Human Reasoning and the Weak Completion Semantics
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Problem 1

Normal logic programs are finite or countably infinite set of clauses of the form Head if Body i.e. $Head \leftarrow A_1 \wedge A_2 \wedge \ldots \wedge A_m \wedge \neg B_1 \wedge \neg B_2 \wedge \ldots \wedge \neg B_n$*,* $Head \leftarrow \top$ *or* $Head \leftarrow \bot$ *where* $Head$ *is a positive literal (also called atom). The right hand side of the implication* $\leftarrow$ *is called the* $Body$*.*
Now, consider the normal logic program P_1: $\{e \leftarrow \top,\ l \leftarrow e \wedge \neg ab_e,\ ab_e \leftarrow \bot\}$. *Clauses are usually taken to be a finite collection of (universally closed) positive or negative literals (i.e. atoms or their negations respectively) which are connected using disjunctions, i.e.* $l_1 \vee l_2 \vee \ldots \vee l_n$*.*
Each of the implications in the program P_1 are logically equivalent to clauses.
a. Can you write down the (equivalent) clausal form for each?
b. *A definite logic program is a special kind of logic program that does not have occurrences of negations or* $\bot$. Then, is P_1 a definite logic program? Why or why not?
c. P_1 is a propositional program. What will be a grounded instance of P_1?

Problem 2

Consider the definite first-order program P_2: $\{p(a) \leftarrow \top,\ p(f(X)) \leftarrow p(X)\}$. X is a variable, a is a constant symbol and f is a function symbol. We only consider two-valued logic.
a. What is the Herbrand Universe? Is it finite?
b. What is the Herbrand Base?
c. What is the grounded program?
d. A Herbrand interpretation maps atoms in the Herbrand Base to true or false. It is this mapping that distinguishes one Herbrand interpretation from another. An Herbrand interpretation is called a (Herbrand) model when it maps each clause in the grounded program to true. In two-valued logic, models can be represented simply using the atoms from the Base which the model has mapped to true. Furthermore, in two-valued logic, the model intersection property holds for definite logic programs. This basically means, the intersection of all Herbrand models of a definite logic program in two-valued logic is again a model. This model is called the least model, as it is minimal and there can be no further minimal models aside from this.
Given all this information, what should be the least (Herbrand) model for the above program?
e. Recall the definition of the dependency function *deps*. What is $deps(P_2,\ p(a))$ and $deps(P_2,\ \neg p(f(f(a))))$?

Problem 3

a. Proposition: *Given a definite logic program P. The model intersection property holds for P, in two-valued logic; in other words P will have a least Herbrand model.* Prove the proposition.

b. Consider what would happen to the model intersection property in case of a normal logic program as defined above, w.r.t. two-valued logic.

Problem 4

Consider the normal first-order logic program P_3: $\{q(X) \leftarrow \neg p(X),\ p(a) \leftarrow \top\}$ and equation $a \approx b$.
a. What will be the Herbrand Universe, and the Herbrand Base?
b. Under the semantics of the three-value Łukasiewicz-logic, what might be the least model of P_3?
c. Can you also think of a model which is not the least one?

Problem 5 (Optional)

Let us revisit the Supression Task. We see how weak completion (under the three valued semantics of Łukasiewicz-logic) helps us adequately model the responses of the experiment. But what if the weak completions are replaced by completions? Do you think it could still sufficiently model the Supression Task?