

Human Reasoning and the Weak Completion Semantics

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Exercise 6

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Problem 1

Consider the program $P: \{q(X) \leftarrow \neg p(X), p(a) \leftarrow \top\}$, and the equational theory $\{a \approx b\}$.

- What is the Herbrand Base, and the grounded program?
- Starting from the empty interpretation $\langle \emptyset, \emptyset \rangle$, please show how the least fixed point of this program can be computed by the (iterative) application of the modified Fitting operator, Φ_P .
- What do you think is the least model of P ?
- What do you think is the least model of the weak completion of P ?
- What observation can be drawn from the above points b, c and d?
- With particular regard to the above point b, is Φ_P *monotonic*? Please state why.
- With particular regard to the above point b, is Φ_P *continuous*? (Hint: You can use propositions from the manuscript to justify your response.)
- What is the set of all *possible interpretations* (not to be confused with *models*) of P ?
- Is the above set (let's call it I) directed? Please state the reason(s) for your answer.
- Is I a complete partial order? Please state the reason(s) for your answer.
- Please write down any two directed subsets of I , and state their least upper bounds.

Problem 2

- Consider the program $P_1: \{q(1) \leftarrow \top, q(X \circ a) \leftarrow q(X)\}$ and the AC1 theory: $\{x \circ 1 \approx X, X \circ Y \approx Y \circ X, (X \circ Y) \circ Z \approx X \circ (Y \circ Z)\}$. Please state the Herbrand Universe, the Herbrand Base, the grounded program, and state the least fixed point by listing the first 5 iterations of the Φ_{P_1} operator.
- Consider the program $P_2: \{q(1) \leftarrow \top, q(X \circ a) \leftarrow q(X), p \leftarrow \neg q(X)\}$, and the above AC1 theory. Please state the Herbrand Universe, the Herbrand Base, the grounded program, and state the least fixed point by listing the first 5 iterations of the Φ_{P_2} operator.
- Is there a difference in the number of iterations between the two operators, before it reaches a fixed point?

Problem 3

Consider the program $\{q(a) \leftarrow \top, r(b) \leftarrow \top, p(X) \leftarrow q(X) \wedge r(X)\}$, and the equation $\{b \approx c\}$. Please state the following,

- Herbrand Universe.
- Herbrand Base.
- Grounded program.
- A bijection between elements from the Herbrand Base, to propositional atoms (of your

choice).

e. The resulting, equivalent propositional program, P .

f. Starting with the empty interpretation, please show the computation of the least fixed point of the propositional program, using Φ_P .

g. Is Φ_P monotonic, and continuous? Please state the reasons for your response.

Problem 4

Please provide a proof sketch of the following proposition: *Let X be a directed (sub)set of interpretations. Then, the interpretation $I = \langle \bigcup X^\top, \bigcup X^\perp \rangle$ is the least upper bound of X .* Note: Here, $\bigcup X^\top$ and $\bigcup X^\perp$ denote the union of all the true and false elements of all interpretations in the (sub)set X , respectively.