are b***	$(\forall X)(a\,X \rightarrow b\,X)$	**Aab**
affirmative existential (I)	***some a are b***	$(\exists X)(a\,X \wedge b\,X)$	**Iab**
negative universal (E)	***no a are b***	$(\forall X)(a\,X \rightarrow \neg\, b\,X)$	**Eab**
negative existential (O)	***some a are not b***	$(\exists X)(a\,X \wedge \neg\, b\,X)$	**Oab**

- **4 figures**

	figure 1	figure 2	figure 3	figure 4
premise 1	**a-b**	**b-a**	**a-b**	**b-a**
premise 2	**b-c**	**c-b**	**c-b**	**b-c**

- **64 pairs of premises**
  - **abbreviated by the first and the second mood and the figure (e.g., IE1)**
- **512 syllogisms**
  - **possible conclusions are the 4 moods instantiated by a-c and c-a**

A Meta-Study

- Khemlani, Johnson-Laird 2012
- **Data from 6 studies**
  - **Humans deviate from FOL reasoning**
- **12 cognitive theories**
  - **None of the 12 theories models human reasoning adequately**
- ***The existence of 12 theories of any scientific domain is a small desaster***
- ***If psychologists could agree on an adequate theory of syllogistic reasoning, then progress towards a more general theory of reasoning would seem to be feasible***
- ***If researchers were unable to account for syllogistic reasoning, then they would have little hope of making sense of reasoning in general***

Three Examples

- OA4: *some b are not a   all b are c*
- IE4: *some b are a   no b are c*
- IA2: *some b are a   all c are b*

	participants	FOL	PSYCOP	mental models	verbal models
OA4	Oca	Oca	Oca	Oca	Oca
			Ica Iac	Oac NVC	NVC
matching percentage		1.0	0.78	0.78	0.89
IE4	Oac NVC	Oac	Oac	Oac NVC	Oac NVC
			Iac Ica	Eac Eca Oca	
matching percentage		0.89	0.67	0.67	1.00
IA2	Ica Iac			Ica Ica	Ica
		NVC	NVC	NVC	NVC
matching percentage		0.67	0.67	0.89	0.78
accuracy			0.77	0.83	0.84

Significance and Accuracy

- **Significance of an Answer**
  - Given 9 possible answers,
    the chance that a conclusion has been chosen randomly is 1/9 = 0.11
  - A binomial test shows that if a conclusion is drawn more than 0.16
    it is unlikely to be a random guess
- **Accuracy of the Predication**
  - For each syllogism
    - Order the nine possible conclusions (Aac, Eac, . . . , Oca, NVC)
    - Consider the list of the participant's conclusions (0, 1, . . . , 1, 0)
    - Compute the list of conclusions predicted by a theory (1, 0, . . . , 1, 1)
    - Compute
      $$comp\ i = \begin{cases} 1 & \text{if both lists have the same value for the } i\text{th element} \\ 0 & \text{otherwise} \end{cases}$$
    - The matching percentage of the syllogism is $\sum_{i=1}^{9} comp\ i/9$
  - The accuracy is the average of the matching percentage of all syllogisms

First Principle: Licenses for Inferences (licences)

- Stenning, van Lambalgen 2008
- **Formalize conditionals by licences for inferences**

$$\textbf{for all } X, \ \textbf{if } q\,X \textbf{ then } p\,X$$

$$\Downarrow$$

$$p\,X \leftarrow q\,X \wedge \neg\, ab\,X$$

$$ab\,X \leftarrow \bot$$

Second Principle: Existential Import or Gricean Implicature (import)

- **Humans normally do not quantify over things that do not exist**
  - **Gricean implicature** Grice 1975
  - **Consequently, *for all* implies *there exists***
- **Likewise, humans seem to require existential import for a conditional to be true**
- **Furthermore, *some a are b* often implies *some a are not b***

Third Principle: Unknown Generalization (unknownGen)

- **Humans seem to distinquish between *some a are b* and *all a are b***
- **If we learn that *some a are b* then**
  - **there must be an object o_1 belonging to *a* and *b* (existential import)**
  - **there must be another object o_2 belonging to *a* and for which it is unknown whether it belongs to *b***
- **This is a new principle!**

Fourth Principle: Converse Interpretation (converse)

- Some humans seem to distinguish between *some a are b* and *some b are a*
- But in FOL $\exists X(a\,X \wedge b\,X) \equiv \exists X(b\,X \wedge a\,X)$
- Nevertheless, we propose that lab implies lba and vice versa

Fifth Principle: Search Alternative Conclusions to NVC (abduction)

- Suppose, NVC is derived
  - Humans may not want to accept this conclusion
  - They proceed to check whether there exists unknown relevant information
  - This information may be explanations for facts
  - The facts will come from existential import
- Skeptical abduction

Sixth Principle: Negation by Transformation (transformation)

- **Logic programs do not allow negative literals as heads of clauses**
- **Replace a negative conclusion $\neg\, p\, X$ by $p'\, X$ and add the clause**

$$p\, X \leftarrow \neg\, p'\, X$$

as well as the weak integrity constraint

$$\mathsf{U} \leftarrow p\, X \wedge p'\, X$$

- **Combined with the principle of licences for inferences we obtain**

$$\begin{array}{rcl} p\, X & \leftarrow & \neg\, p'\, X \wedge \neg\, ab\, X \\ ab\, X & \leftarrow & \bot \\ \mathsf{U} & \leftarrow & p\, X \wedge p'\, X \end{array}$$

Seventh Principle: Blocking by Double Negatives (blocking)

- What conclusions can be drawn from double negatives?
- This appears to be a quite difficult reasoning task for humans
- They seem to avoid drawing conlusions through double negatives
- Example
  - *If not a then b*   *If not b then c*   *a is true*
  - We obtain

$$\begin{array}{rcl} b & \leftarrow & \neg a \wedge \neg ab_{nab} \\ ab_{nab} & \leftarrow & \bot \\ c & \leftarrow & \neg b \wedge \neg ab_{nbc} \\ ab_{nbc} & \leftarrow & \bot \\ a & \leftarrow & \top \end{array}$$

  - The least model of its weak completion is

$$\langle \{a, c\}, \{b, ab_{nab}, ab_{nbc}\} \rangle$$

  - c can be blocked by removing $ab_{nbc} \leftarrow \bot$

Ayz: All y are z

- $\mathcal{P}_{Ayz}$

$$
\begin{array}{rcll}
z\,X & \leftarrow & y\,X \wedge \neg\, ab_{yz}\,X & \text{licenses} \\
ab_{yz}\,X & \leftarrow & \bot & \text{licenses} \\
y\,o & \leftarrow & \top & \text{import}
\end{array}
$$

- **Computing the least model of its weak completion**

$$
\begin{array}{rcl}
\Phi_{\mathcal{P}_{Ayz}} \uparrow 0 & = & \langle \emptyset, \emptyset \rangle \\
\Phi_{\mathcal{P}_{Ayz}} \uparrow 1 & = & \langle \{y\,o\}, \{ab_{yz}\,o\} \rangle \\
\Phi_{\mathcal{P}_{Ayz}} \uparrow 2 & = & \langle \{y\,o, z\,o\}, \{ab_{yz}\,o\} \rangle \quad = \quad \mathcal{M}_{wc\mathcal{P}_{Ayz}}
\end{array}
$$

Eyz: No y are z

- $\mathcal{P}_{Eyz}$

$$\begin{array}{rcll}
z'\,X & \leftarrow & y\,X \wedge \neg\, ab_{ynz}\,X & \text{transformation\&licenses} \\
ab_{ynz}\,X & \leftarrow & \bot & \text{licenses} \\
y\,o & \leftarrow & \top & \text{import} \\
z\,X & \leftarrow & \neg\, z'\,X \wedge \neg\, ab_{nzz}\,X & \text{transformation\&licenses} \\
ab_{nzz}\,o & \leftarrow & \bot & \text{licenses\&blocking} \\
 & & & \\
\mathsf{U} & \leftarrow & z\,X \wedge z'\,X & \text{transformation}
\end{array}$$

- **Computing the least model of its weak completion**

$$\begin{array}{rcl}
\Phi_{\mathcal{P}_{Eyz}} \uparrow 0 & = & \langle \emptyset, \emptyset \rangle \\
\Phi_{\mathcal{P}_{Eyz}} \uparrow 1 & = & \langle \{y\,o\}, \{ab_{ynz}\,o, ab_{nzz}\,o\} \rangle \\
\Phi_{\mathcal{P}_{Eyz}} \uparrow 2 & = & \langle \{y\,o, z'\,o\}, \{ab_{ynz}\,o, ab_{nzz}\,o\} \rangle \\
\Phi_{\mathcal{P}_{Eyz}} \uparrow 3 & = & \langle \{y\,o, z'\,o\}, \{ab_{ynz}\,o, ab_{nzz}\,o, z\,o\} \rangle \quad = \quad \mathcal{M}_{wc\mathcal{P}_{Eyz}}
\end{array}$$

Iyz: Some y are z

- $\mathcal{P}_{Iyz}$

$$\begin{array}{rcll}
z\,X & \leftarrow & y\,X \wedge \neg\, ab_{yz}\,X & \text{licenses} \\
ab_{yz}\,o_1 & \leftarrow & \bot & \text{licenses\&unknownGen} \\
y\,o_1 & \leftarrow & \top & \text{import} \\
y\,o_2 & \leftarrow & \top & \text{unknownGen} \\
y\,X & \leftarrow & z\,X \wedge \neg\, ab_{zy}\,X & \text{converse\&licenses} \\
ab_{zy}\,o_3 & \leftarrow & \bot & \text{converse\&licenses\&unknownGen} \\
z\,o_3 & \leftarrow & \top & \text{converse\&import} \\
z\,o_4 & \leftarrow & \top & \text{converse\&unknownGen}
\end{array}$$

- **Computing the least model of its weak completion**

$$\begin{array}{rcl}
\Phi_{\mathcal{P}_{Iyz}} \uparrow 0 & = & \langle \emptyset, \emptyset \rangle \\
\Phi_{\mathcal{P}_{Iyz}} \uparrow 1 & = & \langle \{y\,o_1, y\,o_2, z\,o_3, z\,o_4\}, \{ab_{yz}\,o_1, ab_{zy}\,o_3\} \rangle \\
\Phi_{\mathcal{P}_{Iyz}} \uparrow 2 & = & \langle \{y\,o_1, y\,o_2, z\,o_3, z\,o_4, z\,o_1, y\,o_3\}, \{ab_{yz}\,o_1, ab_{zy}\,o_3\} \rangle \\
 & = & \mathcal{M}_{wc\mathcal{P}_{Iyz}}
\end{array}$$

Oyz: Some y are not z

- $\mathcal{P}_{Oyz}$

$$
\begin{array}{rcll}
z'\,X & \leftarrow & y\,X \wedge \neg\, ab_{ynz}\,X & \text{transformation\&licenses} \\
ab_{ynz}\,o_1 & \leftarrow & \bot & \text{licenses\&unknownGen} \\
y\,o_1 & \leftarrow & \top & \text{import} \\
y\,o_2 & \leftarrow & \top & \text{unknownGen} \\
z\,X & \leftarrow & \neg\, z'\,X \wedge \neg\, ab_{nzz}\,X & \text{transformation\&licenses} \\
ab_{nzz}\,o_1 & \leftarrow & \bot & \text{licenses\&blocking} \\
ab_{nzz}\,o_2 & \leftarrow & \bot & \text{licenses\&blocking} \\
 & & & \\
\mathrm{U} & \leftarrow & z\,X \wedge z'\,X & \text{transformation}
\end{array}
$$

- **Computing the least model of its weak completion**

$$
\begin{array}{rcl}
\Phi_{\mathcal{P}_{Oyz}} \uparrow 0 & = & \langle \emptyset, \emptyset \rangle \\
\Phi_{\mathcal{P}_{Oyz}} \uparrow 1 & = & \langle \{y\,o_1, y\,o_2\}, \{ab_{ynz}\,o_1, ab_{nzz}\,o_1, ab_{nzz}\,o_2\} \rangle \\
\Phi_{\mathcal{P}_{Oyz}} \uparrow 2 & = & \langle \{y\,o_1, y\,o_2, z'\,o_1\}, \{ab_{ynz}\,o_1, ab_{nzz}\,o_1, ab_{nzz}\,o_2\} \rangle \\
\Phi_{\mathcal{P}_{Oyz}} \uparrow 3 & = & \langle \{y\,o_1, y\,o_2, z'\,o_1\}, \{ab_{ynz}\,o_1, ab_{nzz}\,o_1, ab_{nzz}\,o_2, z\,o_1\} \rangle \\
 & = & \mathcal{M}_{wc\mathcal{P}_{Oyz}}
\end{array}
$$

Entailment of Syllogisms

- Khemlani, Johnson-Laird 2012 **appear to use entailment as defined in FOL**
- **$\mathcal{P}$ entails Ayz (all y are z)**

  iff $\exists X(\mathcal{P} \models_{wcs} y\,X) \wedge \forall X(\mathcal{P} \models_{wcs} y\,X \rightarrow \mathcal{P} \models_{wcs} z\,X)$
- **$\mathcal{P}$ entails Eyz (no y are z)**

  iff $\exists X(\mathcal{P} \models_{wcs} y\,X) \wedge \forall X(\mathcal{P} \models_{wcs} y\,X \rightarrow \mathcal{P} \models_{wcs} \neg\, z\,X)$
- **$\mathcal{P}$ entails Iyz (some y are z)**

  iff $\exists X_1(\mathcal{P} \models_{wcs} y\,X_1 \wedge z\,X_1) \wedge \exists X_2(\mathcal{P} \models_{wcs} y\,X_2 \wedge \mathcal{P} \not\models_{wcs} z\,X_2)$
  $\wedge\, \exists X_3(\mathcal{P} \models_{wcs} z\,X_3 \wedge \mathcal{P} \not\models_{wcs} y\,X_3)$
- **$\mathcal{P}$ entails Oyz (some y are not z)**

  iff $\exists X_1(\mathcal{P} \models_{wcs} y\,X_1 \wedge \neg z\,X_1) \wedge \exists X_2(\mathcal{P} \models_{wcs} y\,X_2 \wedge \mathcal{P} \not\models_{wcs} \neg z\,X_2)$
- **$\mathcal{P}$ entails NVC**

  iff **none of the above is entailed where either $yz = ac$ or $yz = ca$**

Syllogism OA4

- **The premises are Oba (*some b are not a*) and Abc (*all b are c*)**
- **The participants concluded Oca (*some c are not a*)**
- $\mathcal{P}_{OA4}$:

$$
\begin{array}{rcll}
b\ o_1 & \leftarrow & \top & \text{import} \\
b\ o_2 & \leftarrow & \top & \text{unknownGen} \\
a'\ X & \leftarrow & b\ X \wedge \neg\ ab_{bna}\ X & \text{transformation\&licenses} \\
ab_{bna}\ o_1 & \leftarrow & \bot & \text{unknownGen\&licenses} \\
a\ X & \leftarrow & \neg a'\ X \wedge \neg\ ab_{naa}\ X & \text{transformation\&licenses} \\
ab_{naa}\ o_1 & \leftarrow & \bot & \text{blocking\&licenses} \\
ab_{naa}\ o_2 & \leftarrow & \bot & \text{blocking\&licenses} \\
c\ X & \leftarrow & b\ X \wedge \neg\ ab_{bc}\ X & \text{licenses} \\
ab_{bc}\ X & \leftarrow & \bot & \text{licenses} \\
b\ o_3 & \leftarrow & \top & \text{import} \\
\\
\mathrm{U} & \leftarrow & a\ X \wedge a'\ X & \text{transformation}
\end{array}
$$

- $\mathcal{M}_{wc\mathcal{P}_{OA4}} = \langle\{b\ o_1, b\ o_2, b\ o_3, a'\ o_1, c\ o_1, c\ o_2, c\ o_3\}, \{ab_{bna}\ o_1, ab_{naa}\ o_1, ab_{naa}\ o_2, ab_{bc}\ o_1, ab_{bc}\ o_2, ab_{bc}\ o_3, a\ o_1\}\rangle$
- $\mathcal{P}_{OA4}$ **entails** Oca **and nothing else** $\leadsto$ **perfect match 1.0**

Syllogism IE4

- **The premises are Iba (*some b are a*) and Ebc (*no b are c*)**
- **The participants concluded Oac (*some a are not c*) and NVC**
- $\mathcal{P}_{IE4}$:

$b\ o_1$	$\leftarrow$	$\top$	import
$b\ o_2$	$\leftarrow$	$\top$	unknownGen
$a\ X$	$\leftarrow$	$b\ X \wedge \neg\ ab_{ba}\ X$	licenses
$ab_{ba}\ o_1$	$\leftarrow$	$\bot$	licenses&unknownGen
$b\ X$	$\leftarrow$	$a\ X \wedge \neg\ ab_{ab}\ X$	converse&licenses
$ab_{ab}\ o_3$	$\leftarrow$	$\bot$	converse&licenses&unknownGen
$a\ o_3$	$\leftarrow$	$\top$	converse&import
$a\ o_4$	$\leftarrow$	$\top$	converse&unknownGen
$c'\ X$	$\leftarrow$	$b\ X \wedge \neg\ ab_{bnc}\ X$	transformation&licenses
$ab_{bnc}\ X$	$\leftarrow$	$\bot$	licenses
$c\ X$	$\leftarrow$	$\neg\ c'\ X \wedge \neg\ ab_{ncc}\ X$	transformation&licenses
$b\ o_5$	$\leftarrow$	$\top$	import
$ab_{ncc}\ X$	$\leftarrow$	$\bot$	licenses
U	$\leftarrow$	$c\ X \wedge c'\ X$	transformation

- $\mathcal{P}_{IE4}$ **entails Oac and nothing else** $\leadsto$ **partial match 0.89**

Syllogism IA2

- The premises are Iba (*some b are a*) and Acb (*all c are b*)
- The participants concluded Iac and Ica
- $\mathcal{P}_{IA2}$:

$$
\begin{array}{rcll}
a\,X & \leftarrow & b\,X \wedge \neg\, ab_{ba}\,X & \text{licenses} \\
ab_{ba}\,o_1 & \leftarrow & \bot & \text{licenses\&unknownGen} \\
b\,o_1 & \leftarrow & \top & \text{import} \\
b\,o_2 & \leftarrow & \top & \text{unknownGen} \\
b\,X & \leftarrow & a\,X \wedge \neg\, ab_{ab}\,X & \text{converse\&licenses} \\
ab_{ab}\,o_3 & \leftarrow & \bot & \text{converse\&licenses\&unknownGen} \\
a\,o_3 & \leftarrow & \top & \text{converse\&import} \\
a\,o_4 & \leftarrow & \top & \text{converse\&unknownGen} \\
b\,X & \leftarrow & c\,X \wedge \neg\, ab_{cb}\,X & \text{licenses} \\
ab_{cb}\,X & \leftarrow & \bot & \text{licenses} \\
c\,o_5 & \leftarrow & \top & \text{import}
\end{array}
$$

- $\mathcal{M}_{wc\mathcal{P}_{IA2}} = \langle\{a\,o_1, a\,o_3, a\,o_4, b\,o_1, b\,o_2, b\,o_3, b\,o_5, c\,o_5\}$
  $\{ab_{ba}\,o_1, ab_{ab}\,o_3, ab_{cb}\,o_1, ab_{cb}\,o_2, ab_{cb}\,o_3, ab_{cb}\,o_4, ab_{cb}\,o_5\}\rangle$
- $\mathcal{P}_{IA2}$ entails NVC
- Search for alternatives   skeptical abduction

Syllogism IA2 Continued

- **Idea** the heads of existential imports are considered as observation

$$\mathcal{O} = \{b\,o_1,\ a\,o_3,\ c\,o_5\}$$

- The corresponding facts are removed

$$\mathcal{P}^-_{IA2} = \mathcal{P}_{IA2} \setminus \{b\,o_1 \leftarrow \top,\ a\,o_3 \leftarrow \top,\ c\,o_5 \leftarrow \top\}$$

- The minimal and skeptical explanation for $\mathcal{O}$ is

$$\mathcal{X} = \{c\,o_5 \leftarrow \top, c\,o_1 \leftarrow \top, c\,o_3 \leftarrow \top, ab_{ba}\,o_3 \leftarrow \bot\}$$

- Let $\mathcal{P}'_{IA2} = \mathcal{P}^-_{IA2} \cup \mathcal{X}$ and we obtain $\mathcal{M}_{wc\mathcal{P}'_{IA2}} =$

$$\langle\{a\,o_1, a\,o_3, a\,o_4, b\,o_1, b\,o_2, b\,o_3, b\,o_5, c\,o_1, c\,o_3, c\,o_5\}$$
$$\{ab_{ba}\,o_1, ab_{ba}\,o_3, ab_{ab}\,o_3, ab_{cb}\,o_1, ab_{cb}\,o_2, ab_{cb}\,o_3, ab_{cb}\,o_4, ab_{cb}\,o_5\}\rangle$$

- $\mathcal{P}'_{IA2}$ entails Iac and Ica and nothing else $\rightsquigarrow$ perfect match 1.0

The Examples Revisited

- **OA4:** ***some b are not a all b are c***
- **IE3:** ***some b are not a no b are c***
- **IA2:** ***some b are a all c are b***

	participants	FOL	PSYCOP	mental models	verbal models	WCS
OA4	Oca	Oca	Oca	Oca	Oca	Oca
			Ica Iac	Oac NVC	NVC	
		1.0	0.78	0.78	0.89	1.00
IE4	Oac NVC	Oac	Oac	Oac NVC	Oac NVC	Oac
			Iac Ica	Eac Eca Oca		
		0.89	0.67	0.67	1.00	0.89
IA2	Ica Iac			Ica Iac	Ica	Ica Iac
		NVC	NVC	NVC	NVC	
		0.67	0.67	0.89	0.78	1.00
accuracy			0.77	0.83	0.84	0.89

Discussion

- The best possible value achievable by WCS is .925
  - because NVC is entailed only if nothing else is entailed
- WCS is better than any other cognitive theory that I am aware of!
- Open Questions
  - How can we model clusters of reasoners?
  - How shall we define entailment?
  - What exactly is the role of the abnormalities?
  - How important is the sequence in which the premises are presented?
  - Is there a difference between abstract and social syllogisms?

Contextual Reasoning

- The Context Operator
- Contextual Programs
- Properties
- Examples

The Context Operator

- A new truth-functional operator

L	$ctxt\ L$
$\top$	$\top$
$\bot$	$\bot$
U	$\bot$

- Captures locally negation by failure

$$\begin{array}{lll} p & \leftarrow & q \\ p & \leftarrow & \bot \end{array} \qquad\qquad \begin{array}{lll} p & \leftarrow & ctxt\ q \\ p & \leftarrow & \bot \end{array}$$

- Their weak completions have the following minimal models

$$\langle \emptyset, \emptyset \rangle \qquad\qquad \langle \emptyset, \{p\} \rangle$$

Another Example

- Let $\mathcal{P}_1 = \{p\,a \leftarrow \top,\ q\,b \leftarrow r\,b\}$ with $\mathcal{M}_{wc\mathcal{P}_1} = \langle\{p\,a\}, \emptyset\rangle$
  - How is $c\,\mathcal{P}_1$ defined?

$$c\,\mathcal{P}_1 = \{p\,a \leftrightarrow \top,\ p\,b \leftrightarrow \bot,\ q\,a \leftrightarrow \bot,\ q\,b \leftrightarrow r\,b,\ r\,a \leftrightarrow \bot,\ r\,b \leftrightarrow \bot\}$$

- Now consider $\mathcal{P}_2$

$$\begin{array}{rcl} p\,X & \leftarrow & X \approx a \\ q\,X & \leftarrow & X \approx b \wedge r\,b \\ X \approx X & \leftarrow & \top \end{array}$$

  - What is the least model of $wc\,\mathcal{P}_2$?

$$\mathcal{M}_{wc\mathcal{P}_2} = \langle\{a \approx a,\ b \approx b,\ p\,a\}, \emptyset\rangle$$

  - What happens if $\mathcal{P}_3 = \mathcal{P}_2 \cup \{a \approx b \leftarrow \bot,\ b \approx a \leftarrow \bot\}$?

$$\mathcal{M}_{wc\mathcal{P}_3} = \langle\{a \approx a,\ b \approx b,\ p\,a\}, \{a \approx b,\ b \approx a,\ p\,b,\ q\,a\}\rangle$$

  - Is there a problem with $\mathcal{P}_3$?

Another Example – Continued

- Let $\mathcal{P}_4$

$$\begin{array}{rcl} p\,X & \leftarrow & ctxt\,X \approx a \\ q\,X & \leftarrow & ctxt\,X \approx b \wedge r\,b \\ X \approx X & \leftarrow & \top \end{array}$$

  - Can you specify a model of $wc\,\mathcal{P}_4$?

$$\langle \{a \approx a,\ b \approx b,\ p\,a\}, \{p\,b,\ q\,a\} \rangle$$

  - Compare

$$\mathcal{M}_{wc\mathcal{P}_3} = \langle \{a \approx a,\ b \approx b,\ p\,a\}, \{a \approx b,\ b \approx a,\ p\,b,\ q\,a\} \rangle$$

  - This is a local version of negation by failure!

Contextual Programs

- Literals are atoms or negated atoms
- Let L be a literal
- A contextual literal is of the form *ctxt* L or $\neg$ *ctxt* L
- A contextual rule is of the form $A \leftarrow Body$, where A is an atom and $Body$ is a finite conjunction of literals and contextual literals containing at least one contextual literal
- A contextual program is a set of rules, contextual rules, facts, and assumptions containing at least one contextual rule
- Note a program is not a contextual program

Contextual Programs and Models

- $\mathcal{P}$

$$\begin{array}{lcl} p & \leftarrow & \mathit{ctxt}\ q \\ p & \leftarrow & \bot \end{array}$$

- $wc\mathcal{P}$

$$p \leftrightarrow \mathit{ctxt}\ q \vee \bot$$

- How many minimal models has $wc\mathcal{P}$?
- What is

$$\begin{array}{rcl} \langle \emptyset, \emptyset \rangle (wc\mathcal{P}) & = & ? \\ \langle \emptyset, \{p\} \rangle (wc\mathcal{P}) & = & ? \\ \langle \{p, q\}, \emptyset \rangle (wc\mathcal{P}) & = & ? \\ \langle \{p\}, \emptyset \rangle (wc\mathcal{P}) & = & ? \\ \langle \{q\}, \emptyset \rangle (wc\mathcal{P}) & = & ? \end{array}$$

- Does there exist a least model?

Contextual Programs and Supported Models

- Let $\mathcal{P}$ consist of

$$\begin{array}{lcl} p & \leftarrow & ctxt\ q \\ p & \leftarrow & \bot \end{array}$$

- $wc\mathcal{P}$ has two minimal models $\langle \emptyset, \{p\} \rangle$ and $\langle \{p, q\}, \emptyset \rangle$
- Let's apply the semantic operator

$\Phi_{\mathcal{P}}$	$I^{\top}$	$I^{\bot}$	$I^{\top}$	$I^{\bot}$
$\uparrow 0$			p q	
$\uparrow 1$		p	p	
$\uparrow 2$		p		p

- Only $\langle \emptyset, \{p\} \rangle$ is a fixed point
  - It will turn out that it is the only fixed point
  - It will be called supported model

Contextual Programs and Monotonicity

- Let $\mathcal{P}$ consist of

$$p \leftarrow ctxt\,\neg\, p$$

- We find

$\Phi_{\mathcal{P}}$	$I^{\top}$	$I^{\perp}$
$\uparrow 0$		
$\uparrow 1$		p
$\uparrow 2$	p	
$\uparrow 3$		p
$\vdots$	$\vdots$	$\vdots$

- The semantic operator is no longer monotonic
- $wc\mathcal{P} = \{p \leftrightarrow ctxt\,\neg p\}$ is unsatisfiable

Acyclic Contextual Programs

- Let L be a literal
$$lvl\ ctxt\ L = lvl \neg\ ctxt\ L = lvl\ L$$
- A contextual program $\mathcal{P}$ is acyclic with respect to the level mapping lvl if and only if for each rule $A \leftarrow Body$ occurring in $\mathcal{P}$ and each (normal or contextual) literal L occurring in $Body$ we find $lvl\ A > lvl\ L$
- A contextual program $\mathcal{P}$ is acyclic if and only if it is acyclic with respect to some level mapping
- Recall
$$d_{lvl}(I, J) = \begin{cases} \frac{1}{2^n} & I \neq J \text{ and} \\ & I\,A = J\,A \neq \mathsf{U} \text{ for all } A \text{ with } lvl\ A < n \text{ and} \\ & I\,A \neq J\,A \text{ or } I\,A = J\,A = \mathsf{U} \text{ for some } A \text{ with } lvl\ A = n \\ 0 & \text{otherwise} \end{cases}$$
- Proposition 25 still applies: d_{lvl} is a metric
- Proposition 26 still applies: $(\mathcal{I}, d_{lvl})$ is a complete metric space

Contextual Programs and Fixed Points 1

- In the sequel, let $\mathcal{P}$ be a contextual program, $\mathcal{E}$ and equational theory lvl a level mapping for $\mathcal{P}$ and $\mathcal{I}$ the set of interpretations for $\mathcal{P}$
- **Theorem 32** If $\mathcal{P}$ is acyclic with respect to lvl then $\Phi_{\mathcal{P}}$ is a contraction on the metric space $(\mathcal{I}, d_{lvl})$
- **Proof** Let I and J be interpretations, $\Phi = \Phi_{\mathcal{P}}$, and $d = d_{lvl}$
  - We will show $d(\Phi\, I, \Phi\, J) \leq \frac{1}{2} d(I, J)$
  - If $I = J$ then $\Phi\, I = \Phi\, J$ and $d(\Phi\, I, \Phi\, J) = d(I, J) = 0$
  - If $I \neq J$ then we find $n \in \mathbb{N}$ such that $d(I, J) \leq \frac{1}{2^n}$
    - We will show $d(\Phi\, I, \Phi\, J) \leq \frac{1}{2^{n+1}}$
    - i.e. for all ground atoms A with $lvl\, A < n + 1$ we find $\Phi(I)(A) = \Phi(J)(A) \neq$ U
    - Let's take some A with $lvl\, A < n + 1$
    - Because $\mathcal{P}$ is acyclic, for any $A \leftarrow L_1 \wedge \ldots \wedge L_m \in g\,\mathcal{P}$ we find $lvl\, L_i < lvl\, A < n + 1$ for all $1 \leq i \leq m$
    - Because $d(I, J) \leq \frac{1}{2^n}$ we find $I\, L_i = J\, L_i \neq$ U for all $1 \leq i \leq m$
    - Hence, $\Phi(I)(A) = \Phi(J)(A) \neq$ U $\square$

Contextual Programs and Fixed Points 2

- **Proof of Theorem 27** If program $\mathcal{P}$ is acyclic with respect to *lvl* then $\Phi_{\mathcal{P}}$ is a contraction on the metric space $(\mathcal{I}, d_{lvl})$
  - can be proven as before
  - by considering non-contextual programs
- **Corollary 33** If $\mathcal{P}$ is acyclic then $\Phi_{\mathcal{P}}$ has a unique fixed point which can be computed by iterating $\Phi_{\mathcal{P}}$ up to ω times starting with any interpretation
  - Follows from Theorems 32 and 9 (Banach Contraction Mapping Theorem)

Contextual Programs and Fixed Points 3

- **Proposition 34**
  If $\mathcal{P}$ is acyclic then the unique fixed point of $\Phi_{\mathcal{P}}$ is a model of $wc\mathcal{P}$

- **Proof** Let $I = \langle I^\top, I^\bot \rangle$ be the unique fixed point of $\Phi = \Phi_{\mathcal{P}}$ and $A \leftrightarrow F \in wc\mathcal{P}$

  - $I\,A = \top$ We find $A \leftarrow Body \in g\mathcal{P}$ such that $I\,Body = \top$
    - Hence, $I\,F = I(A \leftrightarrow F) = \top$
  - $I\,A = \bot$ We find a clause $A \leftarrow Body \in g\mathcal{P}$ and
    for all clauses $A \leftarrow Body \in g\mathcal{P}$ we find $I\,Body = \bot$
    - Hence, $I\,F = \bot$ and $I(A \leftrightarrow F) = \top$
  - $I\,A = U$ $\rightsquigarrow$ **Exercise**

- **Conjecture** the unique fixed point of $\Phi_{\mathcal{P}}$ a minimal model of $wc\mathcal{P}$

Supported Models

- The unique fixed point of $\Phi_{\mathcal{P}}$ is called supported model of $wc\mathcal{P}$
- It will be denoted by $\mathcal{M}_{wc\mathcal{P}}$
- Formula F follows from an acyclic contextual program $\mathcal{P}$ under WCS in symbols $\mathcal{P} \models_{wcs} F$ iff $\mathcal{M}_{wc\mathcal{P}}$ maps F to true
- Reconsider $\mathcal{P}$

$$\begin{array}{lcl} p & \leftarrow & ctxt\ q \\ p & \leftarrow & \bot \end{array}$$

  - $\mathcal{M}_{wc\mathcal{P}} = \langle \emptyset, \{p\} \rangle$
  - $\mathcal{P} \models_{wcs} \neg p \wedge \neg(p \wedge q)$

The Tweety Scenario Revisited

- Let $\mathcal{P}$ consist of the following clauses:

$$\begin{array}{rcl} \mathit{fly}\ X & \leftarrow & \mathit{bird}\ X \wedge \neg \mathit{ab}_{\mathit{fly}}\ X \\ \mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{kiwi}\ X \\ \mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{penguin}\ X \\ \mathit{bird}\ \mathit{tweety} & \leftarrow & \top \\ \mathit{bird}\ \mathit{jerry} & \leftarrow & \top \end{array}$$

- Iterating the semantic operator yields

$\Phi_{\mathcal{P}}$	$I^\top$	$I^\perp$
$\uparrow 0$		
$\uparrow 1$	*bird tweety* *bird jerry*	$\mathit{ab}_{\mathit{fly}}$ *tweety* $\mathit{ab}_{\mathit{fly}}$ *jerry*
$\uparrow 2$	*bird tweety* *bird jerry* *fly tweety* *fly jerry*	$\mathit{ab}_{\mathit{fly}}$ *tweety* $\mathit{ab}_{\mathit{fly}}$ *jerry*

Tweety is a Penguin

- Suppose we learn that *Tweety is a penguin*
- Let $\mathcal{P}'$ be

$$\begin{array}{rcl}
\mathit{fly}\ X & \leftarrow & \mathit{bird}\ X \wedge \neg \mathit{ab}_{\mathit{fly}}\ X \\
\mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{kiwi}\ X \\
\mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{penguin}\ X \\
\mathit{bird}\ \mathit{tweety} & \leftarrow & \top \\
\mathit{bird}\ \mathit{jerry} & \leftarrow & \top \\
\mathit{penguin}\ \mathit{tweety} & \leftarrow & \top
\end{array}$$

Computing the Supported Model

- Iterating the semantic operator yields

$\Phi_{\mathcal{P}'}$	$I^\top$	$I^\perp$
$\uparrow 0$		
$\uparrow 1$	*bird tweety* *bird jerry* *penguin tweety*	ab_{fly} *tweety* ab_{fly} *jerry*
$\uparrow 2$	*bird tweety* *bird jerry* *penguin tweety* ab_{fly} *tweety* *fly tweety* *fly jerry*	ab_{fly} *jerry*
$\uparrow 3$	*bird tweety* *bird jerry* *penguin tweety* ab_{fly} *tweety* *fly jerry*	ab_{fly} *jerry* *fly tweety*

The Drowning Problem

- **Drowning Problem** if an object belonging to a particular class and being exceptional with respect to some property of the class, becomes exceptional with respect to other or all properties of the class
- **Example**

$$\begin{array}{rcl} \mathit{fly}\ X & \leftarrow & \mathit{bird}\, X \wedge \neg\, \mathit{ab}_{\mathit{fly}}\ X \\ \mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{penguin}\, X \\ \mathit{ab}_{\mathit{fly}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{moa}\, X \\ \mathit{wings}\, X & \leftarrow & \mathit{bird}\ X \wedge \neg\, \mathit{ab}_{\mathit{wings}}\ X \\ \mathit{ab}_{\mathit{wings}}\ X & \leftarrow & \mathit{ctxt}\ \mathit{moa}\, X \\ \mathit{bird}\ t & \leftarrow & \top \\ \mathit{penguin}\ t & \leftarrow & \top \end{array}$$

- **Least model of the weak completion**

$$\langle \{\mathit{bird}\ t,\ \mathit{penguin}\ t,\ \mathit{ab}_{\mathit{fly}}\ t,\ \mathit{wings}\ t\}, \{\mathit{fly}\ t,\ \mathit{ab}_{\mathit{wings}}\ t\} \rangle$$

- **The Weak Completion Semantics does not suffer from the drowning problem**

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