Human Reasoning and the Weak Completion Semantics
Technische Universität Dresden
Exercise 7

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Note: Please consider the equational theory to be empty for reach question, unless stated otherwise.

Problem 1

Consider the program P: $\{p \leftarrow \top,\ q \leftarrow \neg p\}$. The level mapping is, $level(p) = 0,\ level(q) = 1$. Please answer the following questions:
a. Is the program acyclic? Why or why not?
b. Let $I_1 = \langle \emptyset,\ \emptyset \rangle$, $I_2 = \langle \{p\},\ \emptyset \rangle$ and $I_3 = \langle \{p\},\ \{q\} \rangle$.
Please compute $d_{level}(I_1,\ I_2)$ and $d_{level}(I_2,\ I_3)$.

Problem 2

Consider the program P: $\{p \leftarrow r \wedge q,\ q \leftarrow r \wedge p\}$. Please answer the following questions:
a. Is the program acyclic? Why or why not?
b. Is Φ_P a contraction? Why or why not?

Problem 3

Consider the program P: $\{even(0) \leftarrow \top,\ even(successor(X)) \leftarrow \neg even(X)\}$.
Let the level mapping be such that, $level(even(0)) = 0$,
$level(even(successor(0))) = 1$, $level(even(successor(successor(0)))) = 2$ and so on.
Please answer the following questions:
a. Is P acyclic? Why or why not?
b. Starting with the empty interpretation, please show some immediate consequences of Φ_P, namely, I_0 (this is the one *after* the empty interpretation), I_1, I_2, I_3.
c. What is the fixed point of P, I? Are any other fixed points possible?
c. Please compute the following, $d_{level}(I_0,\ I_1), d_{level}(I_1,\ I_2), d_{level}(I_2,\ I_3)$.
d. Please compute, $d_{level}(I_0,\ I_3)$ and $d_{level}(I_2,\ I_3)$.

Problem 4

a. Please state two points of difference between the operator Φ behaving as contraction and otherwise (not a contraction).
b. Based on whatever has been covered in the lectures so far, what is the utility of the Banach Contraction Mapping Theorem?