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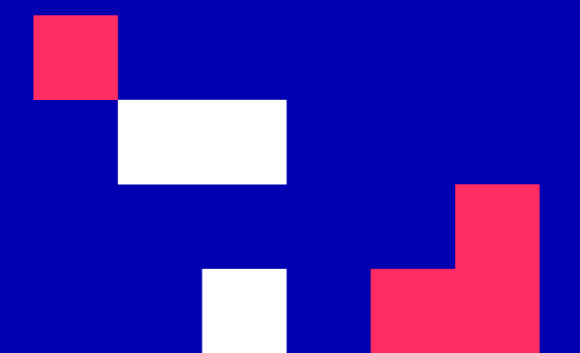


University of Cyprus

MAI643 Artificial Intelligence in Medicine

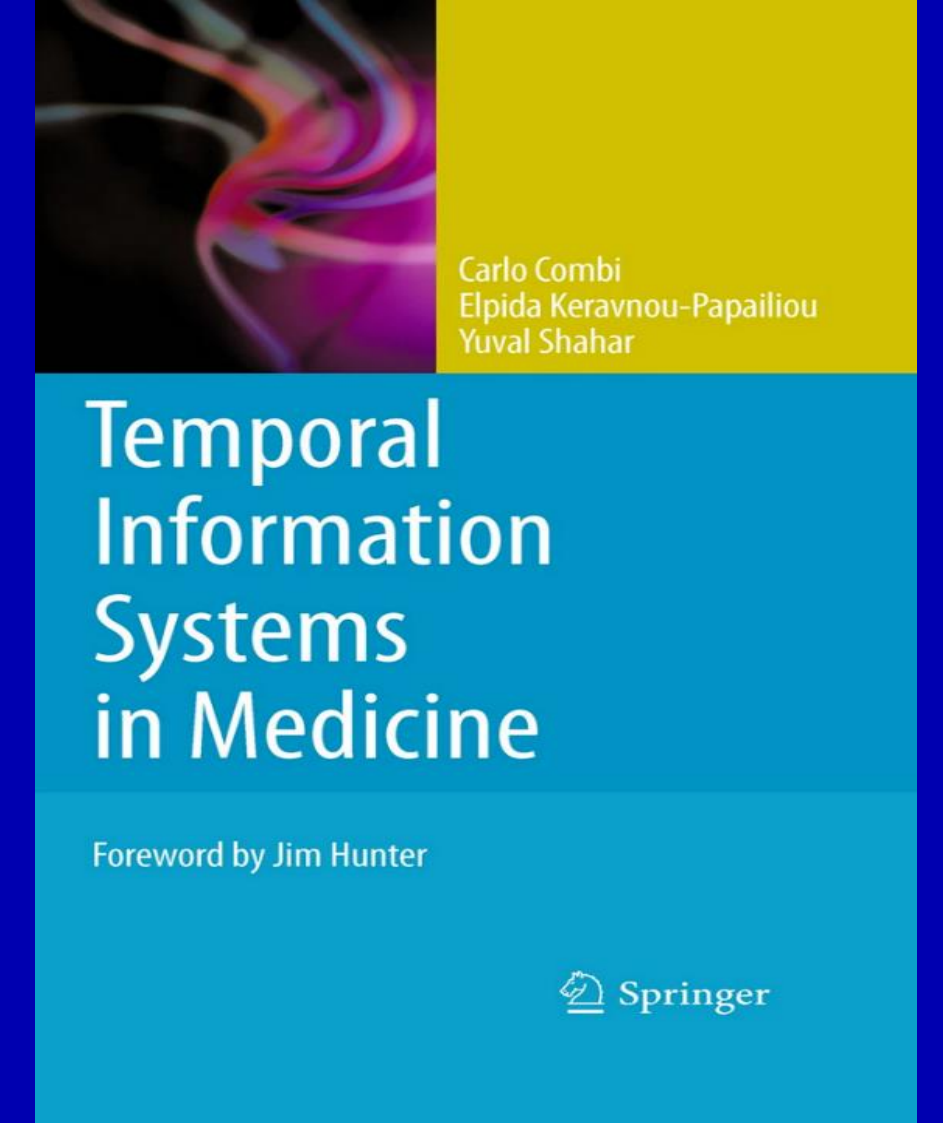
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Time Representation and Temporal Reasoning in Medicine



UNIT 9

Time Representation and Temporal Reasoning

CONTENTS

1. Modeling Temporal Concepts
2. Temporal Reasoning
3. Three Well-Known General Theories of Time and the Medical Domain
4. Temporal Constraints

INTENDED LEARNING OUTCOMES

Upon completion of this unit on time representation and temporal reasoning in medicine, students will be able:

1. To explain the basic notions regarding the modeling of time (time domain, instants/intervals, time structure, absolute/relative, relations, granularity and indeterminacy) and the modeling of temporal entities (kinds, association to time, different semantics).
2. To present the required functionalities for medical temporal reasoning.
3. To discuss the most influential ontologies and models for temporal reasoning (tense logics, time specialist, situation calculus, event calculus, interval-based and point-based temporal logics) and to analyze a simple medical example against three well-known general theories of time.
4. To refer to temporal constraints and their abstract representation.

Modeling Temporal Concepts

A common focus of temporal reasoning, temporal abstraction of clinical data, and modeling and managing clinical data, is the definition or the adoption of a set of basic concepts that enable a description of a time-oriented clinical world.

Several suggestions have emerged from generic fields of computer science or the knowledge and data management areas. This effort has progressed from an ad-hoc definition of concepts supporting a particular application to the adoption of more generic definitions, supporting different clinical applications.

Modeling Temporal Concepts

- ❑ Time manifests in different ways in expressions of medical knowledge and patient information
- ❑ There are two issues here:
 - how to **model time** per se
 - how to **model time-varying situations** or occurrences

Modeling Time

- ❑ Modeling time as a **dense** or **discrete** number line may not provide the appropriate abstraction for medical applications
- ❑ The modeling of time for the management of, or the reasoning about time-oriented clinical data, requires several basic choices to be made, depending on the needs of the domain.

Time Domain

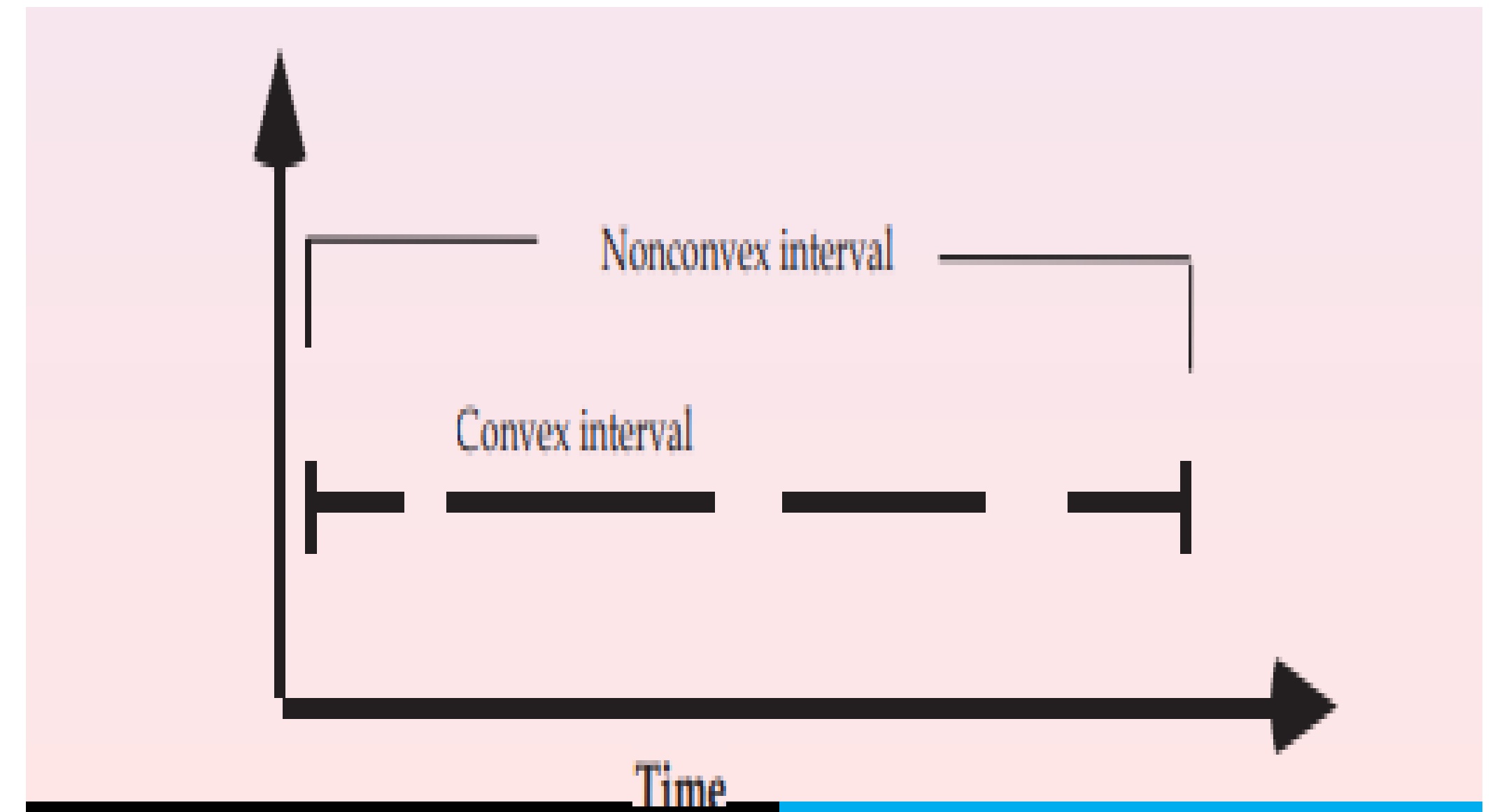
- ❑ The **time domain** consists of the set of primitive time entities used to represent the concept of time
- ❑ **Definition:** A time domain is a pair $(T; \leq)$, where T is a non-empty set of time instants and \leq is a total order on T
- ❑ A time domain is:
 - **bounded** if it contains upper and/or lower bounds with respect to its order relationship
 - **dense** if it is an infinite set and for all $t', t'' \in T$ with $t' < t''$, there exists $t''' \in T$ such that $t' < t''' < t''$
 - **discrete** if every element has both an immediate successor and an immediate predecessor

Instants and Intervals

- ❑ Usually both the (primitive) concepts of **time point** (or **instant**) and **time interval** have been used to represent time
 - In defining basic time entities, time points (i.e., instants) are often adopted
 - Intervals are then represented by their upper and lower temporal bounds (start and end time points)
- ❑ These concepts are usually related to instantaneous events (e.g. myocardial infarction), or to situations lasting for a span of time (e.g. drug therapy)

Time Intervals

- ❑ **Nonconvex intervals** are intervals formed from a union of convex intervals, and might contain gaps
- ❑ In the temporal database community non-convex intervals are usually named **temporal elements**



Linear, Branching and Circular Times

- ❑ Usually time is **linear**, since the set of time points is completely ordered
- ❑ For the tasks of diagnosis, projection, or forecasting a **branching** time might be necessary
 - *only a partial ordering is defined for times*
- ❑ **Circular** (or periodic) time is needed when we have to describe times related to recurrent events
 - *no ordering relations are defined for times*

Relative and Absolute Times

- The position on the time axis of an interval or of an instant can be given as
 - an **absolute** position

Example

“on November, 3 1996”, “from November, 3 1996 for 3 days”

- a **relative** time reference

Example

“after an episode of atrial fibrillation” or “two months after coronary artery angioplasty intervention”

Time Metrics

Absolute times are generally associated to a metric, being its position given as a distance from a given **time origin**. When a metric is defined for the time domain, relative times can be given quantitatively: “three days after birth”.

Modeling Temporal Relationships

- Point/point (i.e., $<$, \leq , $=$, $>$, \geq)
 - Point/interval
 - Interval/point
 - Interval/interval
 - In modeling temporal relationships, Allen's interval algebra has been widely used in medical informatics
 - Extensions to Allen's basic thirteen interval relationships have also been proposed
-
- **qualitative** (e.g., point P1 before interval I2)
 - **quantitative** (e.g., interval I1 two hours before interval I2)

The 13 possible Allen's interval relations

| Relation | Illustration | Interpretation |
|---------------------|--------------|---|
| $X < Y$ $Y > X$ | | X precedes Y Y is preceded by X |
| $X m Y$ $Y mi X$ | | X meets Y Y is met by X (<i>i</i> stands for <i>inverse</i>) |
| $X o Y$ $Y oi X$ | | X overlaps with Y Y is overlapped by X |
| $X s Y$ $Y si X$ | | X starts Y Y is started by X |
| $X d Y$ $Y di X$ | | X during Y Y contains X |
| $X f Y$ $Y fi X$ | | X finishes Y Y is finished by X |
| $X = Y$ | | X is equal to Y |

Modeling Granularities

Definition:

The **granularity** of a given temporal information is the level of abstraction at which information is expressed

- Different units of measure allow one to represent different granularities
 - Calendric time units (years, months, days, ..., minutes, seconds)
 - Domain-related time units (chemotherapy cycles, weeks after the intervention, ...)

Modeling Granularities

A widely accepted definition of temporal granularity, proposed by Bettini et al. has been used both for knowledge representation and for temporal data modelling.

Definition:

A granularity is a mapping G from an index set (e.g., integers) to the power set of the time domain such that:

- if $i < j$ and $G(i)$ and $G(j)$ are non-empty, then all elements of $G(i)$ are less than all elements of $G(j)$, and
- if $i < k < j$ and $G(i)$ and $G(j)$ are non-empty, then $G(k)$ is non-empty

Any $G(i)$ is called a **granule**

Domain-dependent Granularities

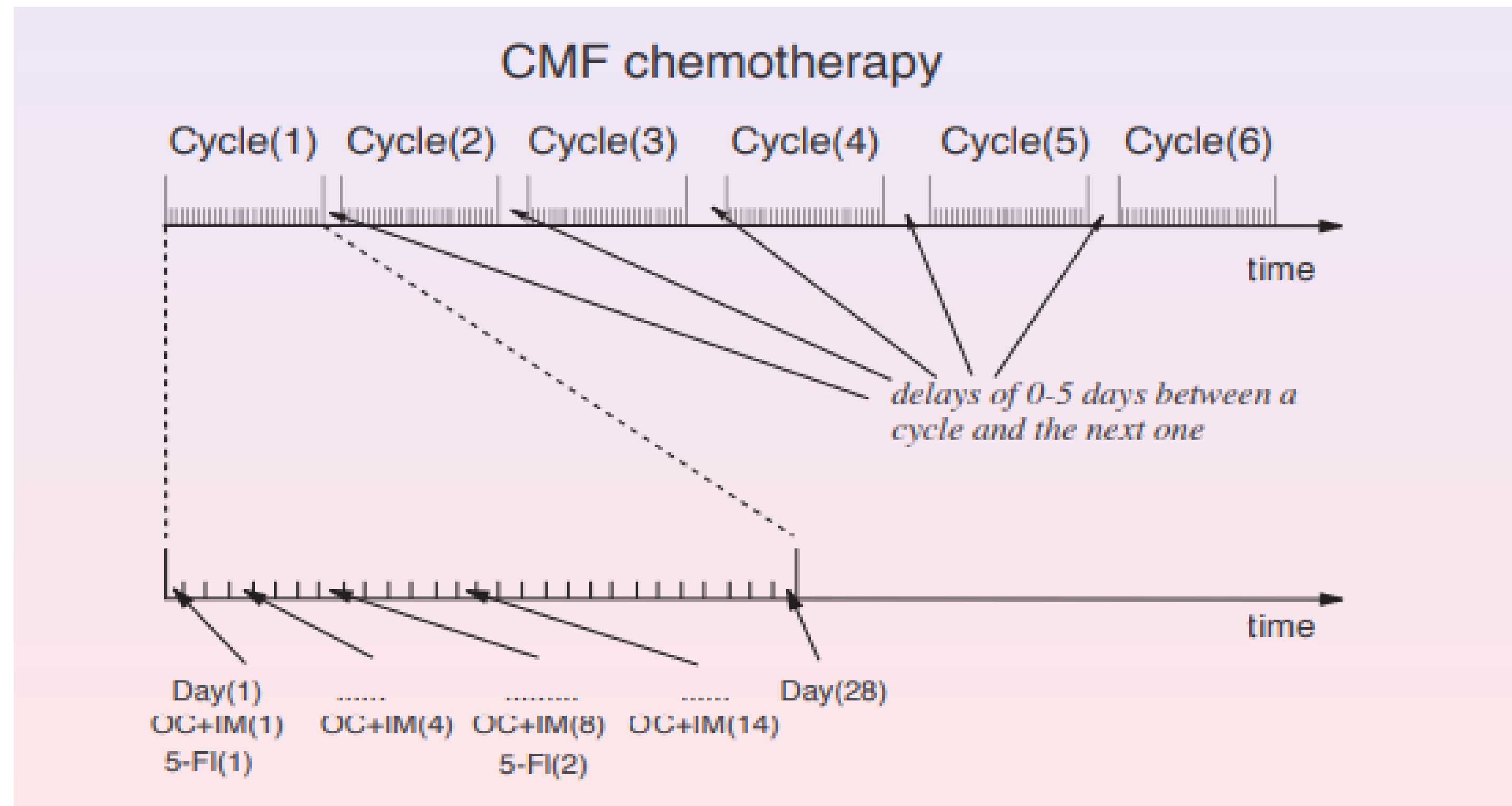
Example:

“The recommended CMF regimen consists of 14 days of oral cyclophosphamide with intravenous methotrexate, and 5-fluorouracil on days 1 and 8. This is repeated every 28 days for 6 cycles”

Moreover, it may happen that the beginning of a cycle is delayed a few days, due to patient’s blood analysis results.

A CMF stands for the chemotherapy based on the drugs Cyclophosphamide, Methotrexate, and 5-Fluorouracil.

Domain-dependent Granularities



Modeling indeterminacy

- **Indeterminacy** is often present in temporal information and is related to incomplete knowledge of when the considered fact happened.
- A frequent need, especially in clinical domains, is the explicit expression of **uncertainty** regarding how long a proposition was true:
 - We might not know precisely when the proposition became true and when it ceased to be true, although we might know that it was true during a particular time interval.
 - Sometimes, the problem arises because the time units involved have different **granularities**.

Modeling indeterminacy

Example:

- “The patient had a severe headache on February 10, 1993, from 2.10-2.30 p.m. to 6-6.15 p.m., lasting between 3 hours 35 minutes and 3 hours 55 minutes”;
- “In 1993 the anticoagulation-therapy was administered for 60-65 days”;
- “The patient had aphasia from 6.30 p.m. to 9 p.m., May 13, 1991”
- “At 1:23 p.m., February 12, 1993, the patient had a stroke”;
- “On December 28, 1997, between 2 and 2.15 p.m., the physician measured the blood-pressure of the patient: it was 190/110 mmHg”;
- “At 5:30 p.m., January 26, 1998, the patient’s atrial fibrillation stopped; it lasted 45 hours”
- “The patient suffered from an altered consciousness lasting for 130 seconds, on January 28, 1998, at 8.34 p.m.”

Modeling indeterminacy

Console and Torasso present a model of time intervals that represents such partial knowledge explicitly:

- The model was proposed in order to represent causal models for diagnostic reasoning
- The authors define a **variable interval** as a time interval / composed of three consecutive convex intervals
- The first interval is *begin(I)*, the second is called *body(I)*, and the third is called *end(I)*

Modeling Temporal Entities

Definition

temporal entities: those concepts/things of the real world which must be represented also for their temporal aspects

- A rich model providing a number of interrelated basic temporal entities is often required when dealing with medical temporal information
- Many representation issues arise with respect to temporal entities

Defining Temporal Entities

- *What are the basic (medical) concepts that have temporal dimension?*
- *How are they interrelated?*

Two different approaches:

- **association of a temporal dimension to existing objects**
 - uses simple, “atomic” temporal entities
 - similar to the one underlying temporal extensions for relational and object-oriented data models
- **creation of model-specific, time-oriented entities**
 - focuses on modeling different temporal features of complex, task-specific entities

Some proposals coming from medical applications

- ❑ in the **HyperLipid system**, patient visits were modeled as instant-based objects called events, while administration of drugs was modeled as therapy objects whose attributes included a time interval; phases of therapy were then introduced to model groups of heterogeneous data.
- ❑ Kahn and colleagues introduced formally the concept of a **Temporal Network (TNET)** and later extended it by the **Extended TNET**, or ETNET model; a T-node (or an ET-node) models task-specific temporal data, such as a chemotherapy cycle, at different levels of abstraction.
- ❑ In the **M-HTP system** for monitoring heart-transplant patients, clinical facts related to a patient are structured in a temporal network (TN) inspired by Kahn's TNET model
- ❑ Keravnou and Washbrook introduce *findings, features, and events* to distinguish various types of instantaneous and interval-based information (patient-specific or general)

Associating Entities to Instants and Intervals

Two main approaches:

- Introduction of both instant-related entities and interval-related entities
- Introduction of clinical entities associated only to a certain type of time concept, usually an interval

Expressing Occurrences of Temporal Entities

□ Absolute and relative temporal occurrences:

- in **absolute** terms, relative to some fixed time point, by specifying its initiation and termination

Example: “Tachycardia on November 3, 1996 from 6:30 to 6:45 p.m.”

- **relative** to other occurrences

Example

- by qualitative relationships: “angina after a long walk” or “several episodes of headache during puberty”
- by quantitative relationships: “angina two hours before headache”

Expressing Occurrences of Temporal Entities

- ❑ **Occurrences with absolute and relative vagueness, duration, and incompleteness:**

Example “an atrial fibrillation episode occurred on December 14th, 1995 between 14:30 and 14:45 and lasted for three-four minutes”

Expressing Occurrences of Temporal Entities

- ❑ Point and interval occurrences
 - ❑ An occurrence may be considered a **point occurrence** in some temporal context if its duration is less than the time unit, if any, associated with the particular temporal context
 - ❑ An occurrence may be considered an **interval occurrence** in some temporal context if its duration is at least equal to the time unit associated with the particular temporal context

Example: A myocardial infarction may be considered both a point occurrence and an interval occurrence, depending on the considered context (follow-up or intensive care)

Semantic Relations between Temporal Entities

- **Compound occurrences:** repeated instantiations of some type of occurrence:
 - **temporal trend**
Example: “increasing blood pressure”
 - **temporal pattern**
Example: “increasing blood pressure overlaps an increase of heart rate”
 - **periodic pattern**
Example: “increasing blood pressure every morning”

Semantic Relations between Temporal Entities

□ Contexts, causality and other temporal constraints

- a **context** represents a state of affairs that, when interpreted (logically) over a time interval, can change the meaning of one or more facts which hold within the context time interval
- **causality** is a central relation between occurrences: changes are explained through causal relations

Temporal Reasoning

- Temporal Reasoning Requirements
- Ontologies and Models for Temporal Reasoning

Temporal Reasoning

- ❑ The ability to reason about time and temporal relations is fundamental to almost any intelligent entity that needs to make decisions
- ❑ It is difficult to represent the concept of taking an action, let alone a series of actions, and the concept of the consequences of taking a series of actions, without explicitly or implicitly introducing the notion of time

Example: Planning actions for a surgery intervention requires reasoning about the temporal order of the actions and about the length of time it will take to perform the actions

Tasks and Temporal Reasoning

- ❑ **Projection** is the task of computing the likely consequences of a set of conditions or actions, usually given as a set of cause-effect relations. Projection is particularly relevant to the **planning** task.
- ❑ **Forecasting** involves predicting particular future values for various parameters given a vector of time-stamped past and present measured values, such as anticipating changes in future hemoglobin-level values, given the values up to and including the present.

Tasks and Temporal Reasoning

- **Planning** consists of producing a sequence of actions for a care provider, given an initial state of the patient and a goal state, or set of states, such that that sequence achieves one of the goal patient states.
- **Interpretation** involves abstraction of a set of time-oriented patient data, either to an intermediate level of meaningful temporal patterns, as is common in the **temporal-abstraction** task or in the **monitoring** task, or to the level of a definite diagnosis or set of diagnoses that explain a set of findings and symptoms, as is common in the **diagnosis** task.

Temporal Reasoning Functionalities

- Mapping the existence of occurrences across temporal contexts
- Determining bounds for entity occurrences
- Consistency detection and clipping of uncertainty
- Deriving new occurrences from other occurrences
- Deriving temporal relations between occurrences
- Deriving the truth status of queried occurrences
- Deriving the state of the world at a particular time

Tense Logics

- ❑ We know that Aristotle was interested in the meaning of the truth value of future propositions
- ❑ Diodorus Chronus, who lived circa 300 B.C., extended Aristotle's inquiries by constructing what is known as **the master argument**

The master argument can be reconstructed in modern terms as follows:

- ❑ Everything that is past and true is necessary (i.e., what is past and true is necessarily true thereafter)
- ❑ The impossible does not follow the possible (i.e., what was once possible does not become impossible)

Tense Logics

- ❑ From these two assumptions, Diodorus concluded that nothing is possible that neither is true nor will be true, and that therefore every (present) possibility must be realized at a present or future time.
- ❑ The master argument leads to **logical determinism**, the central tenet of which is that what is necessary at any time must be necessary at all earlier times.

Tense Logic

The representation of the master argument in temporal terms inspired modern work in temporal reasoning. In particular, Prior attempted to reconstruct the master argument using a modern approach. This attempt led to what is known as **tense logic** - a logic of past and future. In Prior's terms:

Fp: it will be the case that p.

Pp: it was the case that p.

Gp: it will always be the case that p (i.e., $\neg F\neg p$).

Hp: it was always the case that p (i.e., $\neg P\neg p$).

Prior's tense logic is thus in essence a **modal-logic** approach.

Tense Logic

This modal-logic approach has been called a *tenser* approach, as opposed to a *detenser*, or an FOL, approach.

Example:

As an example, in the *tenser* view, the sentence $F(\exists x)f(x)$ is not equivalent to the sentence $(\exists x)Ff(x)$; in other words, if in the future there will be some x that will have a property f , it does not follow that there is such an x now that will have that property in the future.

In the *detenser* view, this distinction does not make sense, since both expressions are equivalent when translated into FOL formulae.

Kahn and Gorry's Time Specialist

Kahn and Gorry built a general temporal-utilities system, the **time specialist**, which was intended not for temporal **reasoning**, but rather for temporal **maintenance** of relations between time-stamped propositions.

A novel aspect of Kahn and Gorry's approach was the use of three different organization schemes; the decision of which one to use was controlled by the user:

- Organizing by **dates** on a date line (e.g., "January 17 1972")
- Organizing by special **reference events**, such as **birth** and **now** (e.g., "2 years after birth")
- Organizing by **before** and **after** chains, for an event sequence (e.g., "the fever appeared after the rash")

Kahn and Gorry's Time Specialist

- ❑ By using a **fetcher** module, the time specialist was able to answer questions about the data that it maintained.
- ❑ The time specialist also maintained the **consistency** of the database as data were entered, asking the user for additional input if it detected an inconsistency.

The Situation Calculus

The **situation calculus** was proposed by McCarthy to describe actions and their effects on the world. The idea is that the world is a set of **states**, or **situations**. Actions and events are functions that map states to states.

Example:

The result of performing the CARE_PROVIDING action in a situation with a suffering patient is a situation where the patient is treated, represented as:

$$\forall s(\text{True}(s, \text{SUFFERING_PATIENT}) \Rightarrow \text{True}(\text{Result}(\text{CARE_PROVIDING}, s), \text{TREATED_PATIENT}))$$

Kowalski and Sergot's Event Calculus

Kowalski and Sergot proposed the **Event Calculus** (EC), a theory of time and change.

- Primitives (and semantics) of the formalism: **events** happen at **time points** and initiate and/or terminate **time intervals** over which **properties** hold.
- Event occurrence: *happens(event, timePoint)* clause.
- Relations between events and properties:
 $initiates(ev1, prop, t) \Leftarrow happens(ev1, t) \wedge holds(prop1, t) \wedge \dots \wedge holds(propN, t)$
 $terminates(ev2, prop, t) \Leftarrow happens(ev2, t) \wedge holds(prop1, t) \wedge \dots \wedge holds(propN, t)$

Kowalski and Sergot's Event Calculus

- The EC model of time and change is concerned with deriving the maximal validity intervals (MVIs) over which properties hold
 - a **validity interval** must not contain any interrupting event for the property
 - a **maximal validity interval** (MVI) is a validity interval which is not a subset of any other validity interval for the property

- Clause *mholds_for(p, [S, E])* returns the MVIs for a given property p: each MVI is given by a pair [S, E], where S (Start) and E (End) are the lower and upper endpoints of the interval.

Kowalski and Sergot's Event Calculus

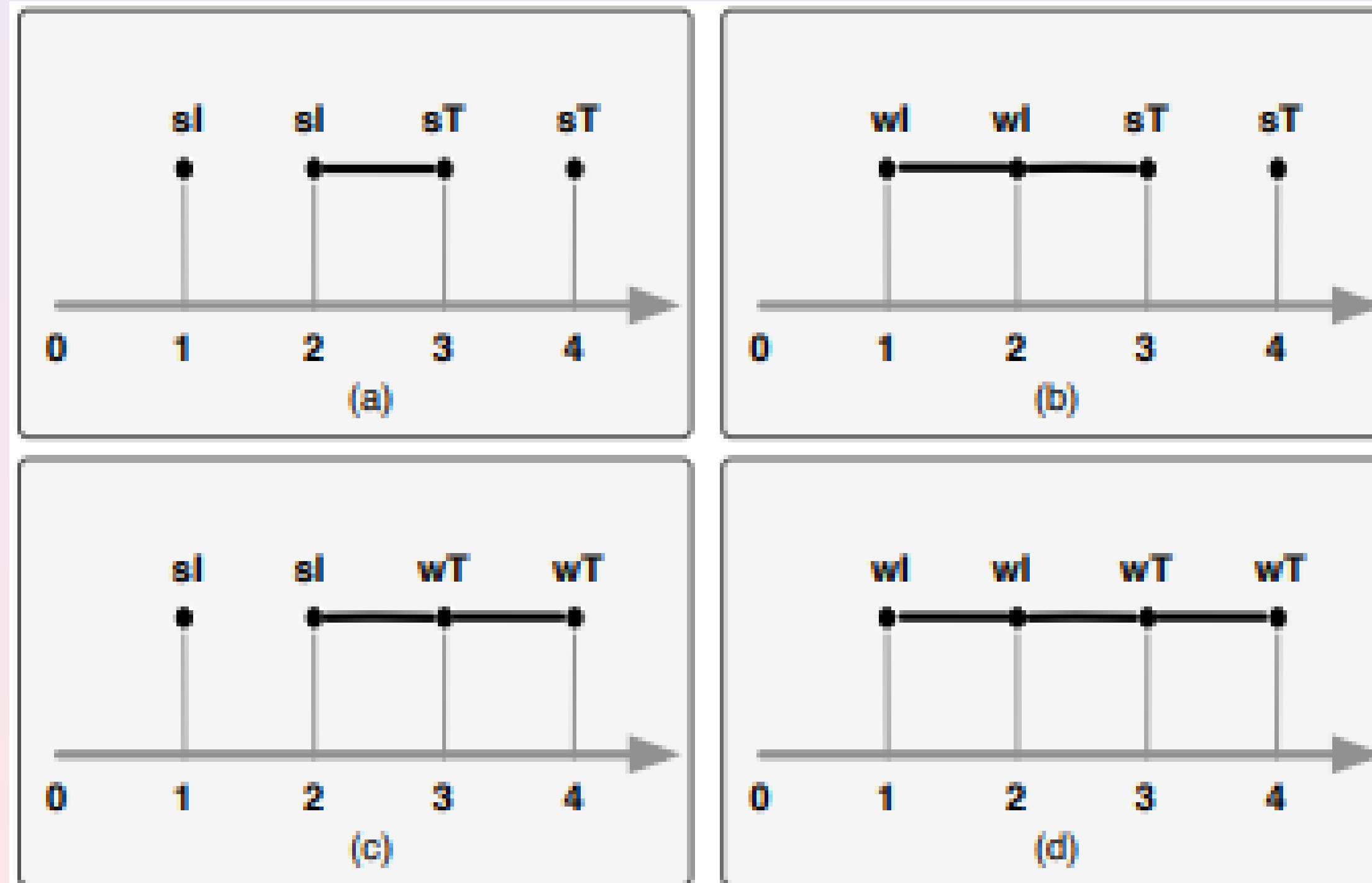
Chittaro and Montanari distinguished two alternative ways of interpreting **initiates** clauses in the derivation of MVIs:

- ❑ **weak interpretation**, only terminating events are considered as interrupting events, and an initiating event e for property p initiates an MVI, provided that p has not been already initiated by a previous event in such a way that p already holds at the occurrence time of e .
- ❑ **strong interpretation** considers also initiating events as interrupting events: therefore, an initiating event e for property p initiates an MVI, provided that there is no subsequent initiating event for p such that p is not terminated between the two events.

The weak and strong interpretation for **terminating events** give **symmetrical** results.

Kowalski and Sergot's Event Calculus

The complete picture of weakly (w) and strongly (s) initiating (I) and terminating (T) events



Allen's Interval-Based Temporal Logic and related extensions

- ❑ Allen has proposed a framework for temporal reasoning, the **interval-based temporal logic**
- ❑ The only ontological temporal primitives in Allen's logic are **intervals**
- ❑ Intervals are also the temporal unit over which **propositions** are interpreted
- ❑ Allen has defined 13 basic binary relations between time intervals, six of which are inverses of the other six: BEFORE, AFTER, OVERLAPS, OVERLAPPED, STARTS, STARTED BY, FINISHES, FINISHED BY, DURING, CONTAINS, MEETS, MET BY, EQUAL TO

Allen's Proposition Types

- **Properties** hold over every subinterval of an interval: *Holds(p, T)*
 - **Example:** “John had fever during last night”

- **Events** hold only over a whole interval and not over any subinterval of it:
Occur(e, T)
 - **Example:** “John broke his leg on Saturday at 6 P.M.”

- **Processes** hold over some subintervals of the interval in which they occur:
Occurring(p, T)
 - **Example:** “John had atrial fibrillation during the last month”

Allen's Interval Logic

- ❑ Allen's logic does not allow **branching time** into the past or the future (unlike, for instance, McDermott's logic)
- ❑ Allen also constructed a transitivity table that defines logical derivation from any two relation sets, and proposed a **sound** (i.e., produces only correct conclusions) but **incomplete** (i.e., does not produce all correct conclusions) algorithm that propagates efficiently ($O(n^3)$) and correctly the results of applying the transitivity rules

McDermott's Point-Based Temporal Logic

- ❑ McDermott suggested a point-based temporal logic. The main goal of McDermott's logic was to model causality and continuous change, and to support planning.
- ❑ McDermott's temporal primitives are **points**, unlike Allen's intervals.
- ❑ Time is continuous: The time line is the set of real numbers.
- ❑ Instantaneous snapshots of the universe are called **states**.
- ❑ States have an order-preserving **date** function to time instants.

McDermott's Types of Propositions

- **Facts** are interpreted over points, and their semantics borrow from the situation calculus
- An **event e** is the set of intervals over which the event exactly happens: $(\text{Occ } s_1 \ s_2 \ e)$ means that event e occurred between the states s_1 and s_2

McDermott's Branching Time

- ❑ McDermott's states are partially ordered and branching into the future, but are totally ordered for the past.
- ❑ This branching intends to capture the notion of a known past, but an indeterminate future. Each maximal linear path in such a branching tree of states is a *chronicle*.
- ❑ A **chronicle** is thus a complete possible history of the universe, extending to the indefinite past and future.

Shoham's Temporal Logic

- ❑ Shoham presented a temporal logic in which the time primitives are **points**, and propositions are interpreted over time **intervals**.
- ❑ Time points are represented as zero-length intervals, $\langle t, t \rangle$.
- ❑ He used **reified** first-order-logic propositions, namely propositions that are represented as individual concepts that can have, for instance, a temporal duration.
- ❑ Shoham provided clear semantics for both the propositional and the first-order-logic cases, using his **reified first-order temporal logic**.

Shoham's Temporal Logic

Shoham pointed out that there is no need to distinguish among particular types of propositions, such as by distinguishing **facts** from **events**. Instead, he defined several relations that can exist between the truth value of a proposition over an interval and the truth value of the proposition over other intervals:

- A proposition type is **downward-hereditary** if, whenever it holds over an interval, it holds over all that interval's subintervals, possibly excluding its end points.
- A proposition is **upward-hereditary** if, whenever it holds for all proper subintervals of some nonpoint interval, except possibly at that interval's end points, it holds over the nonpoint interval itself.
- A proposition type is **gestalt** if it never holds over two intervals, one of which properly contains the other.
- A proposition type is **concatenable** if, whenever it holds over two consecutive intervals, it holds also over their union.
- A proposition is **solid** if it never holds over two properly overlapping intervals.

Shoham's Temporal Logic

Shoham observed that Allen's and McDermott's **events** correspond to **gestalt** propositions, to **solid** ones, or to both, whereas Allen's **properties** are both **upward-hereditary** and **downward-hereditary**.

Projection, Forecasting, and Persistence Uncertainty

- ❑ The probabilistic approach is typically associated with the tasks of interpretation or forecasting of time-stamped clinical data whose values are affected by different sources of uncertainty.
- ❑ Dean and Kanazawa proposed a model of probabilistic temporal reasoning about propositions that **decay** over time.
- ❑ The main idea in their theory is to model explicitly the probability of a proposition P being true at time t , $P(\langle P, t \rangle)$, given the probability of $\langle P, t-\Delta \rangle$.

Projection, Forecasting, and Persistence Uncertainty

- The assumption is that there are events of type E_p that can cause proposition p to be true, and events of type $E_{\neg p}$ that can cause it to be false.
- Thus, we can define a **survivor function** for $P(< P, t >)$ given $< P, t-\Delta >$, such as an exponential decay function.
- Dean and Kanazawa's main intention was to solve the **projection problem**, in particular in the context of the *planning* task.

Three Well-Known General Theories of Time and the Medical Domain

Three Well-Known General Theories of Time

- Allen's interval-based temporal logic
- Kowalski and Sergot's Event Calculus (EC)
- Dean and McDermott's Time Map Manager (TMM)

It should be noted that none of these general theories of time was developed with the purpose of supporting knowledge-based problem solving, let alone medical problem solving.

Evaluation of General Theories of Time against Medical Temporal Requirements

| | <i>Allen's Time-Interval Algebra</i> | <i>Kowalski & Ser-got's Event Cal-culus</i> | <i>Dean & Mc-Dermott's Time-Token Manager</i> |
|---|--------------------------------------|---|---|
| multiple conceptual temporal contexts | X | X | X |
| multiple granularities | X | X | X |
| absolute existences | X | V | V |
| relative existences | V | X | (V) |
| absolute vagueness | X | X | V |
| relative vagueness | V | X | X |
| duration | X | V | V |
| point existences | X | V | V |
| interval existences | V | V | V |
| periodic occurrences | X | X | X |
| temporal trends | X | X | X |
| temporal patterns | (V) | X | (V) |
| structural relations (temporal composition) | X | X | X |
| temporal causality | (V) | (V) | (V) |

Key: X - does not support; (V) - supports partly; V - supports

Evaluation of General Theories of Time against Medical Temporal Requirements

Example (SpondyloEpiphyseal Dysplasia Congenital: SEDC):

“SEDC **presents from birth** and can be lethal. It **persists throughout the lifetime** of the patient. People suffering from SEDC exhibit the following: short stature, due to short limbs, **from birth**; mild platyspondyly **from birth**; absence of the ossification of knee epiphyses **at birth**; bilateral severe coxa-vara **from birth, worsening with age**; scoliosis, **worsening with age**; wide triradiate cartilage **up to about the age of 11 years**; pear-shaped vertebral-bodies **under the age of 15 years**; variable-size vertebral-bodies **up to the age of 1 year**; and **retarded ossification** of the cervical spine, epiphyses, and pubic bones.”

Allen's Interval Logic Primitives

- Recall that the temporal primitive of Allen's interval-based logic is the **time interval**, and seven basic relations (plus the inverses for six of these) are defined between time intervals.
- The other primitives of the logic are **properties** (static entities), **processes** and **events** (dynamic entities), which are respectively associated with predicates holds, occurring and occur as already discussed:

$$\begin{aligned}
 \text{holds}(p, t) &\iff (\forall t' \text{ in}(t', t) \Rightarrow \text{holds}(p, t')) \\
 \text{occurring}(p, t) &\Rightarrow \exists t' \text{ in}(t', t) \wedge \text{occurring}(p, t') \\
 \text{occur}(e, t) \wedge \text{in}(t', t) &\Rightarrow \neg \text{occur}(e, t')
 \end{aligned}$$

Allen's Interval Logic and the Medical Domain

- In order to represent the SEDC knowledge in terms of Allen's logic we need to decide which of the entities correspond to events, which to properties, and which to processes.

- The relevant generic events are easily identifiable:
 - *birth(P)*
 - *age1yr(P)*
 - *age11yrs(P)*
 - *age15yrs(P)*
 - *death(P)*

Allen's Interval Logic and the Medical Domain

$\text{occurring}(\text{SEDC}(P), I) \Rightarrow \text{occur}(\text{birth}(P), B) \wedge \text{occur}(\text{age1yr}(P), O) \wedge$
 $\text{occur}(\text{age11yrs}(P), E) \wedge \text{occur}(\text{age15yrs}(P), F) \wedge \text{occur}(\text{death}(P), D) \wedge$
 $\text{started-by}(I, B) \wedge \text{finished-by}(I, D) \wedge \text{holds}(\text{stature}(P, \text{short}), I) \wedge$
 $\text{holds}(\text{ossification}(P, \text{knee-epiphyses}, \text{absent}), B) \wedge$
 $\text{occurring}(\text{coxa-vara}(P, \text{bilateral-severe}, \text{worsening}), I) \wedge$
 $\text{occurring}(\text{scoliosis}(P, \text{worsening}), I) \wedge$
 $\text{holds}(\text{triradiate-cartilage}(P, \text{wide}), W) \wedge \text{started-by}(W, B) \wedge \text{finished-by}(W, E) \wedge$
 $\text{holds}(\text{vertebral-bodies}(P, \text{pear-shaped}), F') \wedge \text{started-by}(F', B) \wedge \text{before}(F', F) \wedge$
 $\text{holds}(\text{vertebral-bodies}(P, \text{variable-size}), V) \wedge \text{started-by}(V, B) \wedge \text{finished-by}(V, O) \wedge$
 $\text{occurring}(\text{ossification}(P, \text{cervical-spine}, \text{poor}), I) \wedge$
 $\text{occurring}(\text{ossification}(P, \text{epiphyses}, \text{retarded}), I) \wedge$
 $\text{occurring}(\text{ossification}(P, \text{pubic-bones}, \text{retarded}), I)$

Event Calculus (EC) and Medical Domains

Some criticisms:

- In the EC a change in a property is the effect of an event.
- In real-life a symptom may be self-limiting where no event is required to terminate its existence.
- The designers went around this problem by introducing so-called “ghost” events.
- Another limitation encountered was that only instantaneous causality could be expressed. So delayed effects or effects of a limited persistence could not be expressed.
- Limited support for temporal data abstraction.
- Lack of multiple granularities.
- Lack of any vagueness in the expression of event occurrences.

Event Calculus (EC) and Medical Domains

- Recall that the temporal primitive of Kowalski and Sergot's EC is the **event**. Events are instantaneous happenings which initiate and terminate periods over which **properties** hold.
- A property does not hold at the time of the event that initiates it, but does hold at the time of the event that terminates it.
 - **Causality** is not directly modeled, although a rather restricted notion of causality is implied, e.g. an event happening at time t causes the initiation of some property at time $(t+1)$ and/or causes the termination of some (other) property at time t .
 - The calculus can be applied both under a dense or a discrete model of time.

Event Calculus and the SEDC example

The EC representation of the SEDC knowledge consists of a number of clauses such as the following:

$\text{initiates}(\text{birth}(P), \text{ossification}(P, \text{knee-epiphyses}, \text{absent}), t) \Leftarrow$

$\text{happens}(\text{birth}(P), t) \wedge \text{holds}(\text{SEDC}(P), t)$

$\text{initiates}(\text{birth}(P), \text{stature}(P, \text{short}), t) \Leftarrow \text{happens}(\text{birth}(P), t) \wedge \text{holds}(\text{SEDC}(P), t)$

$\text{terminates}(\text{death}(P), \text{stature}(P, \text{short}), t) \Leftarrow \text{happens}(\text{death}(P), t) \wedge \text{holds}(\text{SEDC}(P), t)$

$\text{initiates}(\text{birth}(P), \text{coxa-vara}(P, \text{bilateral-severe}, \text{worsening}), t) \Leftarrow$

$\text{happens}(\text{birth}(P)) \wedge \text{holds}(\text{SEDC}(P), t)$

$\text{terminates}(\text{age15yrs}(P), \text{vertebral-bodies}(P, \text{pear-shaped}), t) \Leftarrow$

$\text{happens}(\text{age15yrs}(P), t) \wedge \text{holds}(\text{SEDC}(P), t)$

The Time Map Manager (TMM)

The temporal primitive of Dean and McDermott's TMM is the **point** (instant). The other temporal entity is the **time-token**, defined to be an interval together with a (fact or event) type.

Definition:

- A **time-token** is a static entity.
- A collection of time-tokens forms a **time map**. This is a graph in which nodes denote instants of time associated with the beginning and ending of events and arcs describe relations between pairs of instants.

TMM and the SEDC example

```
(time-token(SEDC, present) I)
(time-token(coxa-vara, bilateral-severe) C)
(time-token(coxa-vara, worsening) C')
(time-token(ossification, epiphyses, retarded) E)
(time-token(triradiate-cartilage, wide) W)
(time-token(vertebral-bodies, pear-shaped) V)
.....
(elt(distance(begin C) *ref*) 0, 0)
(elt(distance(end C) *ref*), *pos-inf* *pos-inf*)
(elt(distance(begin C') *ref*) ?, ?)
(elt(distance(end C') *ref*) ?, ?) (elt(distance(begin W) *ref*)0, 0)
(elt(distance(end W) *ref*) 10, 11)
(elt(distance(begin V) *ref*) 0, 0)
(elt(distance(end V) *ref*) ?, 14)
(elt(distance(begin E) *ref*) ?, ?)
(elt(distance(end E) *ref*) ?, ?)
```

.....

ref represents the birth year



Temporal Constraints

Temporal Constraints

- ❑ In the AI community there has been substantial interest in **networks of constraints**.
- ❑ In medical tasks such as clinical diagnosis, one needs to address more complex forms of **temporal constraints**.
- ❑ *Our approach*: representation of temporal constraints through an abstract structure.
 - An **Abstract Temporal Graph** (ATG) on one hand places the different types of constraints within the same, and thus unifying, framework and on the other hand enables the analysis and differentiation of the various types of constraints.

Reasoning with Temporal Constraints

Two kinds of problems:

- ❑ Checking the **consistency** of a set of constraints.
- ❑ Deciding the **satisfiability** of some constraint with respect to a set of constraints that are assumed to be mutually consistent.

Abstract Temporal Graph

Definition: An *Abstract Temporal Graph* (ATG) is a graph consisting of a finite set of nodes, n_1, n_2, \dots, n_m , denoting temporal entities (of the same type), and a finite set of directed arcs. A directed arc from n_i to n_j is labeled with a set of temporal constraints, $tc_{ij} \subseteq C$, denoting a disjunctive constraint from n_i to n_j . An ATG has access functions *match* and *propagate* for processing disjunctive constraints.

ATG Temporal Constraints

C is the domain of binary temporal constraints

- The elements of C are mutually exclusive
- C is either a finite or an infinite set

C Access Functions:

- $id : C \times C \rightarrow \{true, false\}$
- $inverse : C \rightarrow C$
- $transit : C \times C \rightarrow 2^C$

ATG Access Functions

● $match : 2^C \times 2^C \rightarrow 2^C$

```

match( $C_i, C_j$ )
   $R \leftarrow \{\}$ 
  for  $c_i \in C_i$  do
    for  $c_j \in C_j$  do
      if  $id(c_i, c_j)$  then  $R \leftarrow R \cup \{c_j\}$ 
  return  $R$ 
  
```

● $propagate : 2^C \times 2^C \rightarrow 2^C$

```

propagate( $C_i, C_j$ )
   $R \leftarrow \{\}$ 
  for  $c_i \in C_i$  do
    for  $c_j \in C_j$  do
       $R \leftarrow R \cup transit(c_i, c_j)$ 
  return  $R$ 
  
```

Fully Connected and Ordered ATG

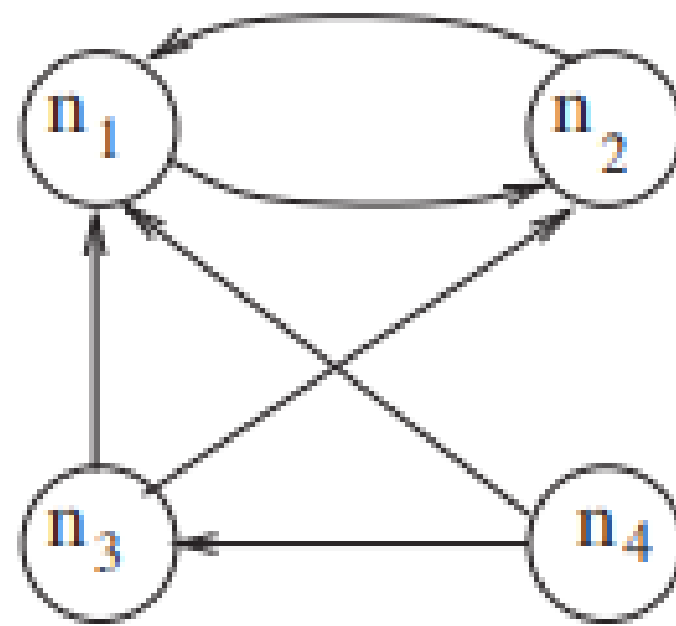
Definition

A *fully connected ATG* is an ATG for which every pair of nodes n_i and n_j such that $i \neq j$ is connected in both directions and each connection is labeled (possibly with the entire set of constraints, \mathcal{C}).

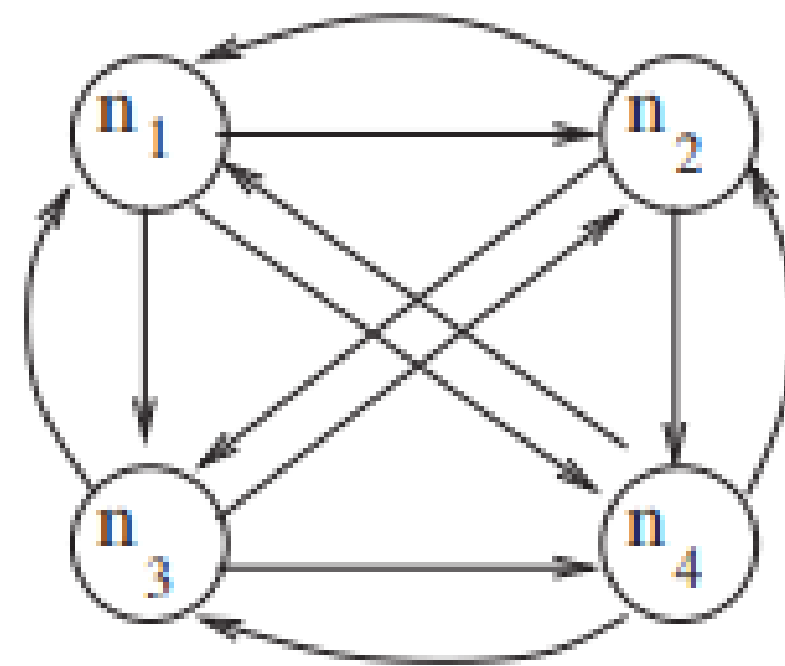
Definition

An *ordered ATG* is an ATG whose nodes n_1, n_2, \dots, n_m form a topological ordering and for every pair of nodes n_i and n_j such that $i < j$ (i.e. n_i precedes n_j in the topological ordering), there is a labeled connection from n_i to n_j . Pairs of nodes, n_i, n_j , such that $i \geq j$ are not connected. The $(m - 1)$ arcs connecting nodes, that are consecutive under the topological ordering, i.e. the arcs from n_i to n_{i+1} , for $i = 1, \dots, m - 1$, are referred to as *basic arcs* because each of them represents the sole path between the given pairs of nodes.

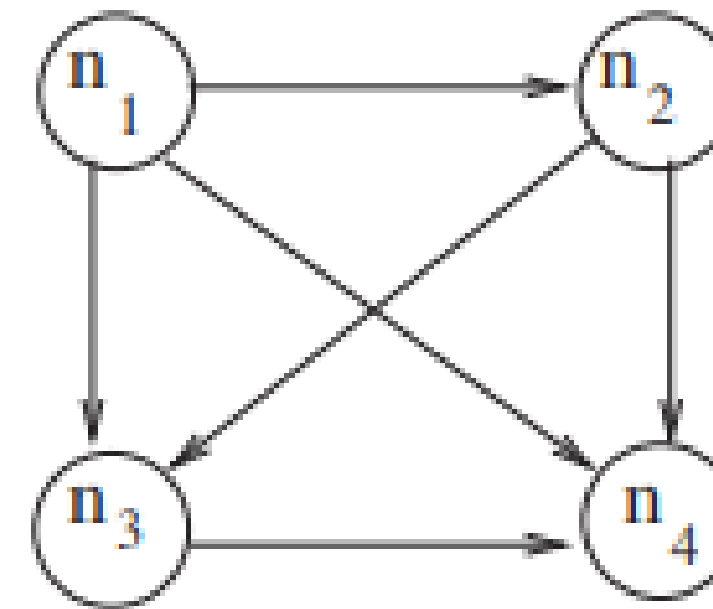
Abstract Temporal Graphs



(a) General ATG



(b) Fully Connected ATG



(c) Ordered ATG

Minimal ATGs

Definition:

A **minimal ATG** is an ordered ATG whose arcs have labels that cannot be further reduced.

Checking the consistency of a set of constraints

- ❑ Informally, checking the consistency of a general ATG is obtained by turning it into its **minimal form** by propagation and matching of constraints.
- ❑ If during the execution of these algorithms, function *match* returns an empty set denoting a complete mismatch, **a conflict is raised**.
 - Complete mismatch means that the disjunctive constraint relating two temporal entities, obtained via some route in the ATG, is in complete disagreement with the constraint, for the same pair of entities, obtained via another route in the ATG.
- ❑ The minimization algorithms detect the presence of some inconsistency but do not say which of the (original) constraints are responsible for it.

Deciding the satisfiability of some constraint

Deciding the satisfiability of some constraint with respect to a set of constraints that are assumed to be mutually consistent, is based on the assumption that the queried constraint and the ATG representing the set of constraints are of the same form.

- Let n_i and n_j be the temporal entities implicated in the queried constraint, and let qc be the (disjunctive) constraint itself (from n_i to n_j).
- The temporal entities, n_i and n_j , could respectively denote the start and end of some symptom, or the starts of two distinct symptoms, etc.

Deciding the satisfiability of some constraint

We distinguish the following cases:

- Both n_i and n_j appear as nodes in the ATG.
- Only one or none of these temporal entities appears as a node in the ATG.

In the first case, the solution is given as follows:

*convert the ATG to minimal form
if there is an arc from n_i to n_j in the ATG
then if $\text{match}(tc_{ij}, qc) \neq \{\}$
 then the queried constraint, qc , is satisfied
 else it is not satisfied
else if $\text{match}(tc_{ji}, qc^{-1}) \neq \{\}$
 then the queried constraint, qc , is satisfied
 else it is not satisfied*

Deciding the satisfiability of some constraint

In the second case:

- the liberal approach is to say that the queried constraint is satisfied by default (especially if the sentence denoted by the queried constraint expresses normality),
- the strict approach is to say that it is not satisfied (except again when the queried constraint expresses normality).

Summary

1. Modeling temporal concepts
2. Temporal reasoning
3. The medical domain and three well-known general theories of time
4. Temporal constraints