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Visual Computing Group

University of Cyprus – MSc Artificial Intelligence

MAI644 – COMPUTER VISION

Lecture 15: Camera Calibration

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Last time

- Perspective projection
- Vanishing points
- Full camera model

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Today’s Agenda

- Full camera model in matrix form
- Camera calibration
- Calibration – Projective camera model
- Calibration – Affine camera model

[material based on Paris Kaimakis]

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Remember - Full camera model

It consists of three transformations:

1. The **Euclidean** (**Rigid**) **transformation** between the camera and the world, i.e., the translation and rotation of the camera with respect the world origin – takes points from 3D world coordinates to 3D camera coordinates

2. The **perspective projection** onto the camera plane – takes points from 3D camera coordinates to 2D image coordinates

3. **CCD imaging**, i.e., the geometry of the CCD array (the size and shape of the pixels) and its position with respect to the optical axis – takes points from 2D image coordinates to 2D pixel coordinates

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Remember - Full camera model – Putting it all together

So here is the full camera model combining all three transformations

$$\left.\begin{aligned} u &= u_0 + k_u x \\ v &= v_0 + k_v y \end{aligned}\right\} \Rightarrow \left.\begin{aligned} u &= u_0 + k_u \frac{f}{Z_c} X_c \\ v &= v_0 + k_v \frac{f}{Z_c} Y_c \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} u &= u_0 + k_u f \frac{r_{11}X + r_{12}Y + r_{13}Z + T_x}{r_{31}X + r_{32}Y + r_{33}Z + T_z} \\ v &= v_0 + k_v f \frac{r_{21}X + r_{22}Y + r_{23}Z + T_y}{r_{31}X + r_{32}Y + r_{33}Z + T_z} \end{aligned}\right\}$$

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Full camera model in matrix form - Euclidean

- How can we express all three transformations in the form of matrices ?
  - Homogeneous coordinates
- Let's start from the Euclidean (rigid) transformation connecting the world and camera coordinate systems.
- This is composed of a rotation (3DOF) expressing the camera pose with respect to the world coordinate frame, and a translation (3DOF) expressing the camera location with respect to the world origin.

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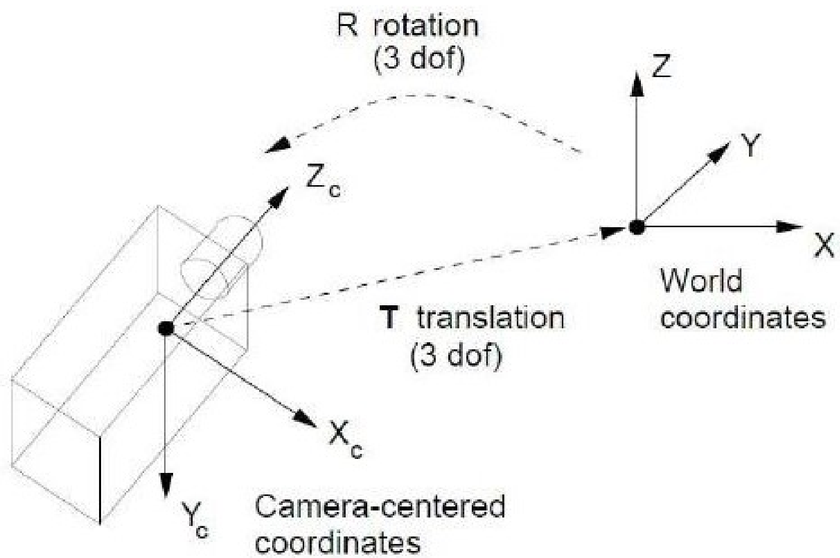
Full camera model in matrix form - Euclidean

- We want to express a ‘world’ 3D point **X**(X, Y, Z) as a 3D point $\mathbf{X_C}$ (X_C, Y_C, Z_C) in camera coordinates
- In homogeneous coordinates we have $\tilde{X}$(λX,λY,λZ,λ) and $\widetilde{X_C}$(λX_C,λY_C,λZ_C,λ) – λ can be set to 1 as it has no effect on $\mathbf{X_C}$, the cartesian equivalent of $\widetilde{X_C}$
- The Euclidean transformation can now be expressed as:

$$\begin{bmatrix} \lambda X_C \\ \lambda Y_C \\ \lambda Z_C \\ \lambda \end{bmatrix} = \begin{bmatrix} r11 & r12 & r13 & T_x \\ r21 & r22 & r23 & T_y \\ r31 & r32 & r33 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda X \\ \lambda Y \\ \lambda Z \\ \lambda \end{bmatrix}$$

- Or equivalently:

$$\tilde{\mathbf{X}}_c = \mathbf{P}_e \tilde{\mathbf{X}} \quad \text{where} \quad \mathbf{P}_e = \left[\begin{array}{ccc|c} & \mathbf{R} & & \mathbf{T} \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$



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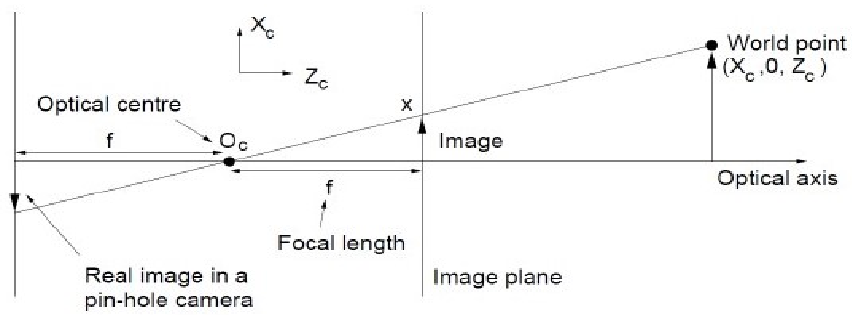
Full camera model in matrix form - Perspective

- We want to compute the projection of a 3D point $\mathbf{X_C}$ (X_C, Y_C, Z_C) expressed in camera coordinates onto a 2D point $\mathbf{x}(x,y)$ lying on the image plane
- In homogeneous coordinates we have $\widetilde{X_C}(\lambda X_C, \lambda Y_C, \lambda Z_C, \lambda)$ and $\tilde{x}(sx,sy,s)$ – again λ and s can be set to 1
- Perspective projection can now be expressed as:

$$\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda X_c \\ \lambda Y_c \\ \lambda Z_c \\ \lambda \end{bmatrix}$$

$$\left. \begin{aligned} x &= \frac{f}{Z_c} X_c \\ y &= \frac{f}{Z_c} Y_c \end{aligned} \right\}$$



- Or equivalently:

$$\tilde{\mathbf{x}} = \mathbf{P}_p \, \tilde{\mathbf{X}}_c$$

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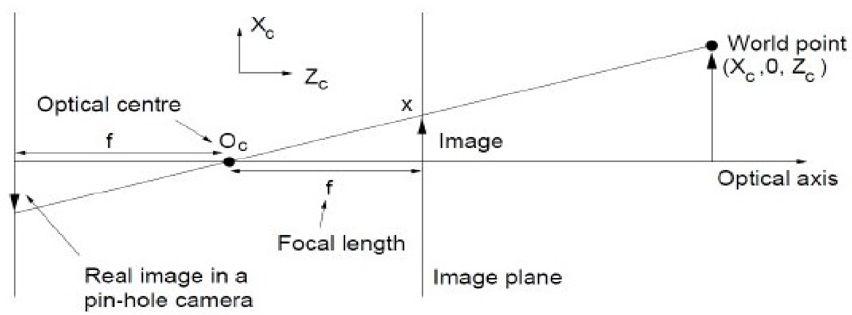
Full camera model in matrix form - Perspective

- We can confirm that the expression in the previous slide is equivalent to the perspective equation by recovering **x**, the cartesian equivalent of $\tilde{x}$

$$\tilde{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} f\lambda X_C \\ f\lambda Y_C \\ \lambda Z_C \end{bmatrix} \Rightarrow x = \begin{bmatrix} sx/s \\ sy/s \end{bmatrix} = \begin{bmatrix} f\lambda X_C/\lambda Z_C \\ f\lambda Y_C/\lambda Z_C \end{bmatrix} = \begin{bmatrix} fX_C/Z_C \\ fY_C/Z_C \end{bmatrix}$$

$$\left.\begin{aligned} x &= \frac{f}{Z_c} X_c \\ y &= \frac{f}{Z_c} Y_c \end{aligned}\right\}$$

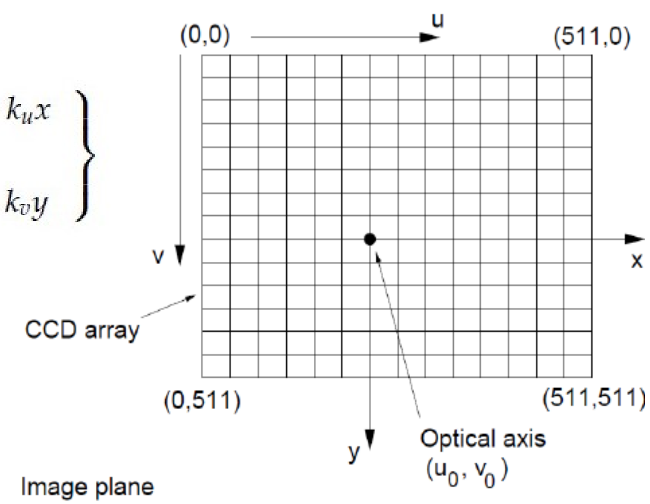


Full camera model in matrix form – CCD Imaging

- Last, wee want to express 2D point **x**(x,y) lying on the image plane in pixel coordinates **w**(u,v)
- In homogeneous coordinates we have $\tilde{x}$(sx,sy,s) and $\widetilde{w}$(su,su,s) – again s can be set to 1
- This is a translation and scaling which can be expressed as:

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx \\ sy \\ s \end{bmatrix}$$

$$\left.\begin{aligned} u &= u_0 + k_u x \\ v &= v_0 + k_v y \end{aligned}\right\}$$



- Or equivalently:

$$\tilde{\mathbf{w}} = \mathbf{P}_c \tilde{\mathbf{x}}$$

Full camera model in matrix form – all together

We can now express the whole process, from $\tilde{X}$ to $\tilde{w}$ as a single transformation P_{ps}

$$
\begin{aligned}
\tilde{\mathbf{w}} &= \mathbf{P}_{ps}\,\tilde{\mathbf{X}} \\
\text{where}\quad \mathbf{P}_{ps} &= \mathbf{P}_c\,\mathbf{P}_p\,\mathbf{P}_e \\
&= \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \left[\begin{array}{ccc|c} & \mathbf{R} & & \mathbf{T} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
\end{aligned}
$$

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Full camera model in matrix form – all together

- The transformation P_{ps} is not a general 3x4 matrix, because it has a special structure composed of P_e, P_p,and P_c.
- We can simplify P_{ps} into an upper-triangular matrix K composed of P_e and P_p, and a matrix representing the Euclidean transformation.

$$\mathbf{P}_{ps} = \mathbf{K}\,[\mathbf{R}\,|\,\mathbf{T}\,]$$

$$= \begin{bmatrix} m_u & 0 & u_0 & 0 \\ 0 & m_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r11 & r12 & r13 & T_x \\ r21 & r22 & r23 & T_y \\ r31 & r32 & r33 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$m_u = k_u\, f$$
$$m_v = k_v\, f$$

Full camera model in matrix form – all together

- The matrix **K** is called the camera calibration matrix and it contains all the viewing parameters (**intrinsics**) coming from inside the camera.
- The Euclidean transformation matrix contains all the viewing parameters (**extrinsics**) that are not controlled by the camera lens or sensor.

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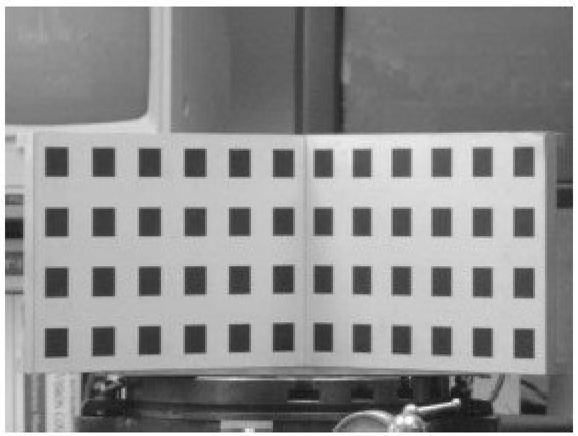
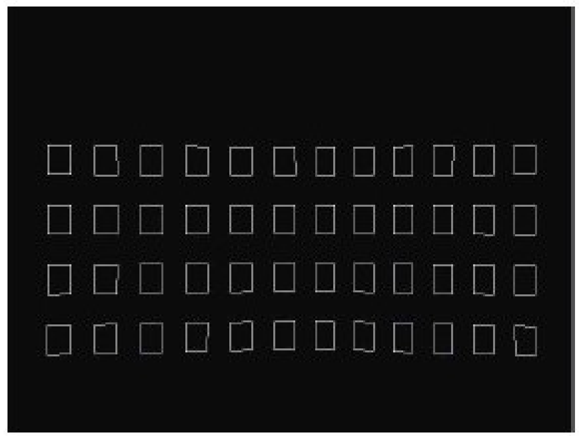
Camera calibration

- Camera calibration is the name given to the process of discovering the parameters inside our camera model, i.e., the values inside the camera projection matrix.
- This is done by using an image of a controlled scene.
- We may want to use a scene with some sort of regular pattern.





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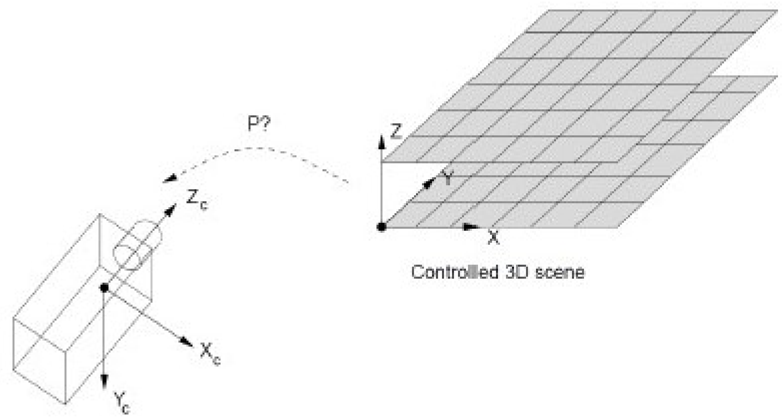

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Camera calibration – the projective camera

Remember that the **perspective** camera projection matrix P_{ps} is not a general 3x4 matrix, as it has a special structure composed of P_e, P_p,and P_c.

$$\tilde{\mathbf{w}} = \mathbf{P}_{ps}\tilde{\mathbf{X}}$$

$$\text{where } \mathbf{P}_{ps} = \mathbf{P}_c\,\mathbf{P}_p\,\mathbf{P}_e$$

$$= \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \left[\begin{array}{ccc|c} & \mathbf{R} & & \mathbf{T} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Camera calibration – the projective camera

- Calibrating the perspective camera model may therefore be difficult. We can instead use the projective camera model which is described by a general 3x4 matrix.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

- The projective camera has **11 degrees of freedom**, since the overall scale of P does not matter when using homogeneous coordinates.
- It is more convenient to deal with a projective camera than a perspective one, since we don't have to worry about placing nonlinear constraints on the elements of P

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Camera calibration – the projective camera

- Using a projective camera, the whole imaging process is described by

$$\tilde{\mathbf{w}} = \mathbf{P}\tilde{\mathbf{X}}$$

$$\Rightarrow \begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- We must estimate 11 parameters (the overall scale does not matter, so let's set $p_{34} = 1$

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Camera calibration – the projective camera

- Because the scene is controlled, we **know** the location of the world points $\mathbf{X}_i$
- We can also find the location of their projections $\mathbf{w}_i$ on the image, e.g., using corner detection.
- Each point we observe gives us two equations

$$u_i = \frac{su_i}{s} = \frac{p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14}}{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + 1}$$

$$v_i = \frac{sv_i}{s} = \frac{p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24}}{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + 1}$$

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Camera calibration – the projective camera

Rearranging them gives us two linear equations in the unknown parameters of the projection matrix

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & u_iX_i & u_iY_i & u_iZ_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & v_iX_i & v_iY_i & v_iZ_i \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \end{bmatrix} = \begin{bmatrix} -u_i \\ -v_i \end{bmatrix}$$

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Camera calibration – the projective camera

- As there are 11 unknowns, we need at least 6 points to calibrate this camera model to get enough equations (rows) in the linear system Ax=b.
- Each observed points adds two equations (rows) to the matrix A
- We can solve the system of equations using linear least squares (pseudo-inverse of A)

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{b} \\ \Rightarrow \quad \mathbf{x} &= (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \end{aligned}$$

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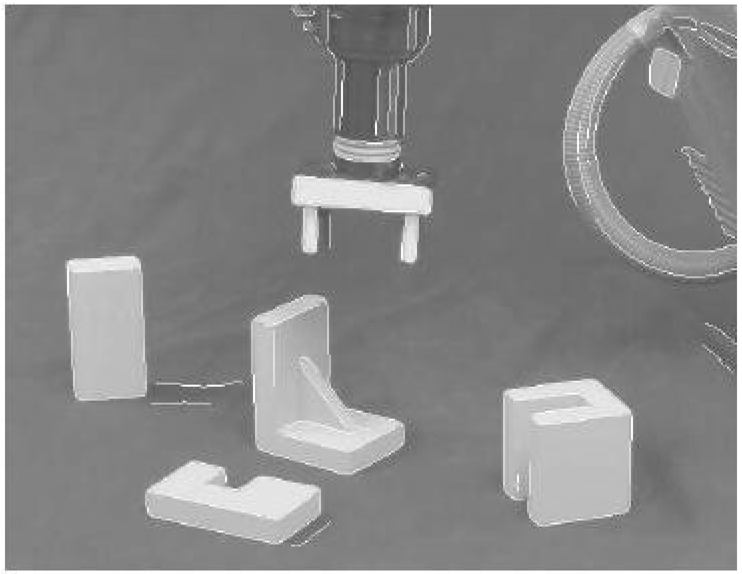
Parallel Projection

When the depth difference ΔZ_C of objects in the scene is small compared to their distance Z_C to the camera, the resulting image is not described well by perspective projection.



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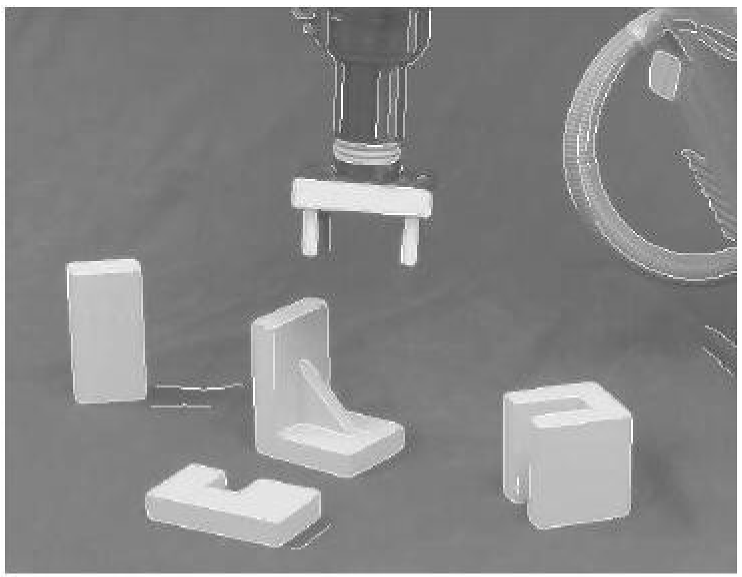
Parallel Projection

For example, parallel lines on the left image remain parallel after projection. In this case, the 3D to 2D projection is better described by **parallel projection**.



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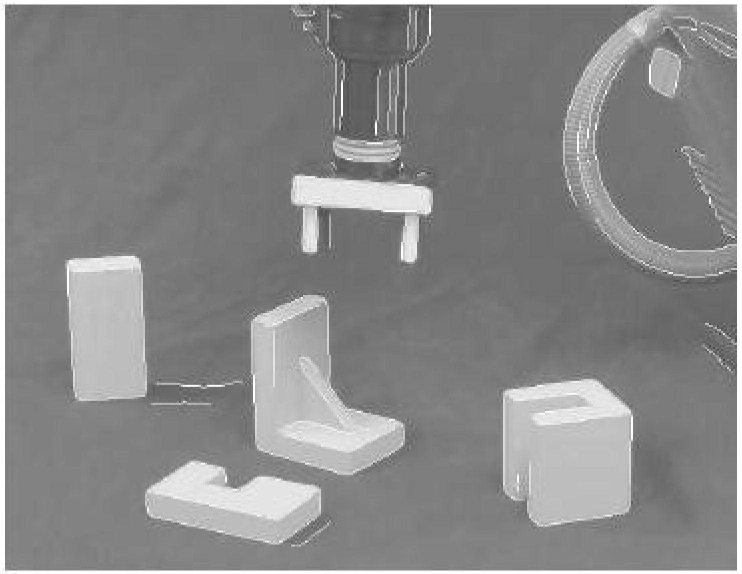
Parallel Projection

A camera which creates images like the one on the left is known as a **weak-perspective camera.**



Parallel Projection

What changes in our imaging transformations is the perspective projection matrix. Remember P_p:

$$\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

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Parallel Projection

What changes in our imaging transformations is the perspective projection matrix.

If the depth variation in the scene is small, then $Z_C \approx Z_C^{avg}$ and the perspective projection matrix P_p can be changed to the parallel projection matrix P_{pll}

$$\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & Z_c^{\text{avg}} \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{x}} = \mathbf{P}_{pll} \tilde{\mathbf{X}}_c$$

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Weak-Perspective Camera Model

This leads to the **weak perspective** camera model:

$$\tilde{\mathbf{w}} = \mathbf{P}_{wp}\,\tilde{\mathbf{X}}$$

where

$$\begin{aligned}\mathbf{P}_{wp} &= \mathbf{P}_c\,\mathbf{P}_{pll}\,\mathbf{P}_e \\ &= \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & Z_c^{\text{avg}} \end{bmatrix} \left[\begin{array}{ccc|c} & \mathbf{R} & & \mathbf{T} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]\end{aligned}$$

Weak-Perspective Camera Model

P_{wp} is the projection matrix for a weak-perspective camera. It is a 3x4 matrix with a special structure composed of P_e, P_{pll}, and P_c.

$$\tilde{\mathbf{w}} = \mathbf{P}_{wp}\,\tilde{\mathbf{X}}$$

where

$$\begin{aligned}\mathbf{P}_{wp} &= \mathbf{P}_c\,\mathbf{P}_{pll}\,\mathbf{P}_e \\ &= \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & Z_c^{\text{avg}} \end{bmatrix} \left[\begin{array}{ccc|c} & \mathbf{R} & & \mathbf{T} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]\end{aligned}$$

Camera calibration – the affine camera

- The special structure of the weak-perspective camera model makes it difficult to calibrate. This is why the affine camera model is often used instead:

$$\mathbf{P}_{aff} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 0 & p_{34} \end{bmatrix}$$

- P_{aff} is the projection matrix for the affine camera. It has 8 degrees of freedom (remember the overall scale does not matter so we can set p_{34} to 1).
- It can be calibrated in the same way as the projective camera. There are eight degrees of freedom, so we need a minimum of 4 points.

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