Human Reasoning and the Weak Completion Semantics
Technische Universität Dresden
Exercise 5

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Problem 1

a. Give an example of a partially ordered set where the *least upper bound* does not exist.
b. Give an example of a partially ordered set where the *greatest lower bound* does not exist.

Problem 2

With reference to the notions of *monotonic* and *continuous* functions as discussed in the lecture and the manuscript, give an example of a partially ordered set S (please also mention the partial order itself)*, and specify a function $f : S \to S$ if possible, which is:
a. Monotonic.
b. Non-Monotonic.
c. Continuous.
d. Not Continuous.
f. Monotonic and Continuous.
e. Continuous and Non-Monotonic.
***Note**: You are free to use different partially ordered sets for each question, if you like.

Problem 3

Consider the set $S = \{0, 1, 2, 3, \ldots, \omega, \omega + 1, \omega + 2, \ldots, \omega + \omega\}$, and the relation $\leq$. Here, the symbol ω is the *first limit ordinal* that occurs after the set of natural numbers, followed by the non-limit ordinals $\omega + 1, \omega + 2$ etc. The symbol $\omega + \omega$ is the *second limit ordinal*. Consider a function $f : S \to S$, such that $f(x) = x$ if $x < \omega$ and $f(x) = \omega + \omega$ if $x \geq \omega$. Now, please answer the following questions:
a. Is the set S partially ordered?
b. Is the function f monotonic? Why or why not?
c. Is the function f continuous? Why or why not?