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University of Ruse “Angel Kanchev”

# MULTIAGENT SYSTEM WITH ARTIFICIAL INTELLIGENCE

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LECTURE 10

# Multiagent Systems Coalitions

1. Coalitions
2. Shapley
3. Coalition Games

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# Forming Coalitions

Coalitional games model scenarios where agents can benefit by cooperating. Sandholm (et. al., 1999) identified the following stages:

**Coalitional Structure Generation**

Deciding in principle who will work together. It asks the basic question:

*Which coalition should I join?*

The result: partitions agents into disjoint coalitions. The overall partition is a coalition structure.

**Solving the optimization problem of each coalition**

Deciding how to work together, and how to solve the "joint problem" of a coalition. It also involves finding how to maximise the utility of the coalition itself, and typically involves joint planning etc.

**Dividing the benefits**

Deciding "who gets what" in the payoff. Coalition members cannot ignore each other's preferences, because members can defect:

*…if you try to give me a bad payoff, I can always walk away…*

We might want to consider issues such as fairness of the distribution.



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# Characteristic Function Games

The objective is to connect a coalition that the agent cannot question to - this includes also calculating the characteristic function for different games.

Each coalition has its own payoff value, defined by the function

$\mathbf{v}(C) = k$ then the coalition will get the payoff $k$ if they cooperate

on some task

Sandholm (1999) proposed:

- If the game is **superadditive**:
  - if $\mathbf{v}(U) + \mathbf{v}(U) < \mathbf{v}(U \cup V)$
  - The coalition that maximises social welfare is the Grand Coalition
- If the game is **subadditive**:
  - if $\mathbf{v}(U) + \mathbf{v}(U) > \mathbf{v}(U \cup V)$
  - The coalitions that maximis social welfare are singletons
- However as some games are neither subadditive or superadditive:
  - the characteristic function value calculations need to be determined for each of the possible coalitions!
  - This is exponentially complex



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# Characteristic Function Games

- Accepting that we know the characteristic work and the payoff vector, what consolidation ought to an agent join?
- An outcome x for a coalition C in game ⟨Ag, **ν**⟩ is a vector of payoffs to members of C, such
- that x = ⟨x1, . . . , xk⟩ which represents an efficient distribution of payoff to members of Ag
- Where "efficient" means:
- Example: if **ν**({1, 2}) = 15, then possible outcomes are: ⟨15,0⟩, ⟨14,1⟩, ⟨13,2⟩ … ⟨1,14⟩, ⟨0,15⟩
- Thus, the agent should only join a coalition C which is:
    - *Feasible*: the coalition C really could obtain some payoff than an agent could not object to; and
    - *Efficient*: all of the payoff is allocated



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# Characteristic Function Games

- In any case, there may be numerous coalitions
- Each contains a diverse characteristic function
- Operators lean toward coalitions that are as beneficial as possible
- Hence a consolidation will as it were shape in the event that all the individuals lean toward to be in it
- I.e. they don’t imperfection to a more ideal fusion

- Therefore:
  - “which consolidation ought to I join?” can be decreased to “is the amalgamation stable?”
    - Is it levelheaded for all individuals of fusion C to remain with C, or seem they advantage by abandoning from it?
    - There's no point in me joining a amalgamation with you, unless you need to create one with me, and bad habit versa



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# Stability

- The concept of stability can be reduced to the concept of the core.
  - Stability is a necessary but not sufficient condition for coalition formation
  - i.e. unstable coalitions will never form, but stable coalitions are not guaranteed to form
- The core of a coalitional game is the set of feasible payoff distributions to coalition members that no sub-coalition can reasonably object to
  - Intuitively, a coalition C objects to an outcome if there is some other outcome that makes all of them strictly better off
- The idea is that an outcome will not happen if someone objects to it!
  - i.e. if the core is empty, then no coalition could form

# Sharing the Benefits of Cooperation

The Shapley value is best known attempt to define how to divide benefits of cooperation fairly.

- It does this by taking into account how much an agent contributes.
- The Shapley value of agent i is the average amount that i is expected to contribute to a coalition.
- The Shapley value is one that satisfies the axioms opposite!

**Symmetry**
Agents that make the same contribution should get the same payoff, i.e. the amount an agent gets should only depend on their contribution.

**Dummy Player**
These are agents that never have any synergy with any coalition, and thus only get what they can earn on their own.

**Additivity**
If two games are combined, the value an agent gets should be the sum of the values it gets in the individual games.



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# Shapley Axioms: Symmetry

- Agents that make the same contribution should get the same payoff
- The amount an agent gets should only depend on their contribution
- Agents i and j are interchangeable if their marginal contribution are the same for each coalition
- The symmetry axiom states:
- If i and j are interchangeable, then their Shapley value is equal



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# Shapley Axioms: Dummy Player

- Agents that never have any synergy with any coalition, and thus only get what they can earn on their own.
- An agent is a dummy player if he/she only adds to a coalition what it could get on its own

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# Shapley Axioms: Additivity

- If two games are combined, the value an agent gets should be the sum of the values it gets in the individual games
- I.e. an agent doesn't gain or loose by playing more than once



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# Shapley value

- Recall that we stated:
- The Shapley value for an agent is based on the marginal contribution of that agent to a coalition (for all permutations of coalitions)
- The marginal contribution can be dependent on the order in which an agent joins a coalition
- This is because an agent may have a larger contribution if it is the first to join, than if it is the last!
- For example, if Ag = {1,2,3} then the set of all possible orderings, is given as {(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)



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# Representing Coalitional Games

- It is important for an agent to know if the core of a coalition is non-empty
- Problem: a naive, obvious representation of a coalitional game is exponential in the size of Ag.
- Now such a representation is:
  - utterly infeasible in practice; and
  - so large that it renders comparisons to this input size meaningless
- An n-player game consists of 2n-1 coalitions
  - e.g. a 100-player game would require 1.2 x 1030 lines

```
% Representation of a Simple
 % Characteristic Function Game

 % List of Agents
1,2,3
% Characteristic Function
1 = 5
2 = 5
3 = 5
1,2 = 10
1,3 = 10
2,3 = 10
1,2,3 = 25
```

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# References

1. Michael Wooldridge, An Introduction to Multiagent Systems, 2009, Chapter 6
2. Russel, S. and Norvig, P. Artificial Intelligence: A Modern Approach, fourth edition, Pearson, 2022
3. David Poole, Alan Mackworth, Artificial Intelligence: Foundations of Computational Agents, second edition, Cambridge University Press 2017 (Available at https://artint.info/index.html)
4. T. Sandholm, K. Larson, M. Anderson, 0. Shehory and F. Tohme, "Coalition structure generation with worst case guarantees", Artifi. Intell., vol. 10, pp. 209-238, July 1999.
5. M. J. Wooldridge, S. Parsons and T.R. Payne, Lecture Notes in MAS, University of Liverpool, 2018
6. Ieong, Samuel & Shoham, Yoav. (2005). Marginal contribution nets: A compact representation scheme for coalitional games. Proceedings of the ACM Conference on Electronic Commerce. 193-202.



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