



University of Cyprus – MSc Artificial Intelligence

# MAI644 – COMPUTER VISION

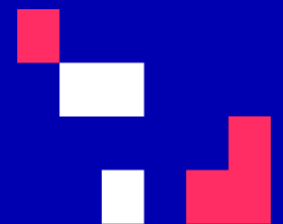
## Lecture 5: Filters – Convolution

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# Last time

- Image basics
  - What is an image – addressing pixels
  - Image as a function – image coordinates
- Image interpolation
  - Nearest neighbor
  - Bilinear
  - Bicubic
- Image resizing
  - Enlarge
  - Shrink

# Today's Agenda

- Averaging vs Interpolation
- Systems - filters
- Convolution
  - Box Filter
  - Gaussian
  - Cross correlation vs Convolution
- Examples of filters

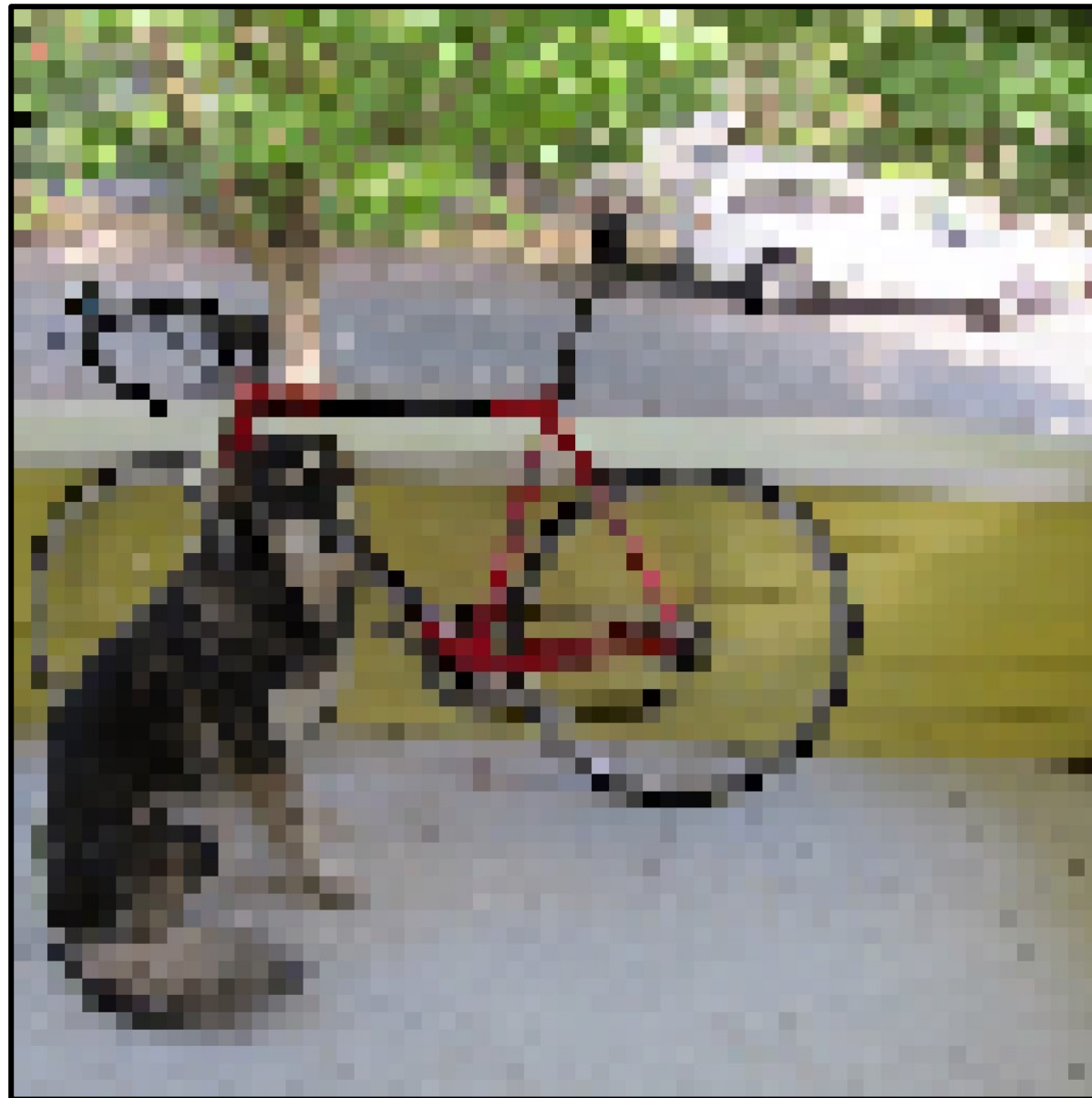
**[material based on Joseph Redmon's course]**

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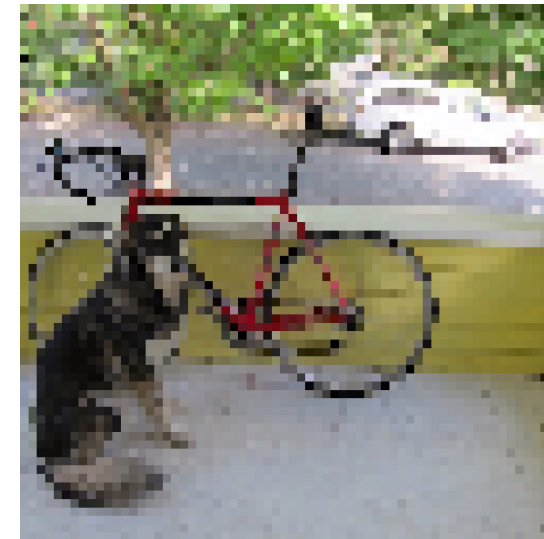


## Is this all there is ??



## Lots of issues

- NN and Bilinear only look at small area
- Lots of artifacting
- Staircase pattern on diagonal lines
- We'll fix this with filters!

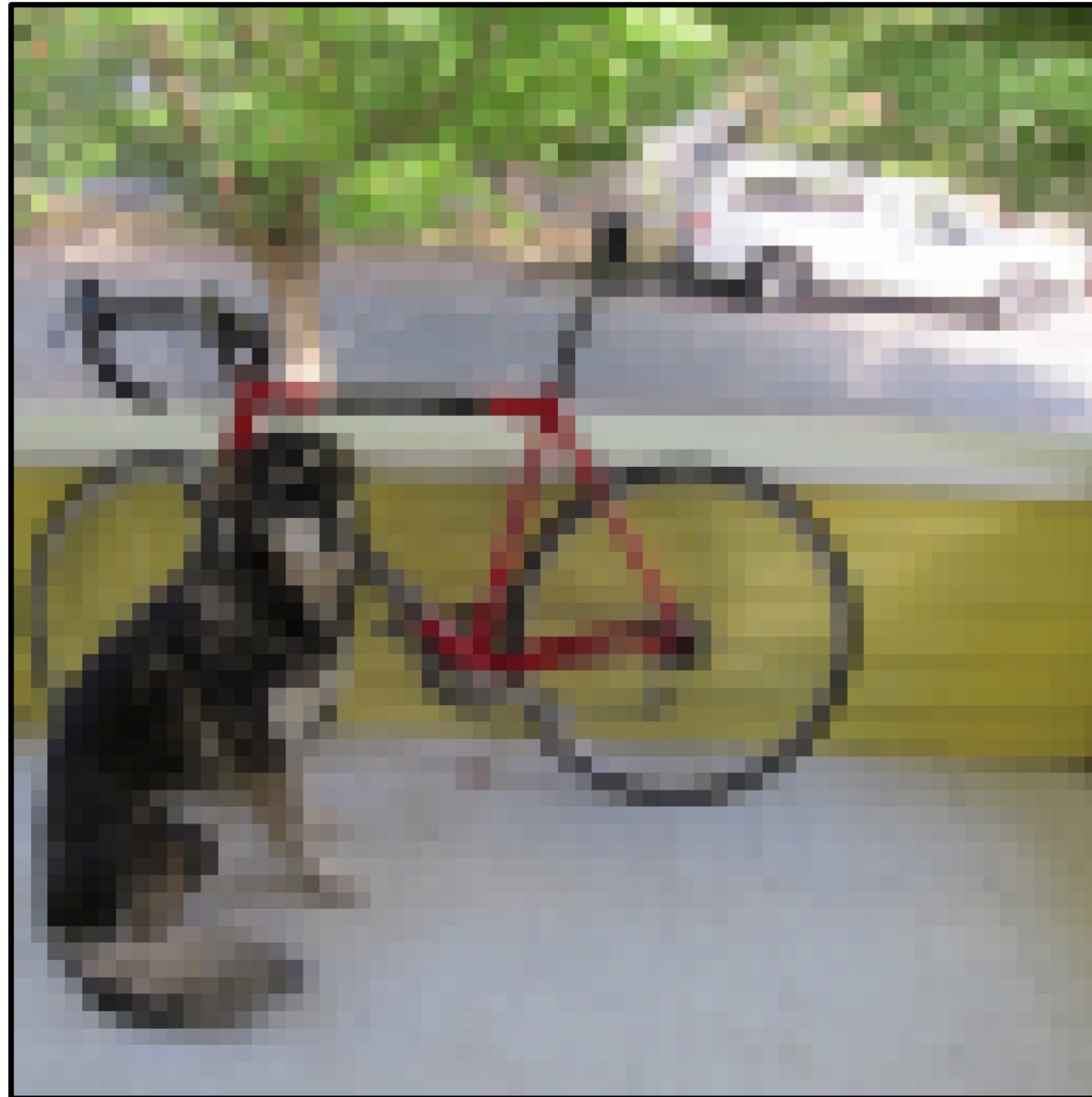


NN



Bilinear

# There is a better way!



## Look at how much better

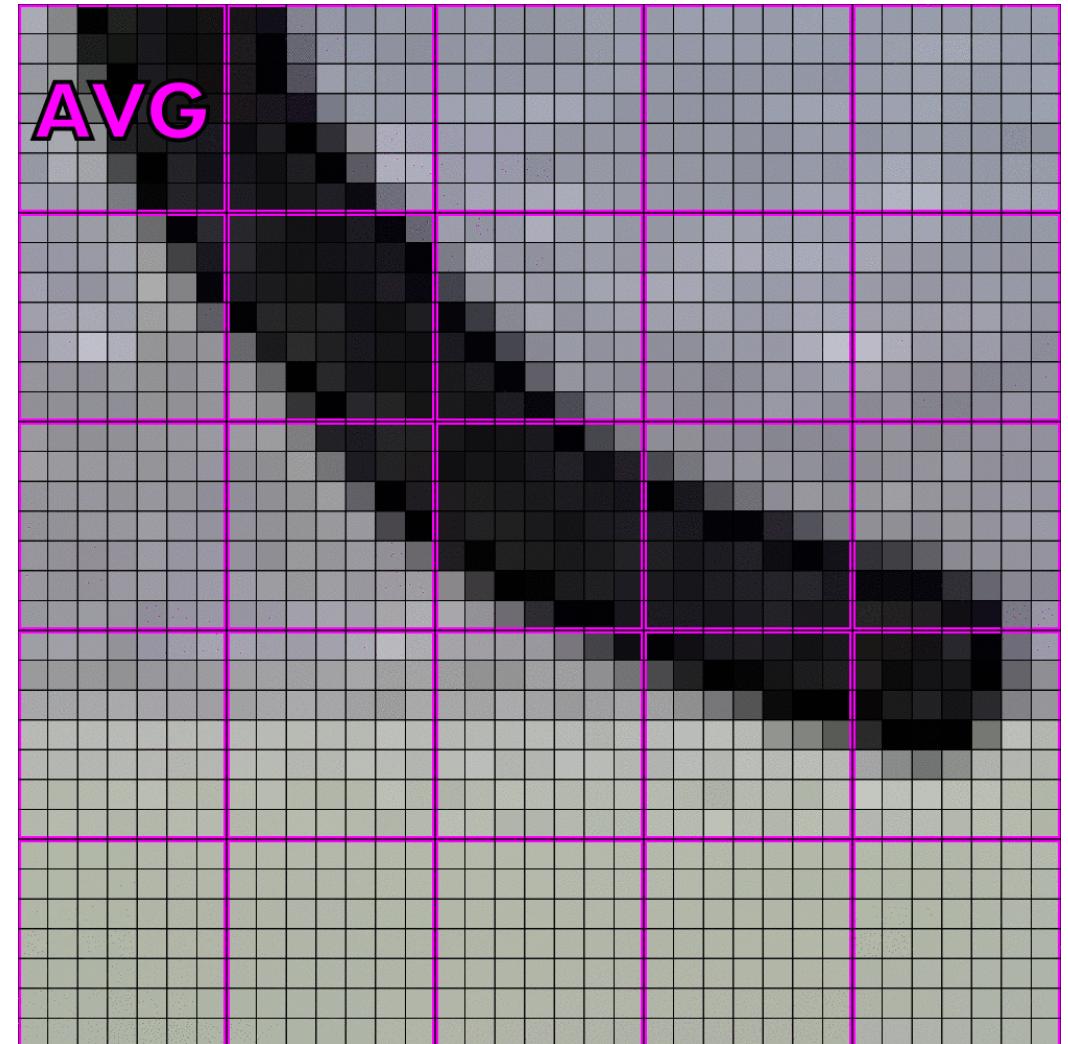
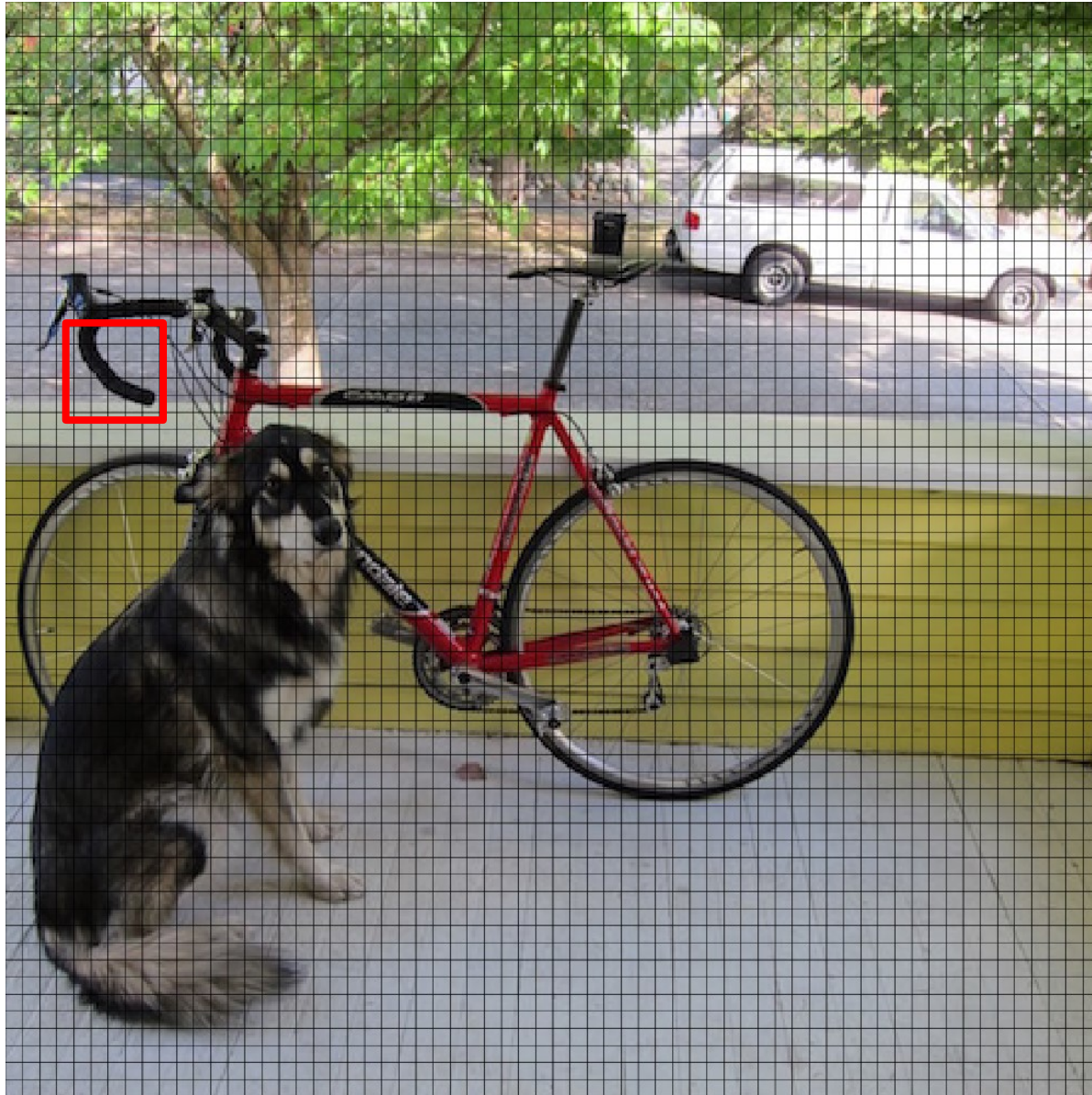




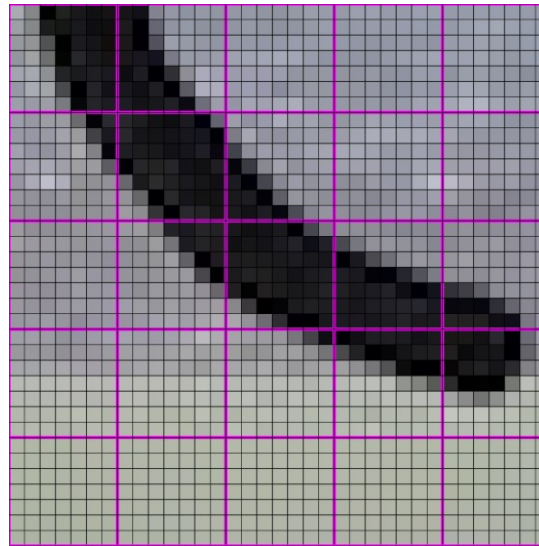
## How?



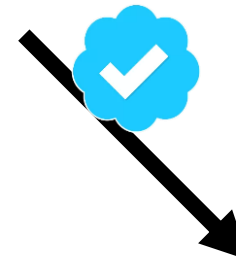
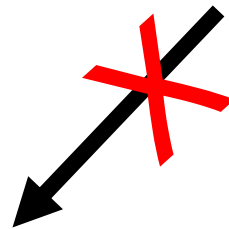
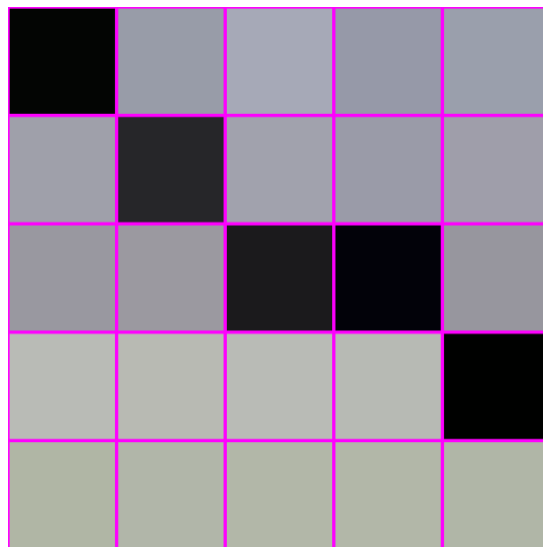
## How? Averaging!



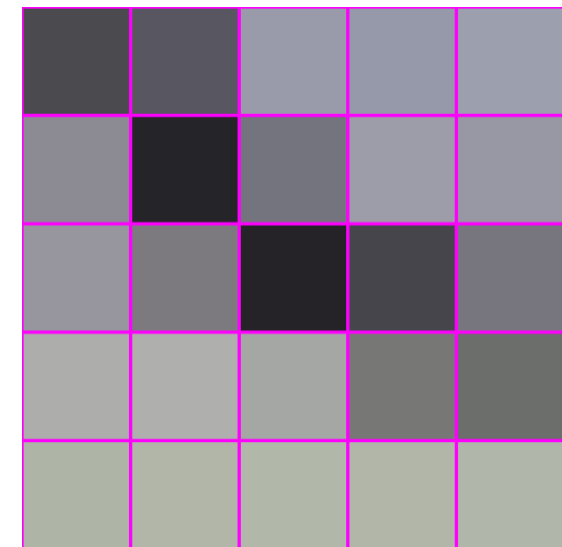
# How? Averaging!



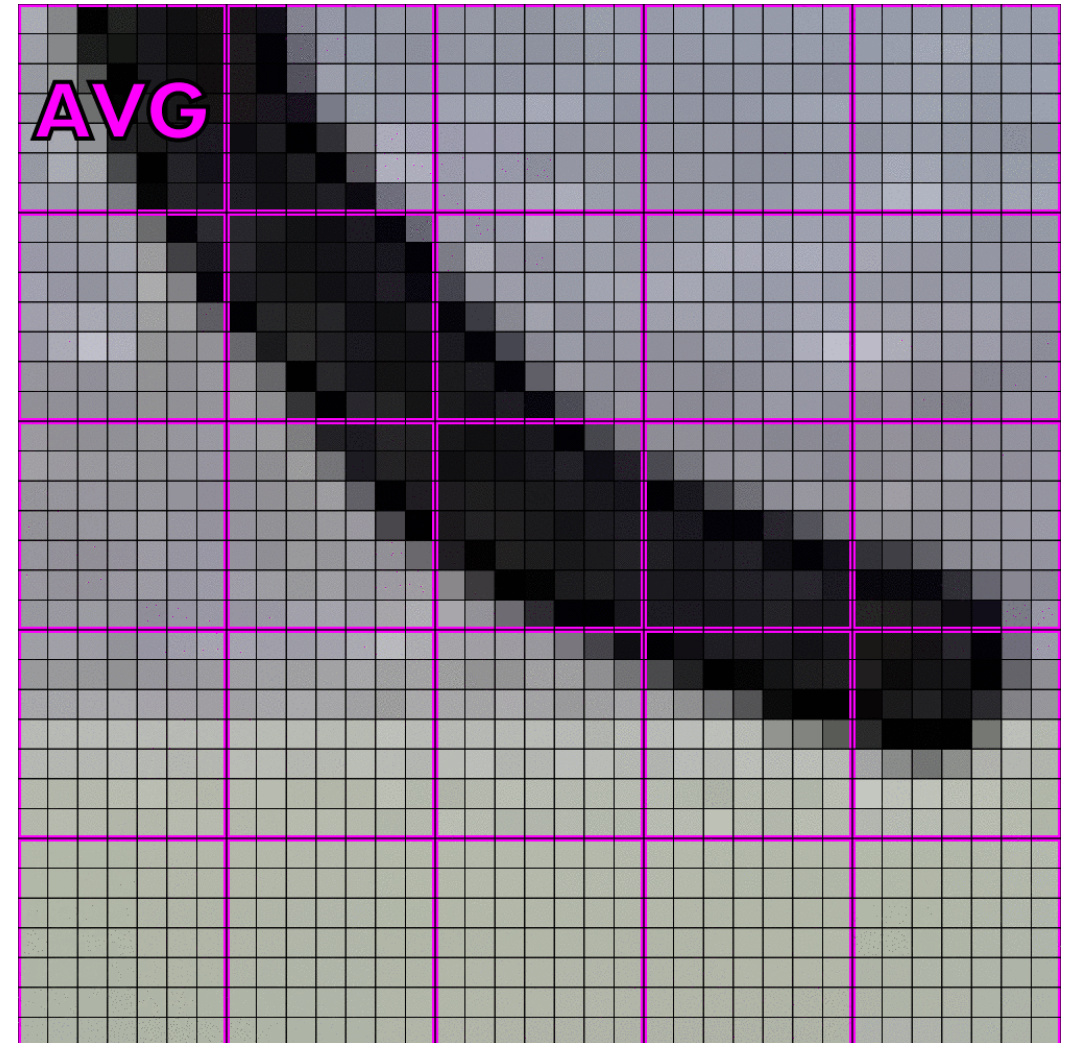
“interpolation”



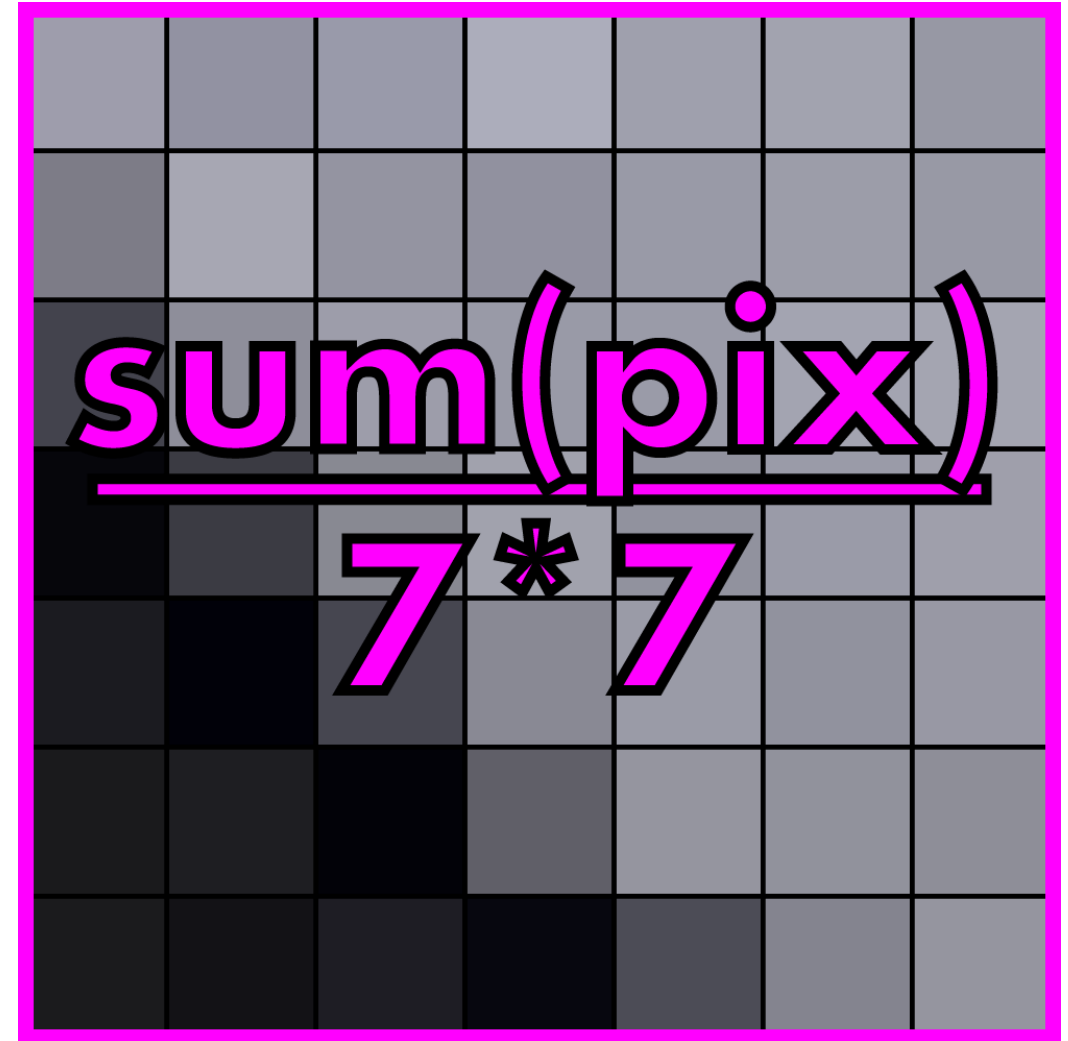
averaging



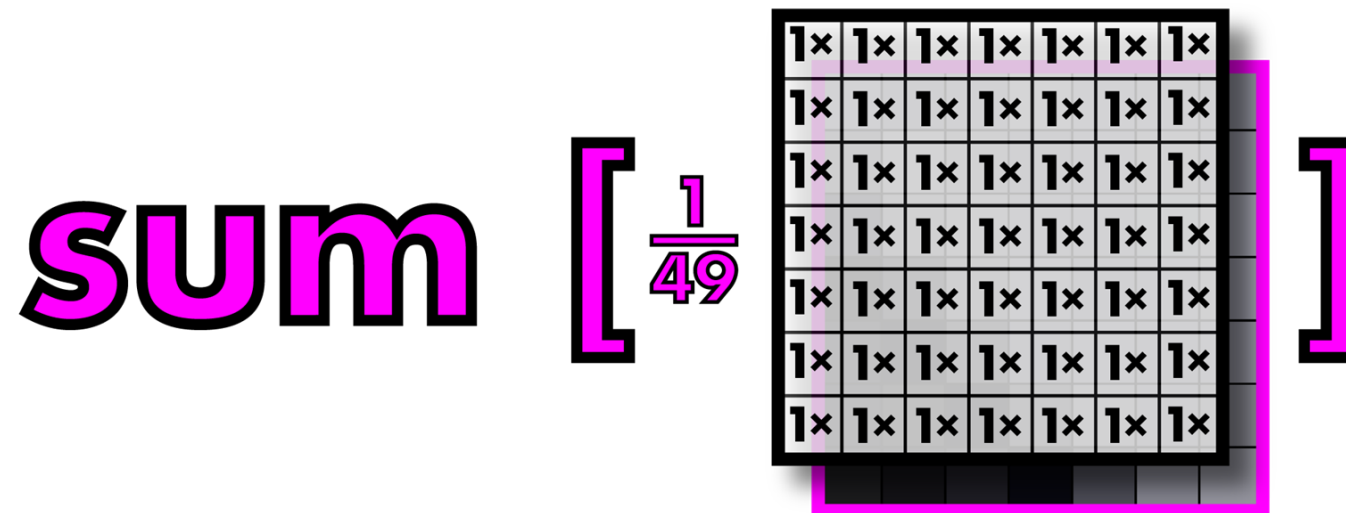
## What is averaging?



# What is averaging? A weighted sum



# What is averaging? A weighted sum



What are the weights here ?



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# Moving average is a filter

*Filter or kernel*

$$\frac{1}{49}$$

1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×
1×	1×	1×	1×	1×	1×	1×





# Filtering

- Filtering
  - Forming a new image whose pixel values are transformed from original pixel values
- Goal is to extract useful information from images, or transform images into another domain where we can modify/enhance image properties
  - Features (edges, corners, blobs...)
  - Applications: super-resolution (resizing); in-painting; de-noising;

[Slide by Niebles]

# Applications

### De-noising



Salt and pepper noise

### Super-resolution



### In-painting



Bertamio et al

[Slide by Niebles]

# Systems

- We define a **system** as a unit that converts an input function  $f[x,y]$  into an output (or response) function  $g[x,y]$ , where  $(x,y)$  are the independent variables.
  - In the case of images,  $(x,y)$  represents the **spatial position in the image**.

[Slide by Niebles]

# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$


[Slide by Seitz]

# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10							

[Slide by Seitz]

# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10	20						

[Slide by Seitz]

# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10	20	30					

[Slide by Seitz]



# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10	20	30	30				

[Slide by Seitz]



# Moving average - example

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

[Slide by Seitz]

# Properties of systems

- Amplitude properties

- Additivity  $S[f_i[n, m] + f_j[n, m]] = S[f_i[n, m]] + S[f_j[n, m]]$

- Homogeneity  $S[\alpha f_i[n, m]] = \alpha S[f_i[n, m]]$

- Superposition  $S[\alpha f_i[n, m] + \beta f_j[n, m]] = \alpha S[f_i[n, m]] + \beta S[f_j[n, m]]$

- Stability  $|f[n, m]| \leq k \implies |g[n, m]| \leq ck$

- Invertibility  $S^{-1}[S[f_i[n, m]]] = f_i[n, m]$

[Slide by Niebles]

# Properties of systems

- Spatial properties

- Causality

for  $n < n_0, m < m_0$ , if  $f[n, m] = 0 \implies g[n, m] = 0$

- Shift invariance

$f[n - n_0, m - m_0] \xrightarrow{\mathcal{S}} g[n - n_0, m - m_0]$

[Slide by Niebles]

# Linear Systems - filters

- Linear filtering
  - Form a new image whose pixels are a weighted sum of original pixel values
  - Use the same set of weights at each point
- $S$  is a linear system (function) iff  $S$  satisfies

$$S[\alpha f_i[n, m] + \beta f_j[h, m]] = \alpha S[f_i[n, m]] + \beta S[f_j[h, m]]$$

superposition property

[Slide by Niebles]

# Linear Shift Invariant Systems

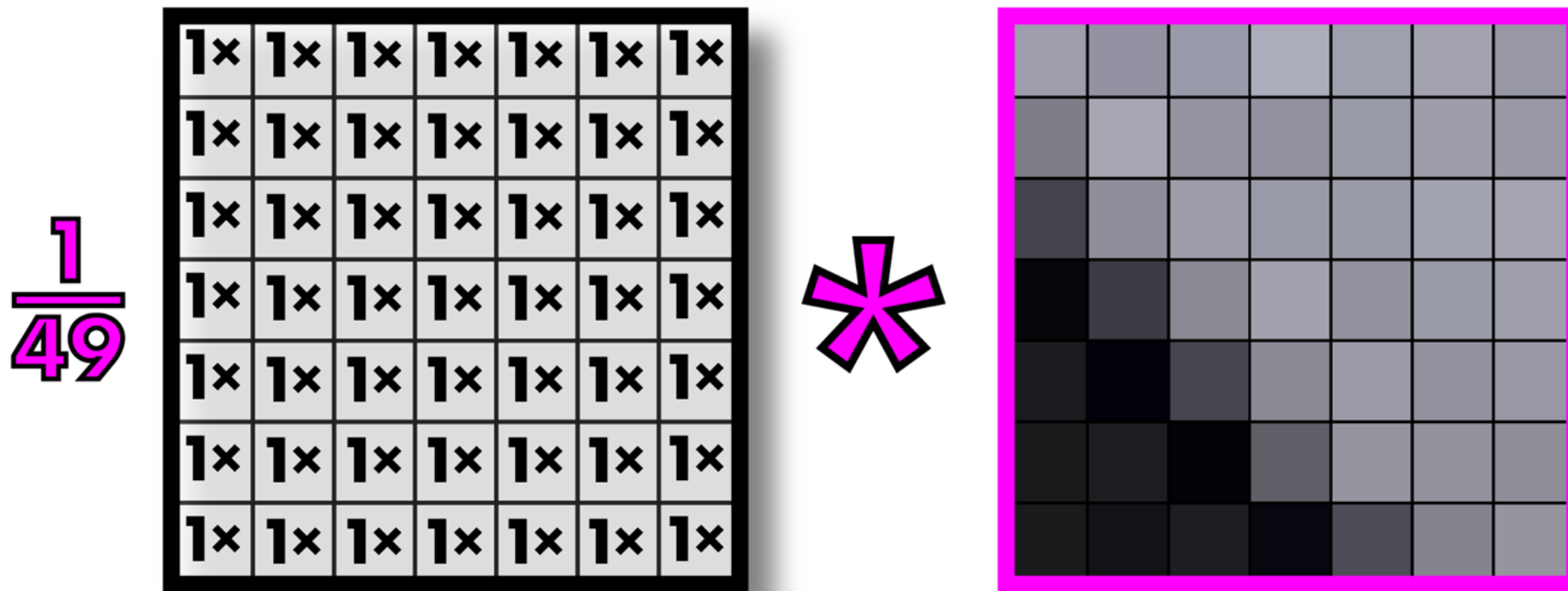
- We call systems which satisfy the superposition and shift-invariant property *Linear Shift Invariant Systems* (LSI)
- Not all filters are LSI
  - Is thresholding linear? 
$$g[n, m] = \begin{cases} 1, & f[n, m] > 100 \\ 0, & \text{otherwise.} \end{cases}$$
  - Consider:
    - $f_1[n, m] + f_2[n, m] > T$
    - $f_1[n, m] < T$
    - $f_2[n, m] < T$
- LSI systems can be described by the **convolution** operation

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Call this operation “*convolution*”

*Filter or kernel*



Note: multiplying an image section by a filter is actually called “correlation” and convolution involves inverting the filter first

# Convolutions on larger images

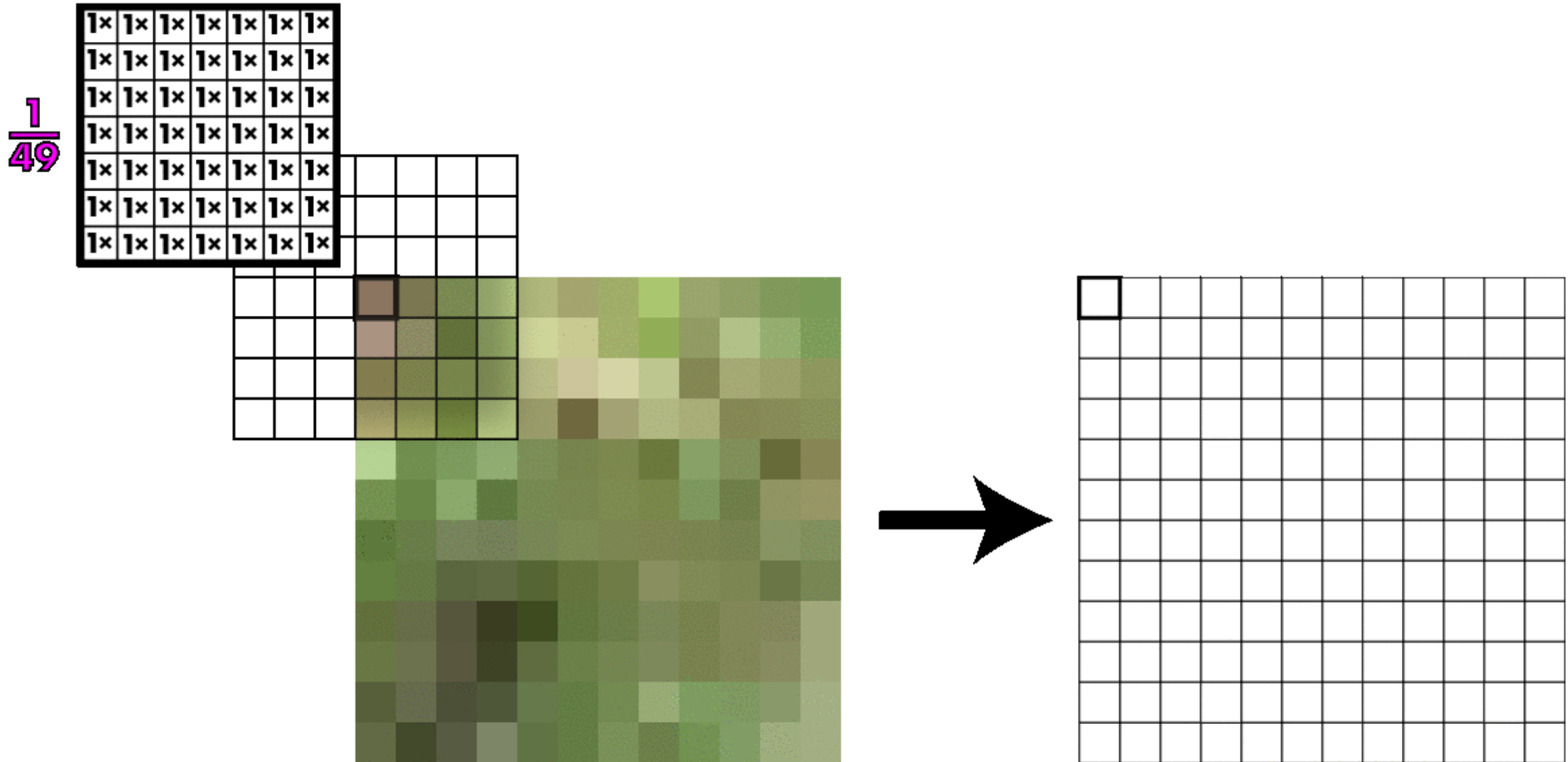
$\frac{1}{49}$

1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x
1x	1x	1x	1x	1x	1x	1x

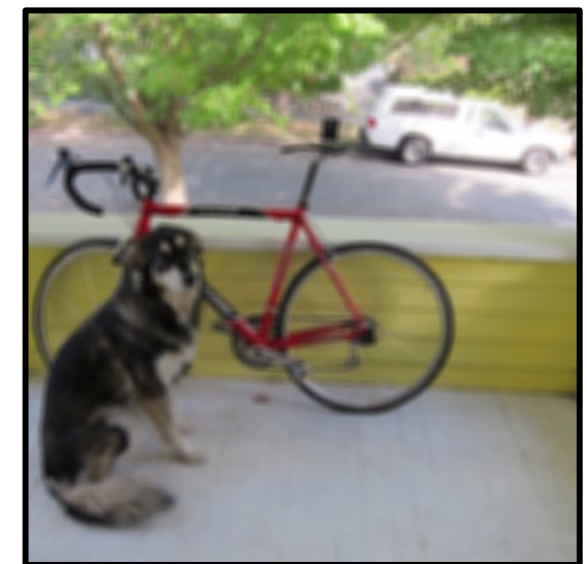
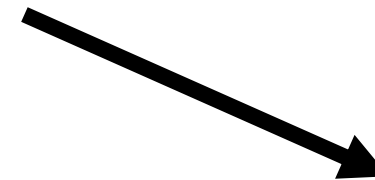
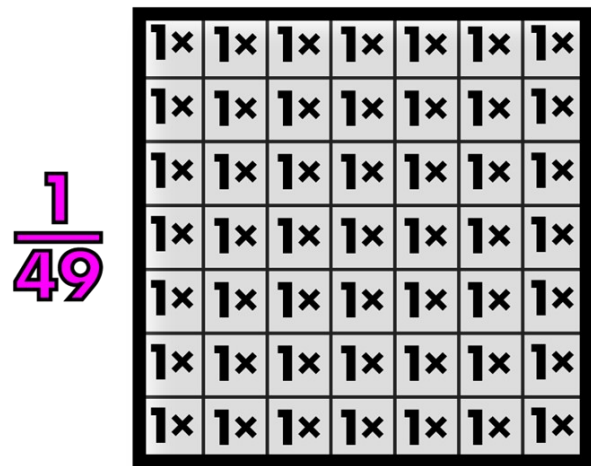




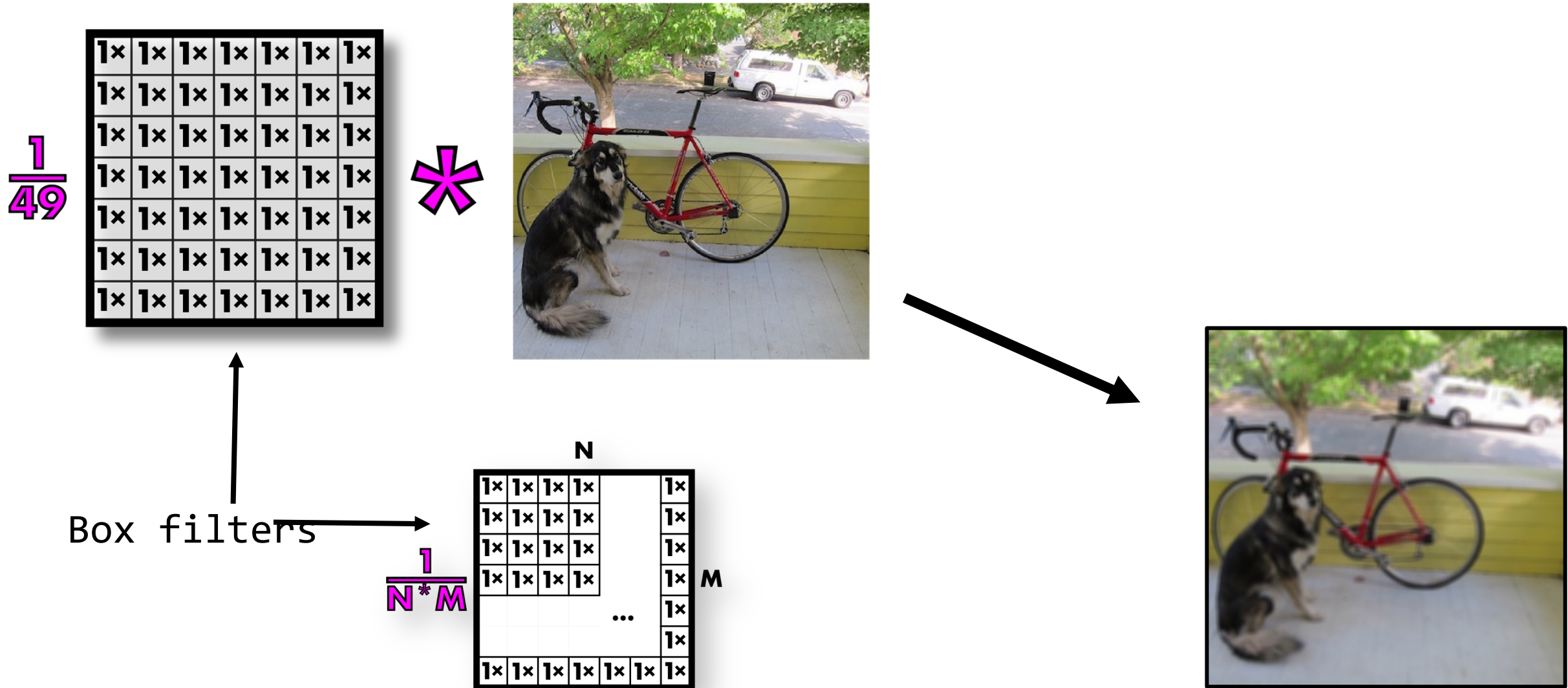
## Kernel slides across image



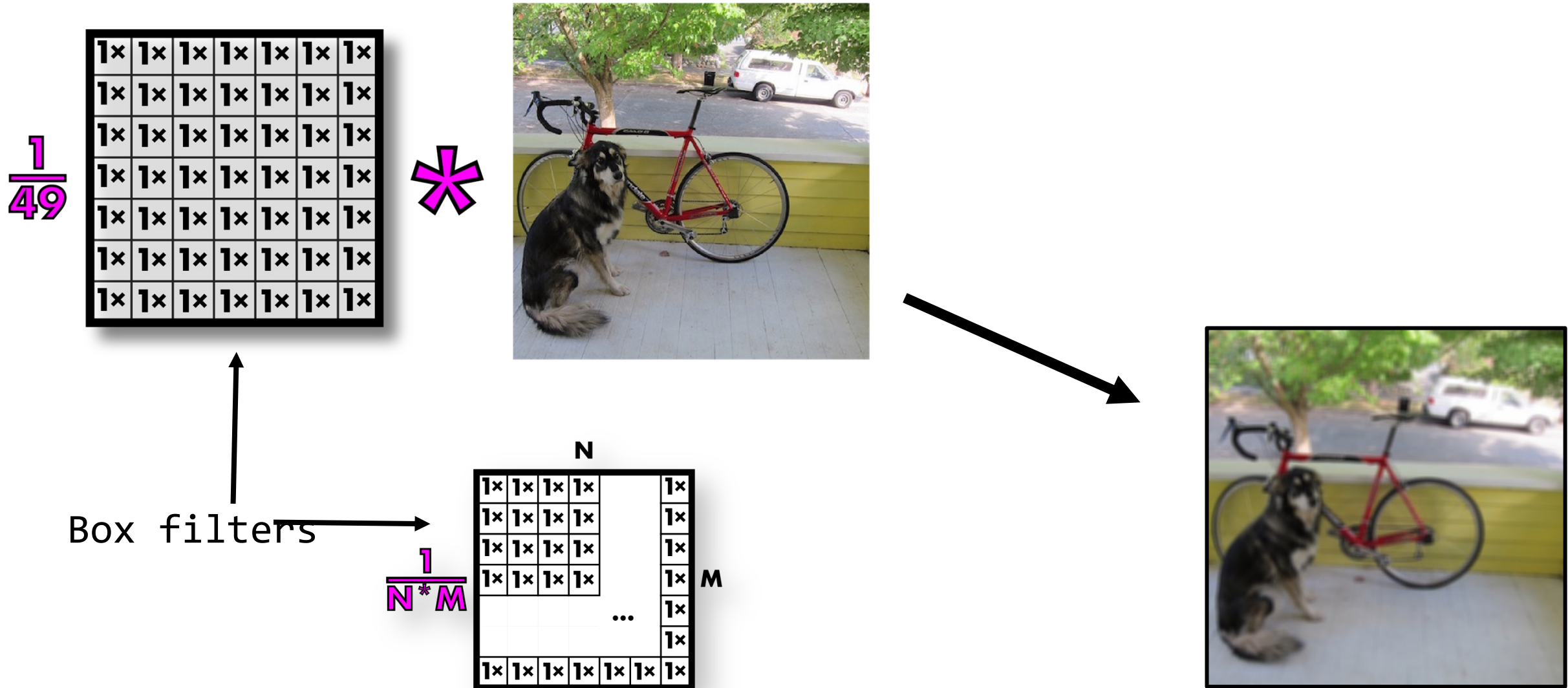
# Convolutions on larger images



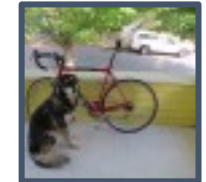
# This is called box filter



# Box filters smooth image

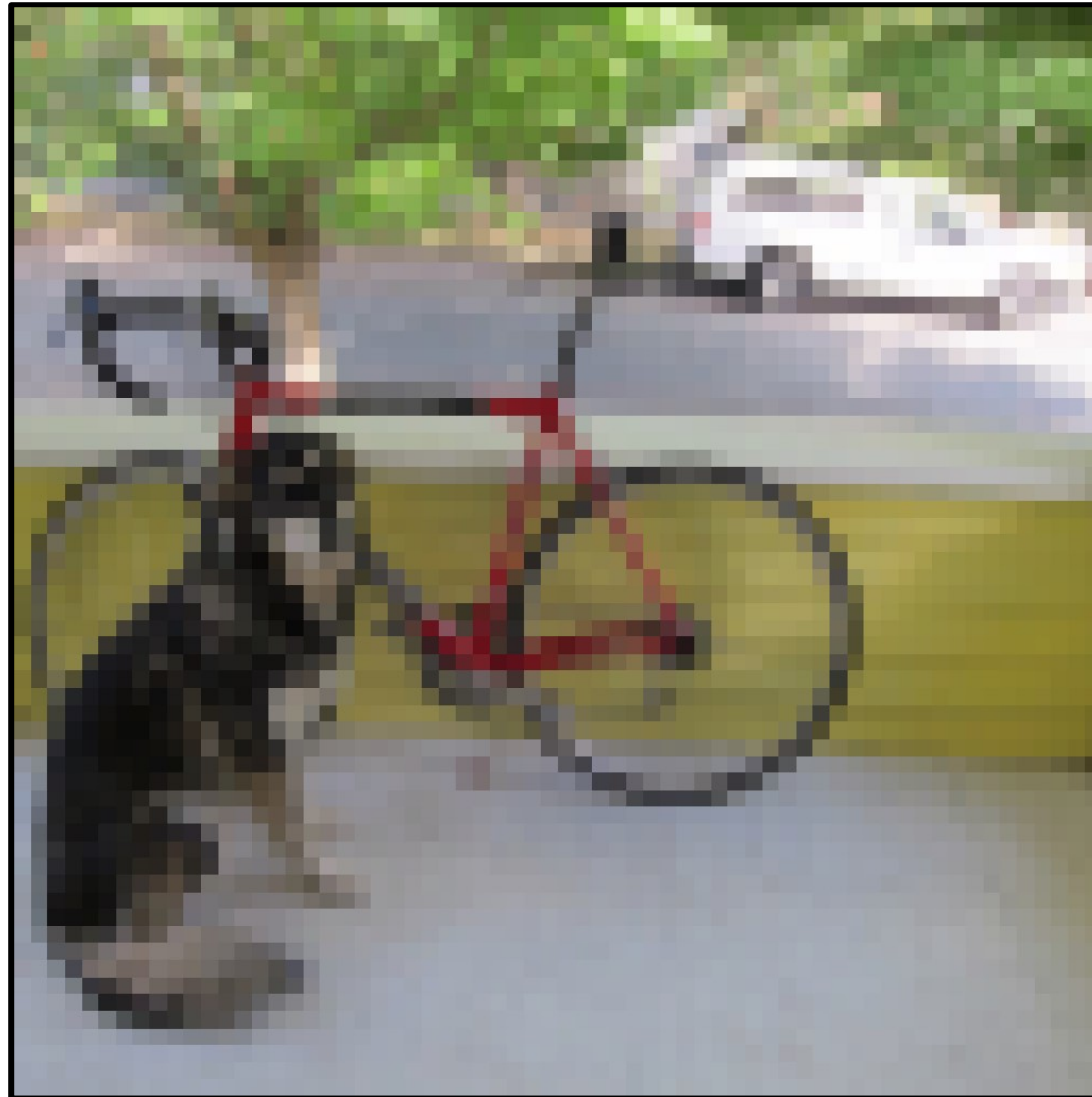


## Now we resize our smoothed image

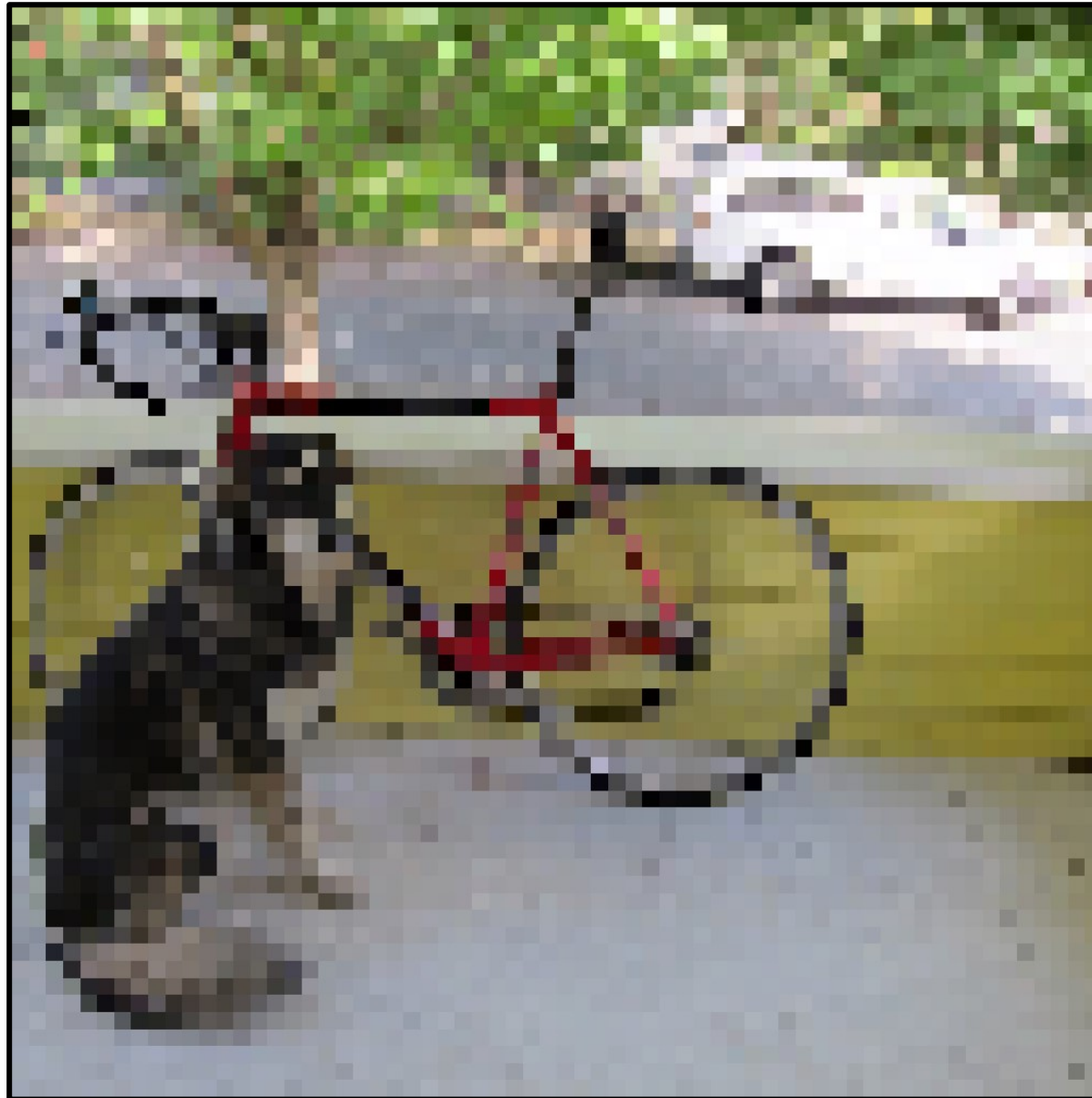




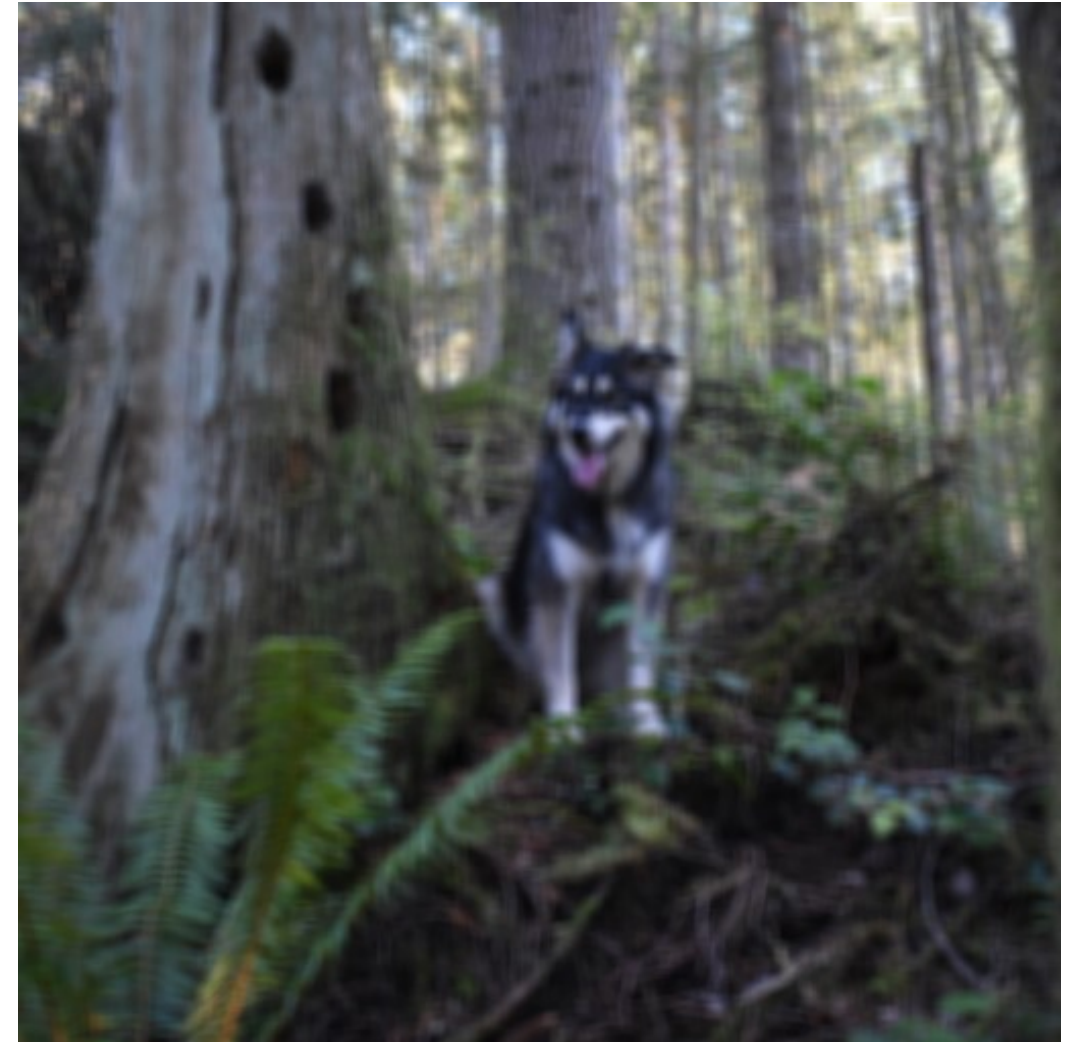
## So much better!



## Compare to interpolation



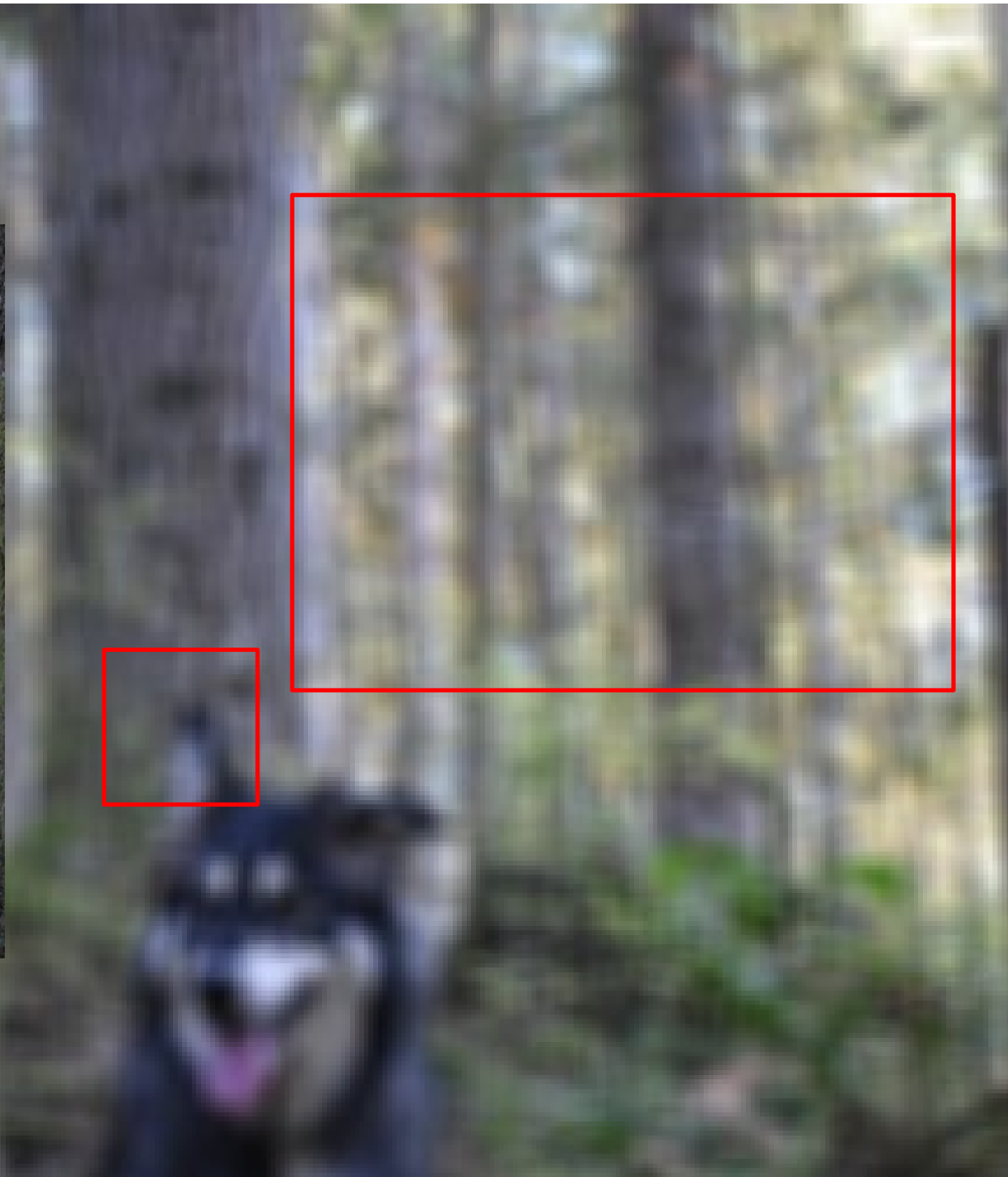
## Box filters have artifacts



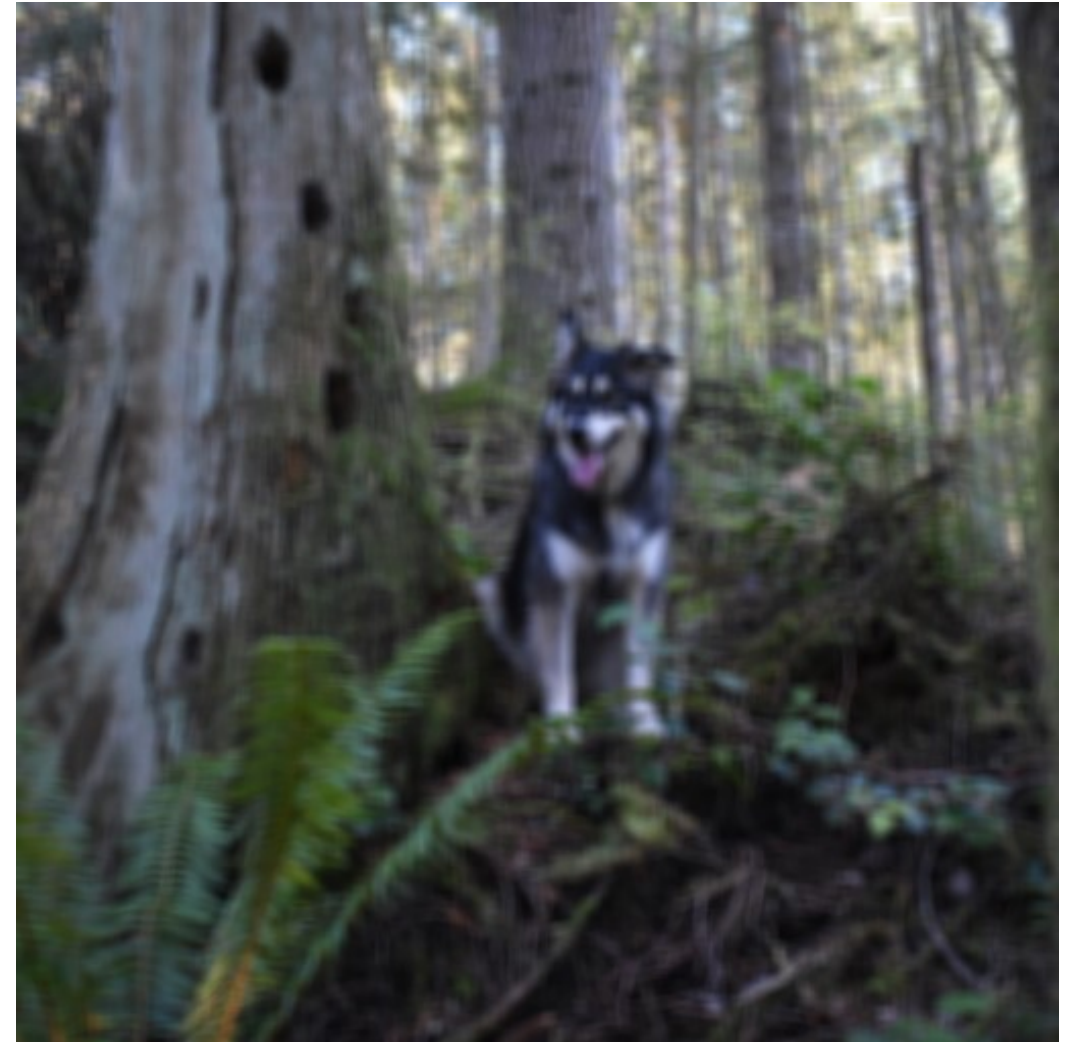
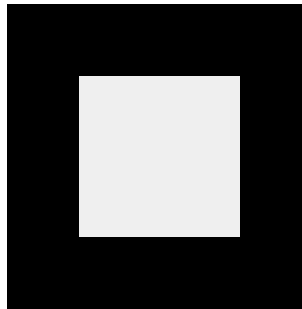




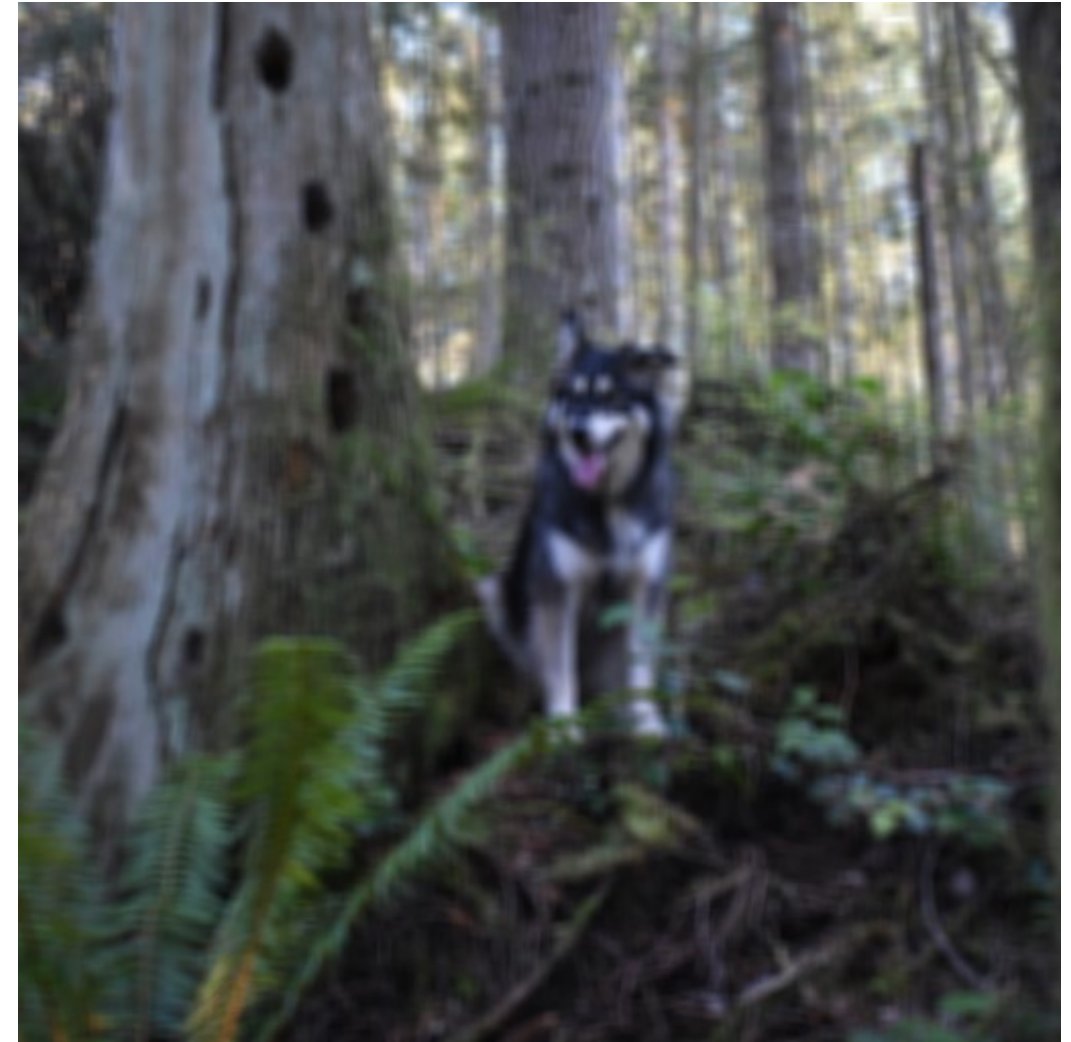
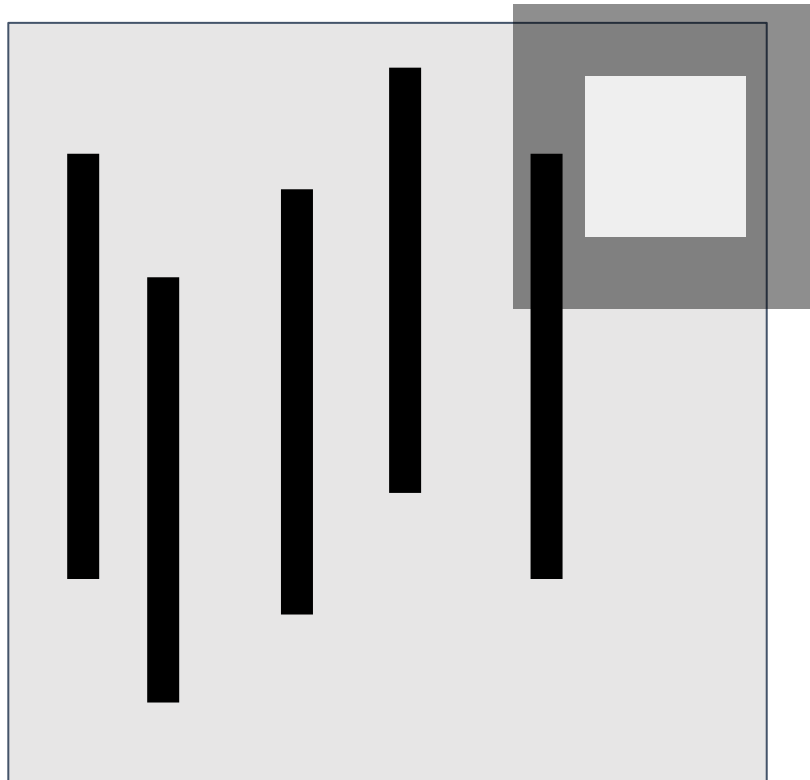
# Box filters have artifacts



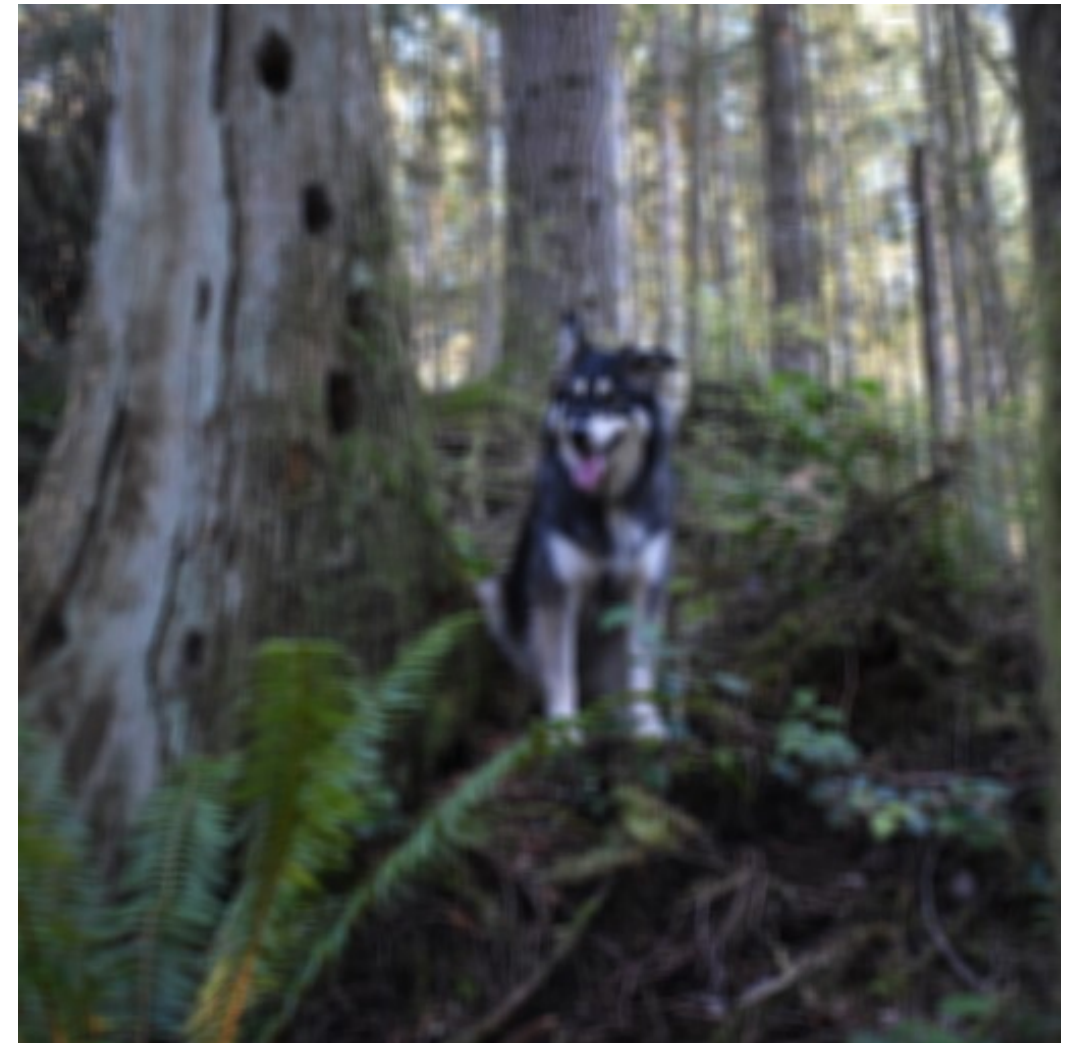
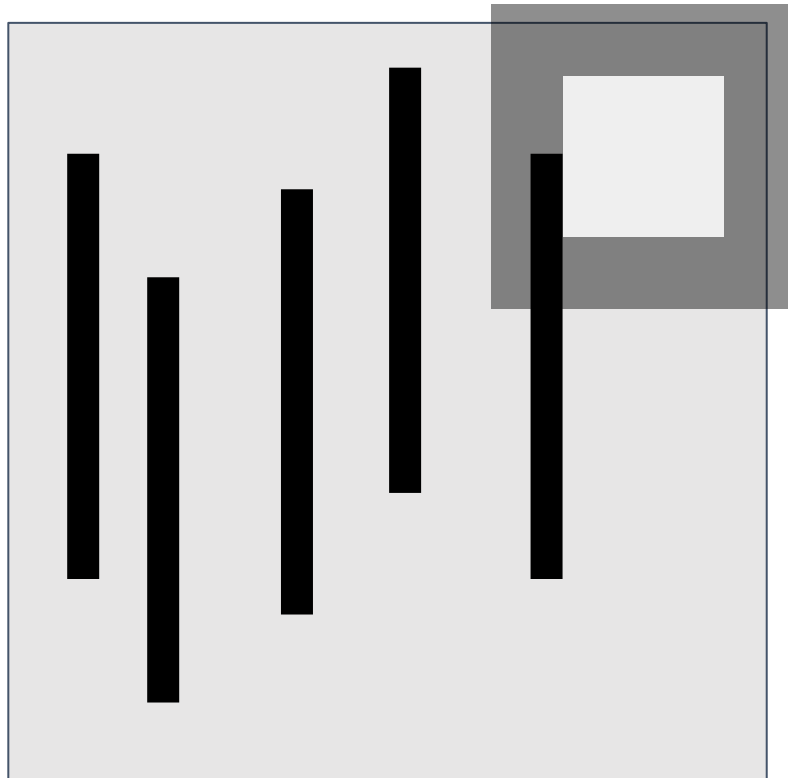
## Box filters: vertical + horizontal streaking



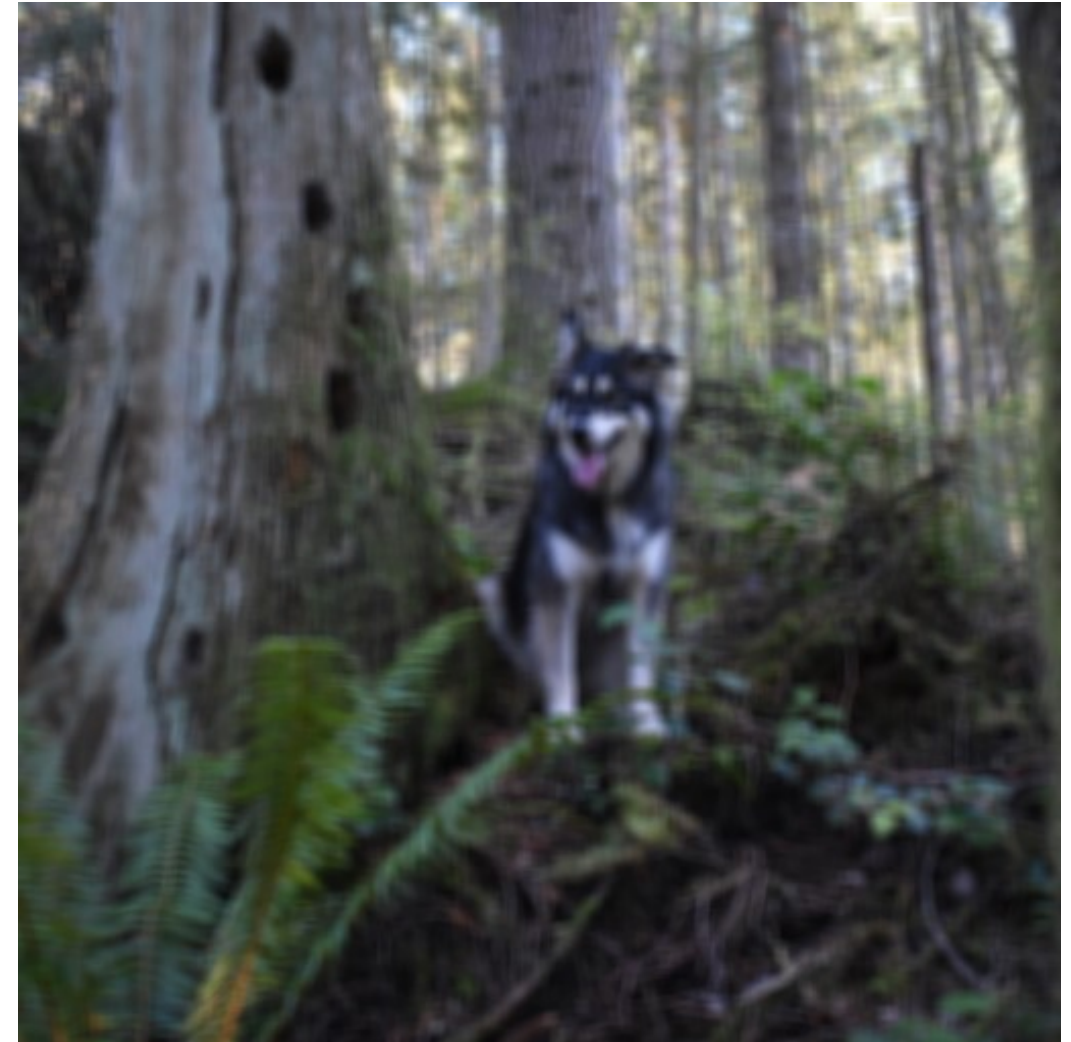
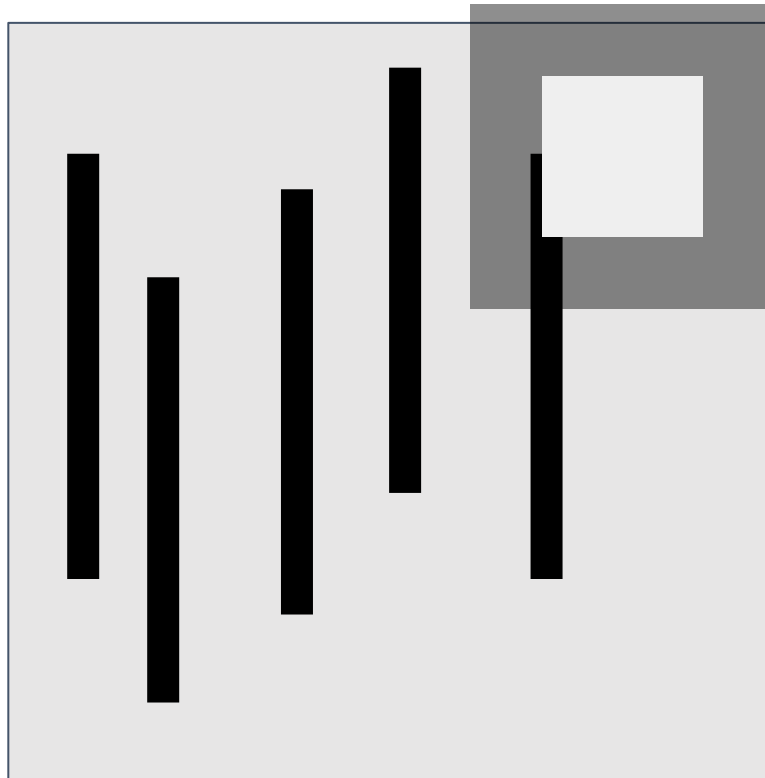
## Box filters: vertical + horizontal streaking



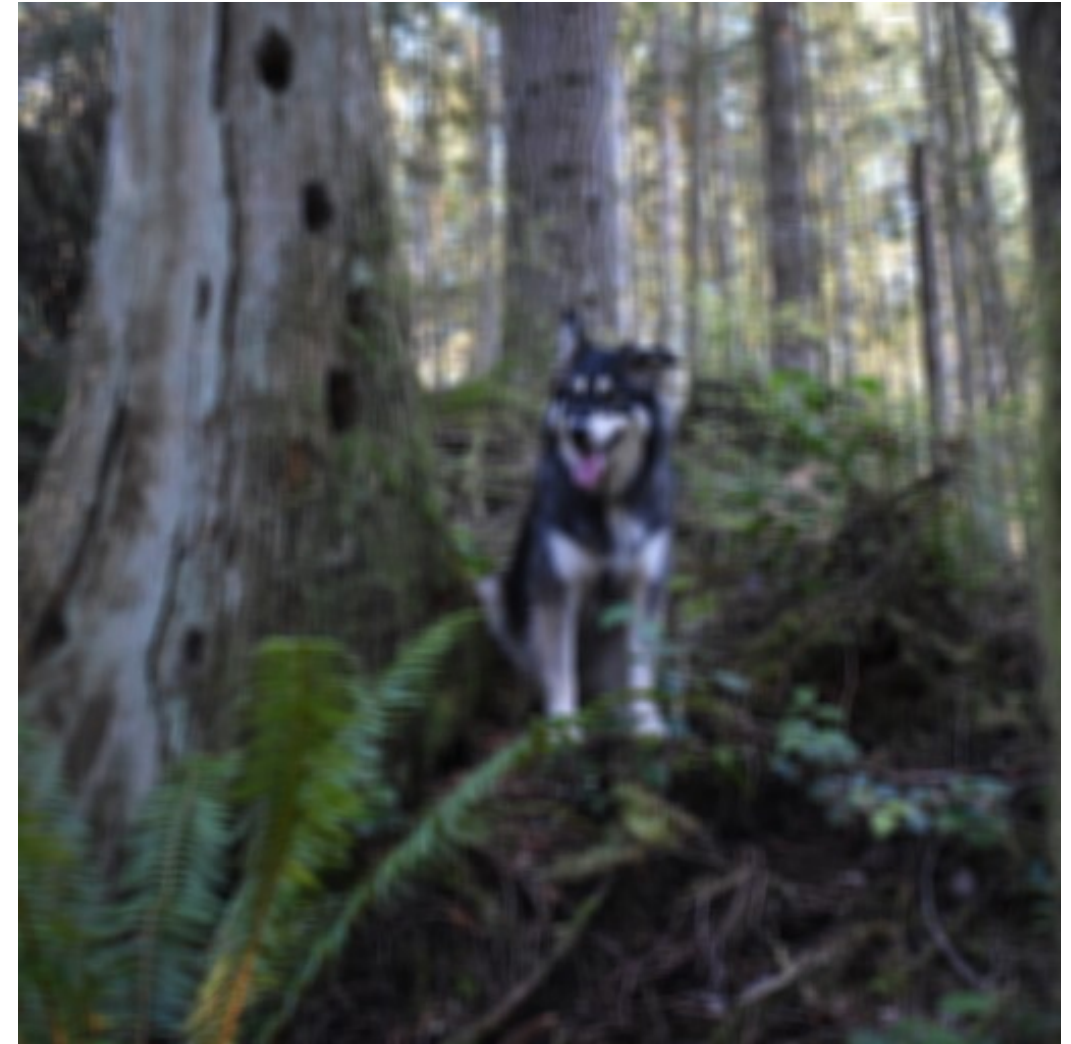
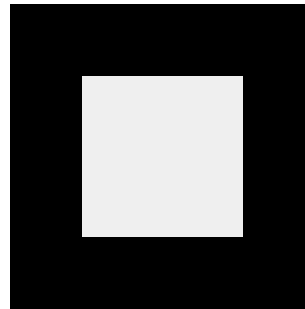
## Box filters: vertical + horizontal streaking



## Box filters: vertical + horizontal streaking



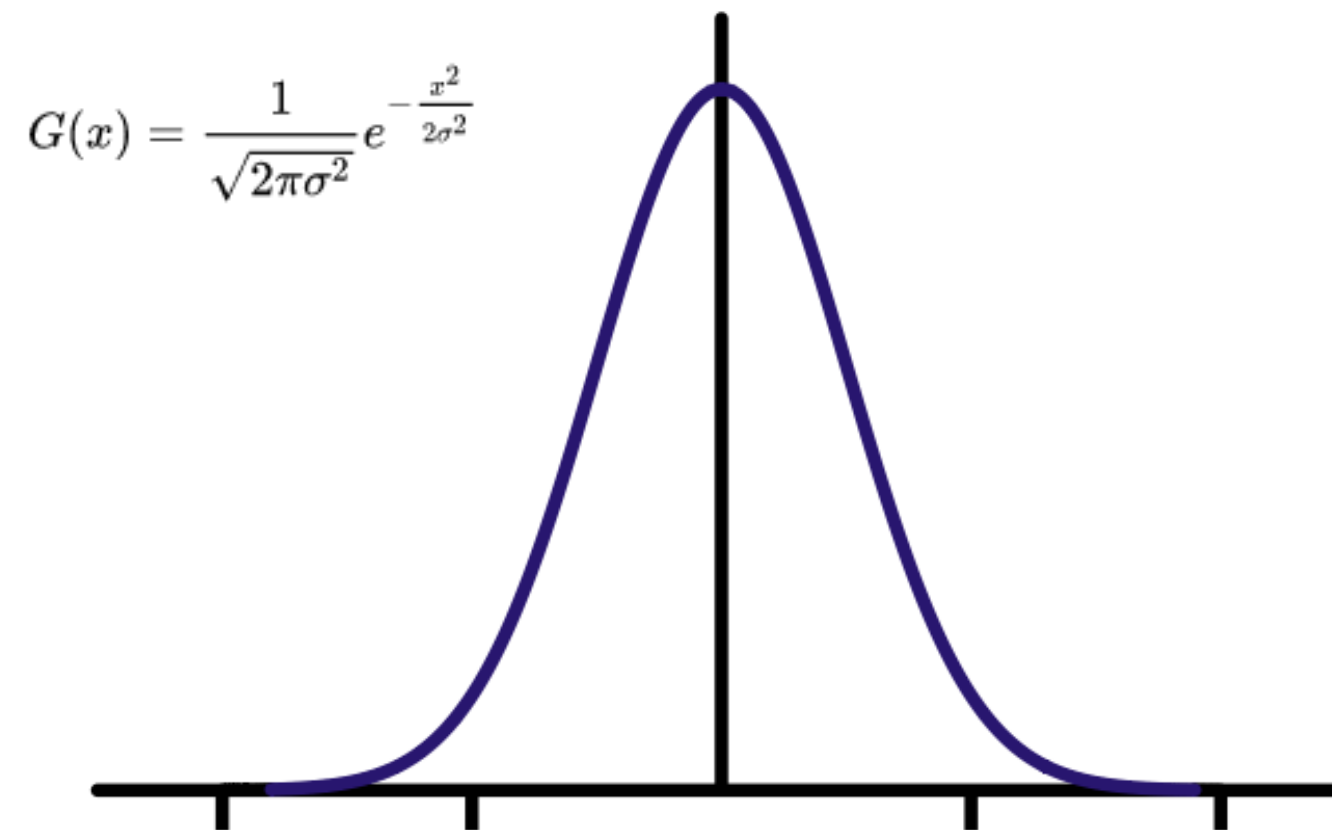
## We want a smoothly weighted kernel



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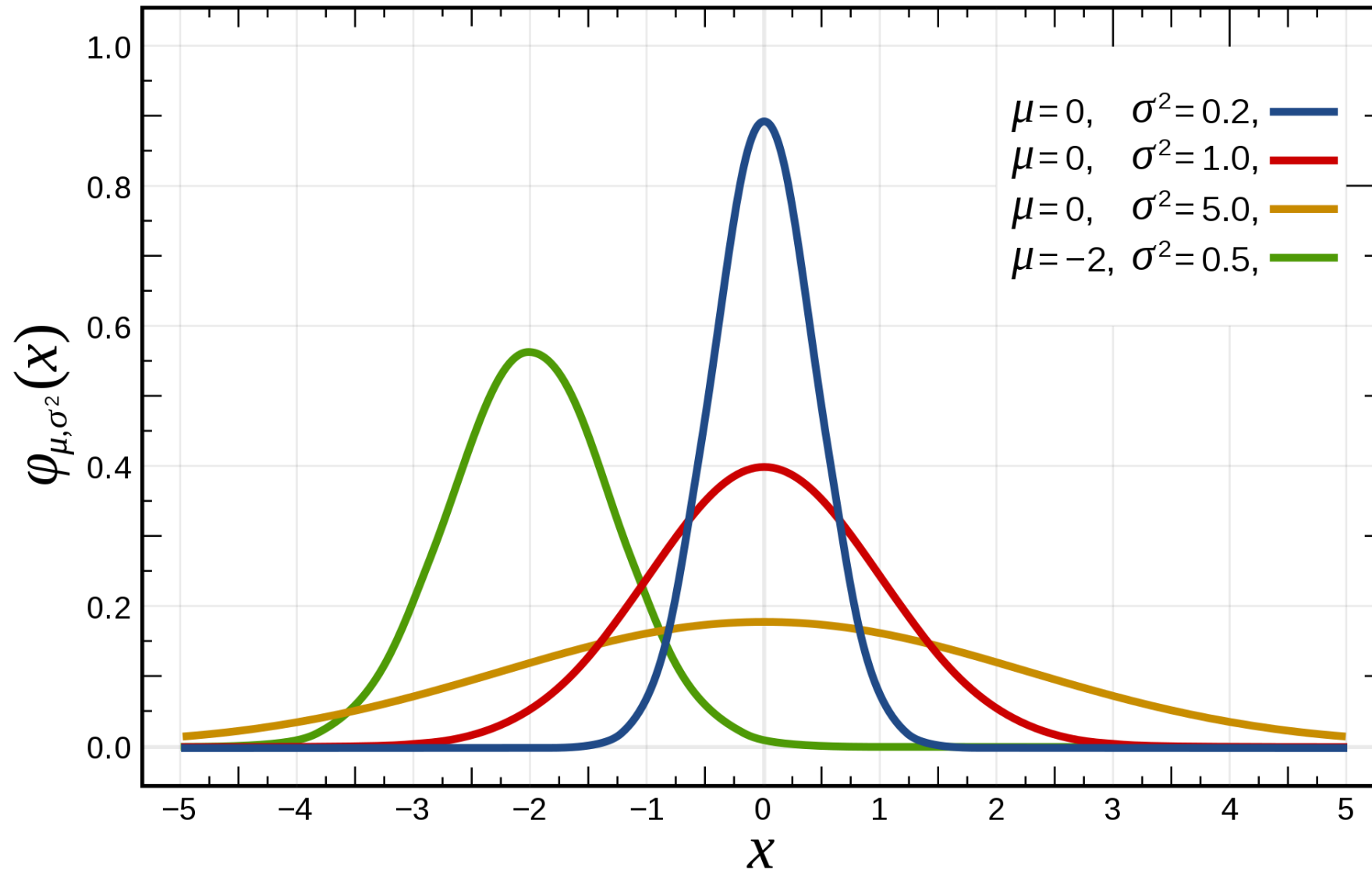
# Gaussians





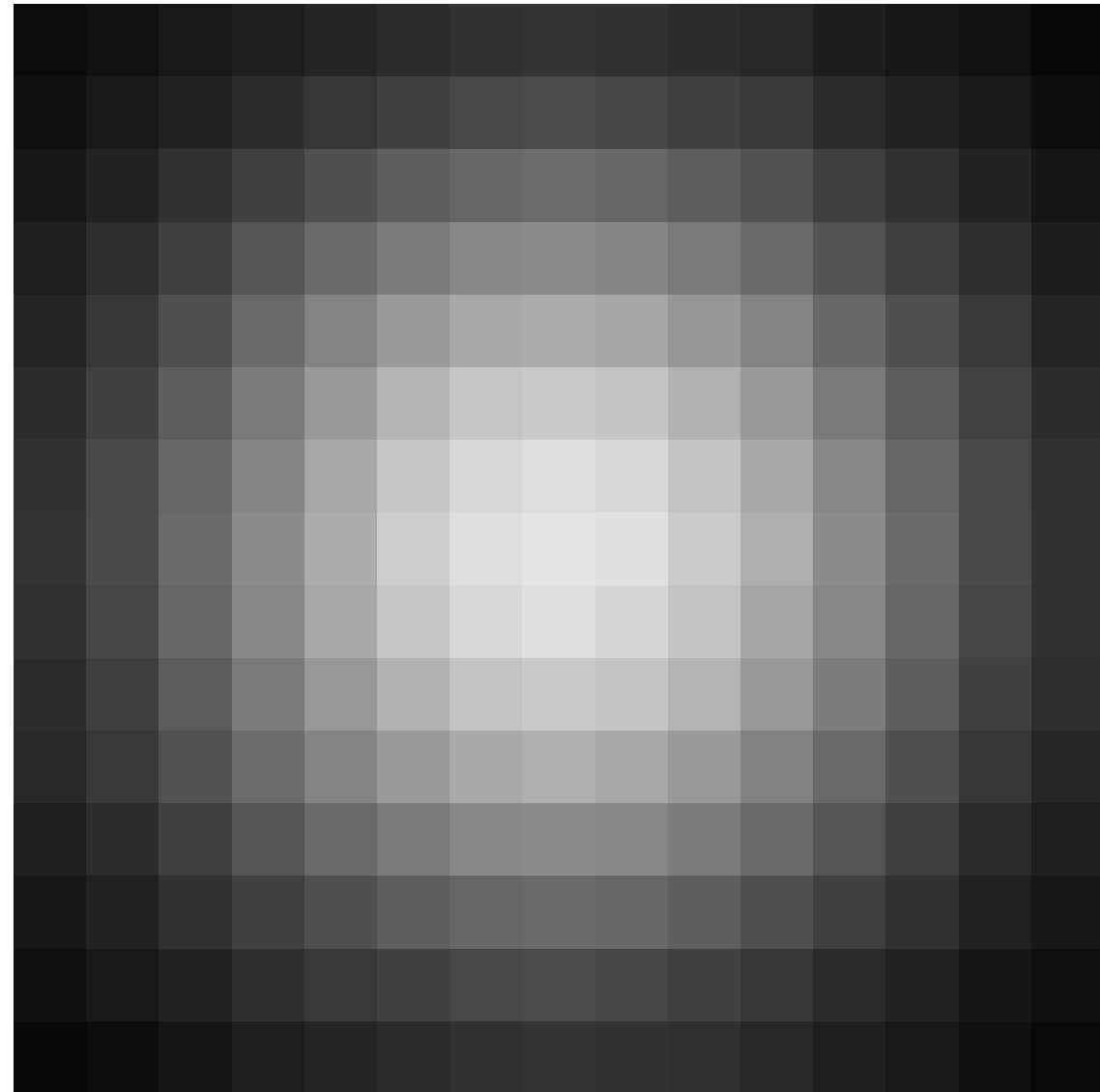
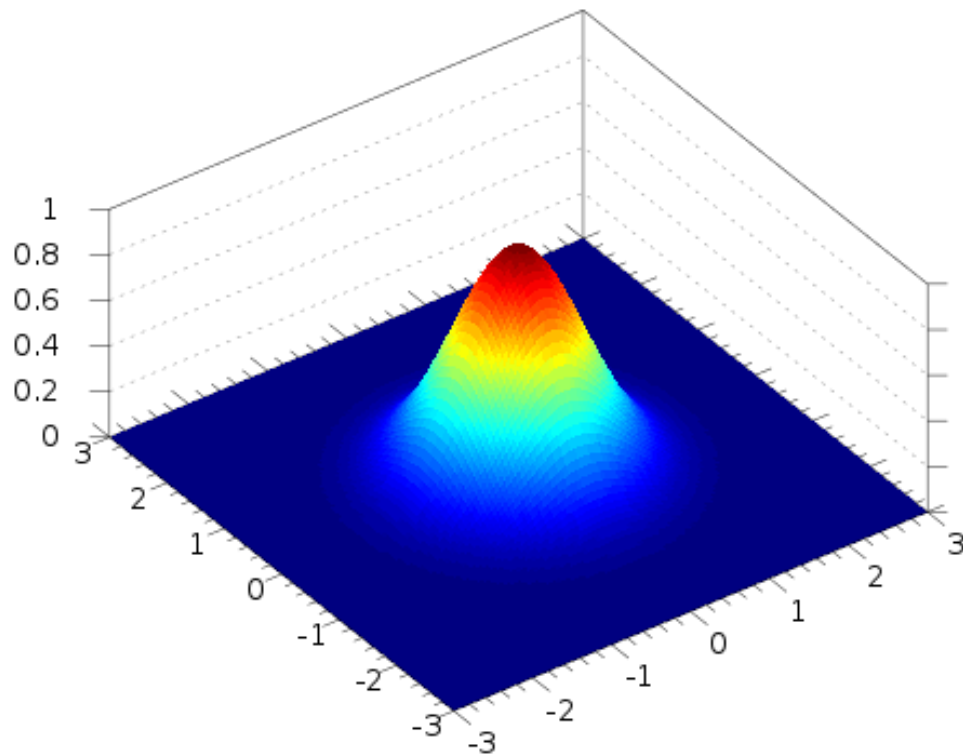


# Gaussians – how $\sigma$ affects the shape



# 2D Gaussian

$$g(x, y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2+y^2}{2\sigma^2}}$$



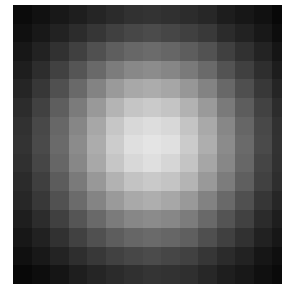
# Example 7x7 Gaussian

	0.000	0.000	0.001	0.001	0.001	0.000	0.000	
	0.000	0.002	0.012	0.020	0.012	0.002	0.000	
	0.001	0.012	0.068	0.109	0.068	0.012	0.001	
	0.001	0.020	0.109	0.172	0.109	0.020	0.001	
	0.001	0.012	0.068	0.109	0.068	0.012	0.001	
	0.000	0.002	0.012	0.020	0.012	0.002	0.000	
	0.000	0.000	0.001	0.001	0.001	0.000	0.000	

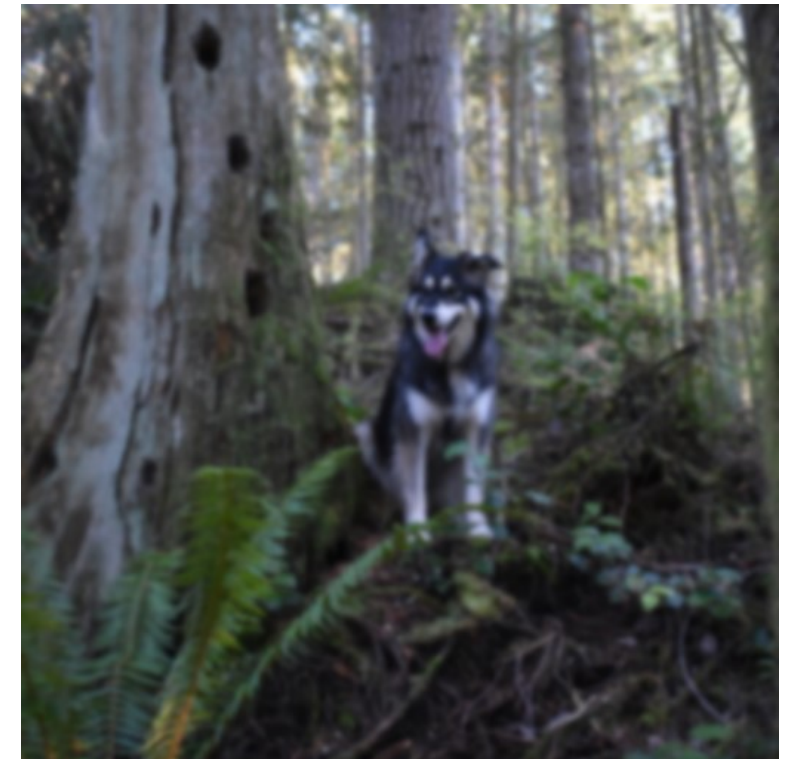
# Better smoothing with Gaussians



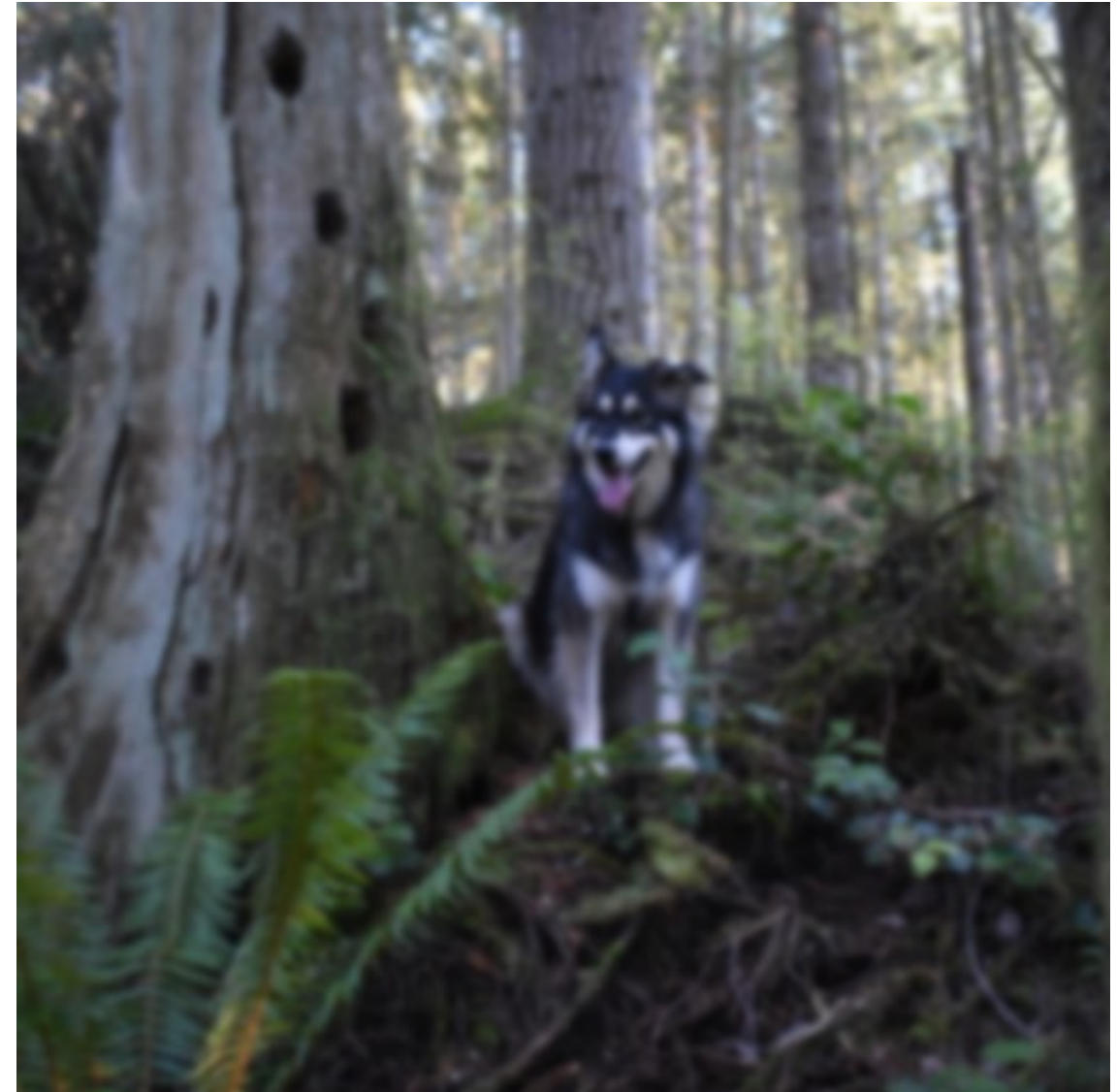
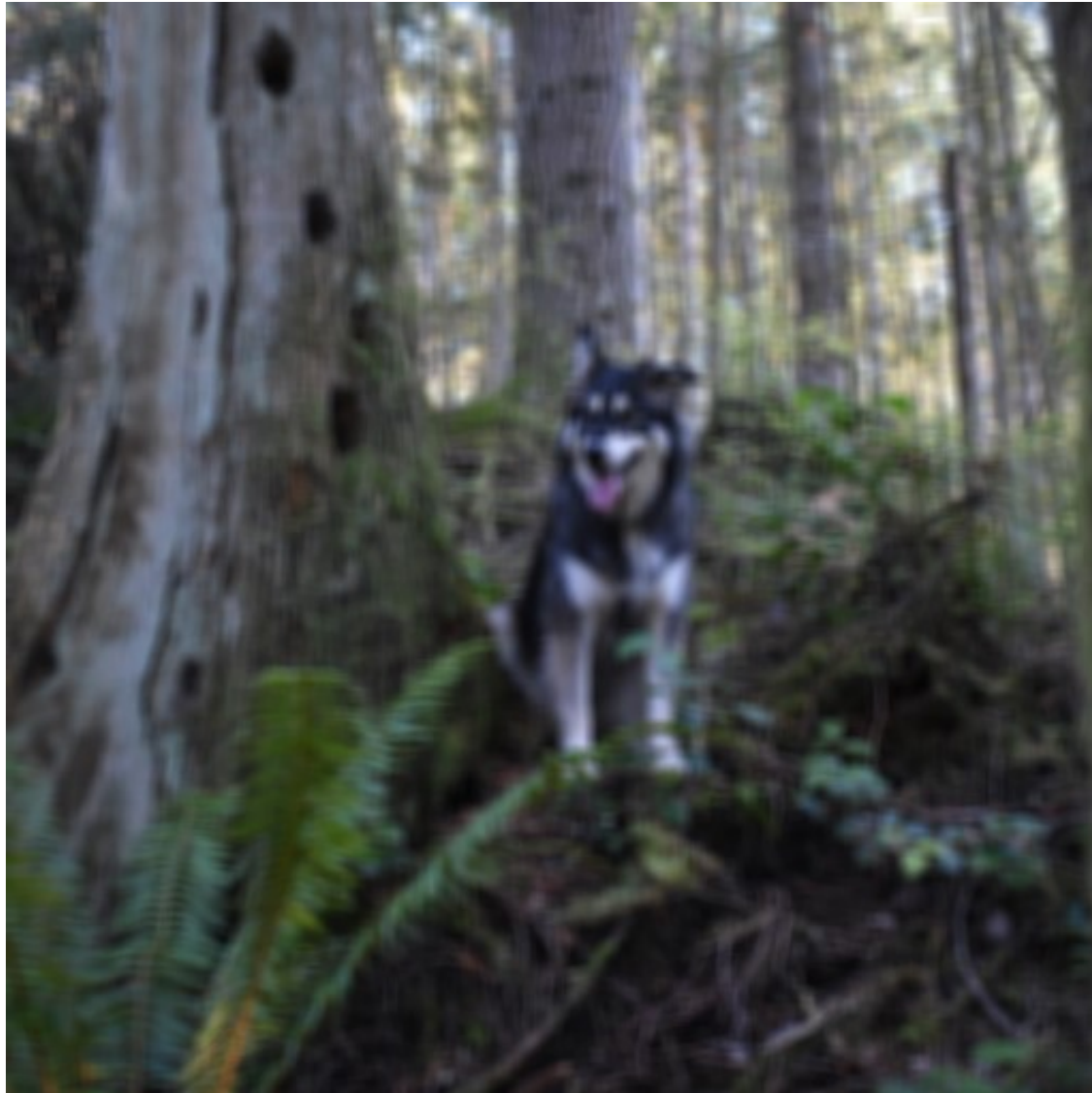
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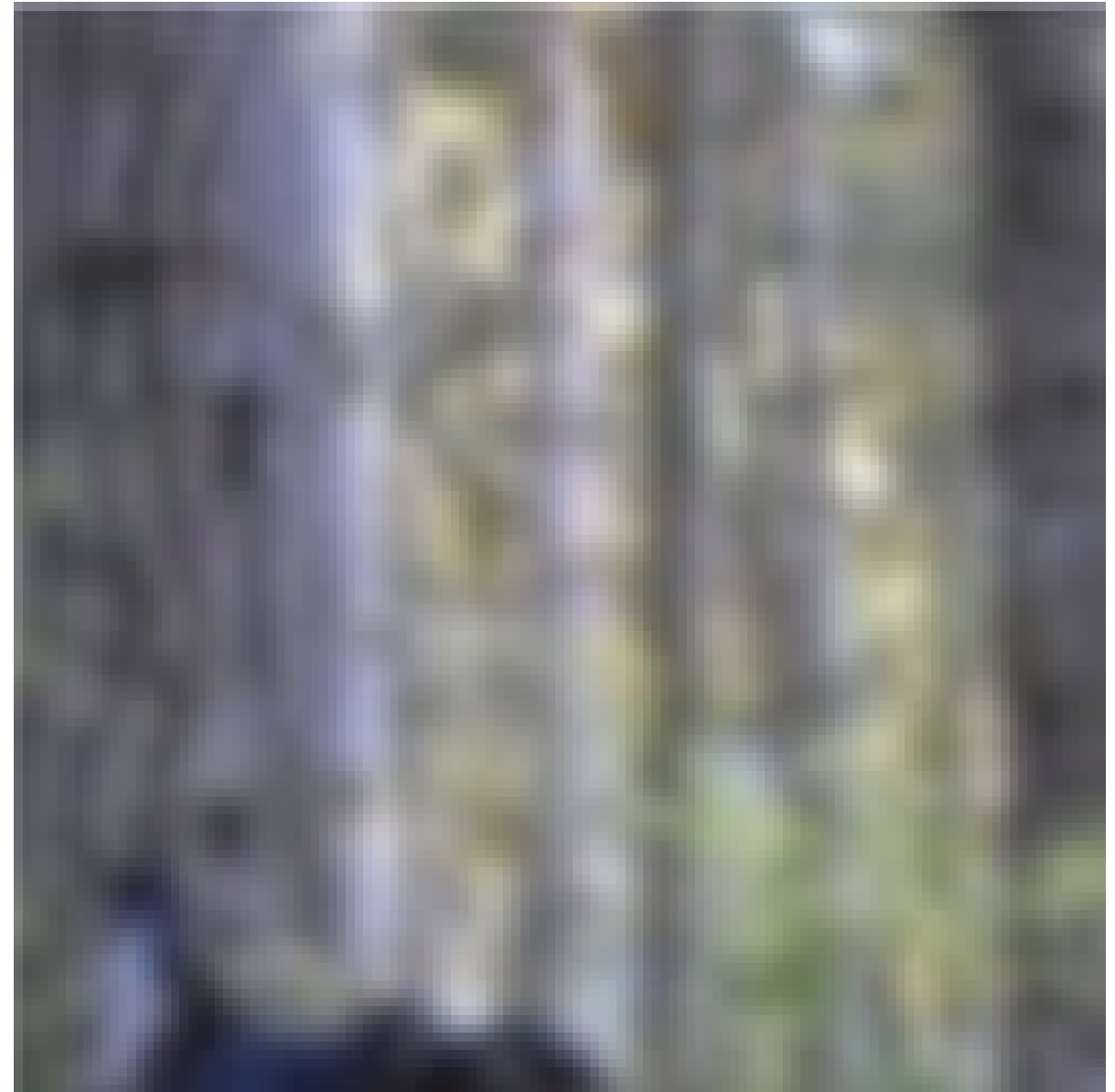
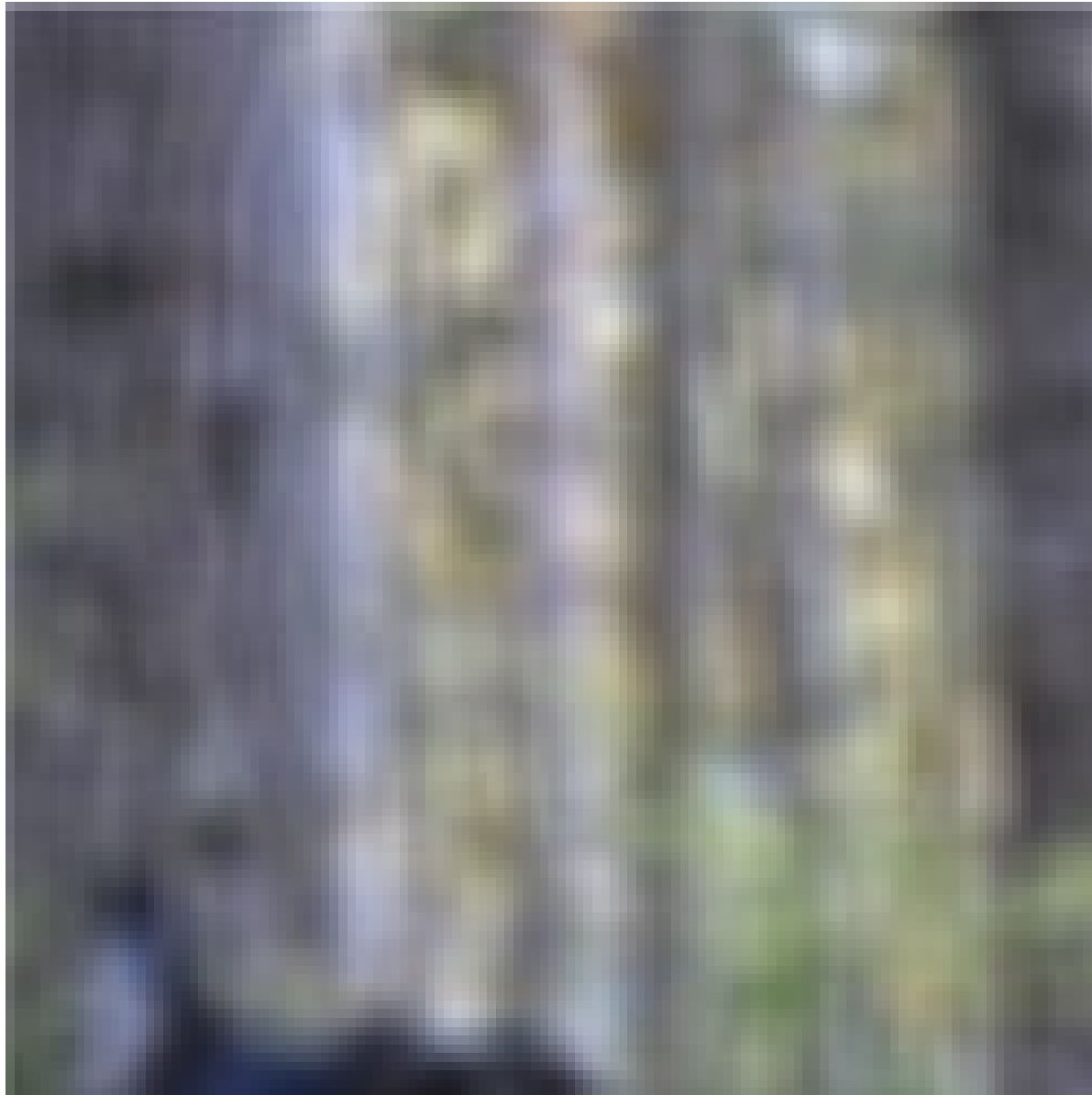
=



# Better smoothing with Gaussians



# Better smoothing with Gaussians

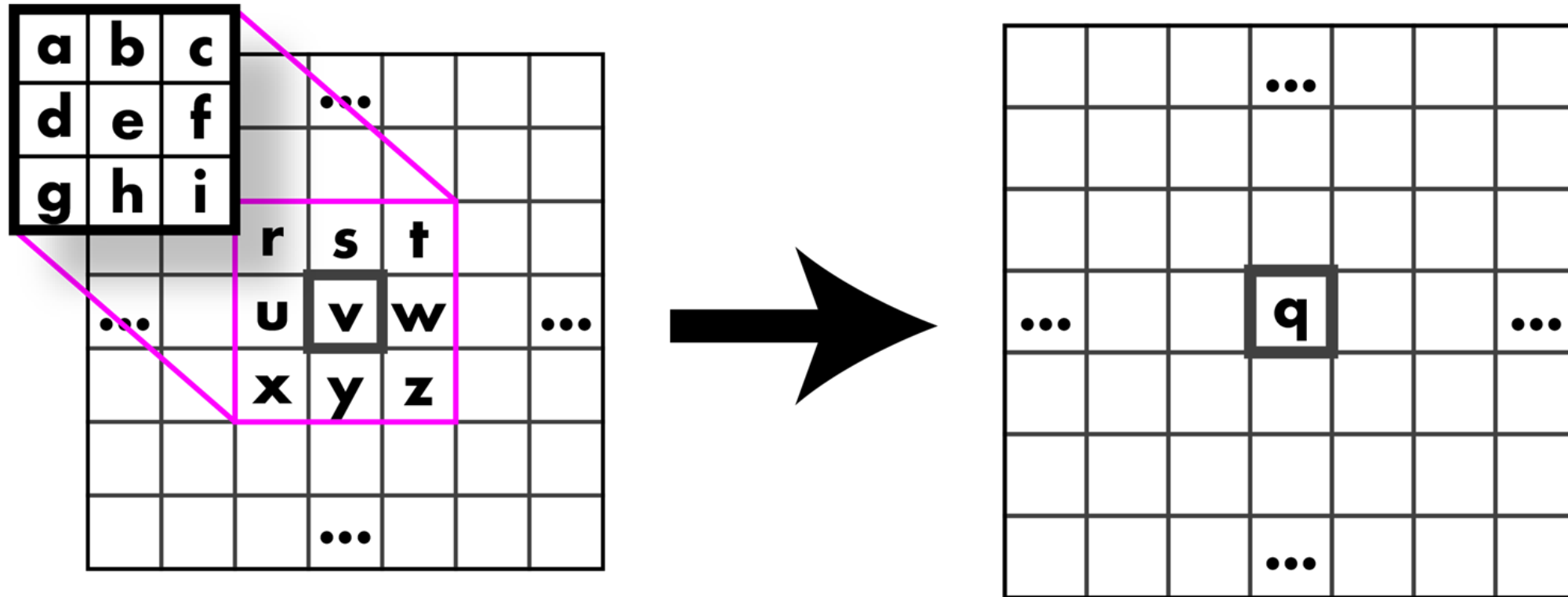


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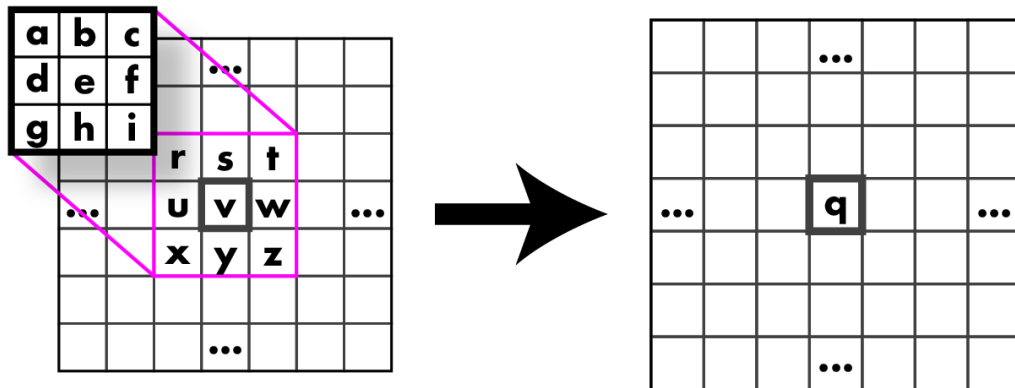
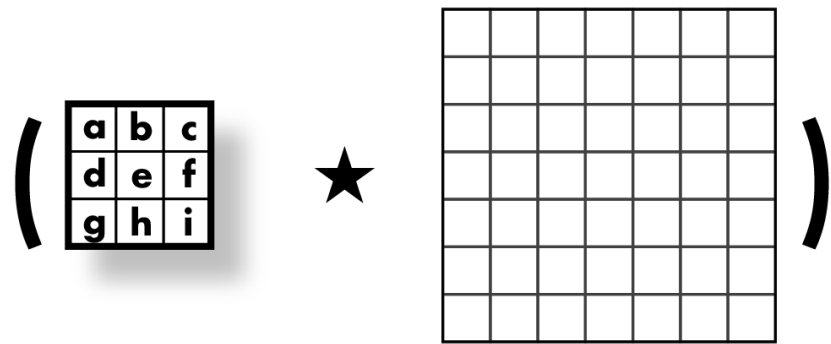
# So what is convolution??





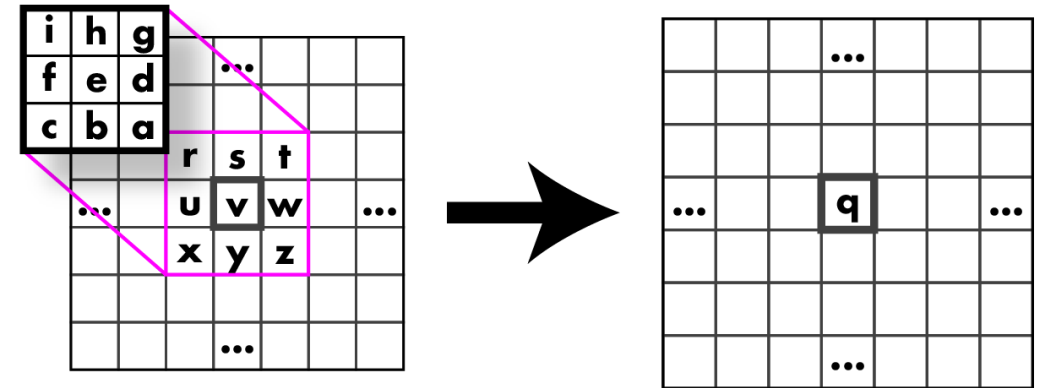
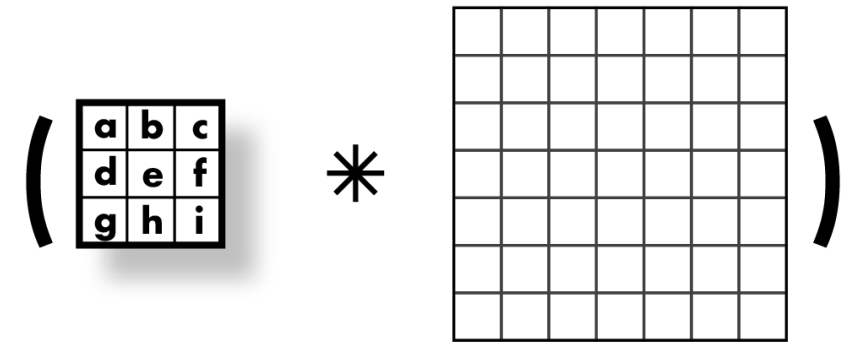
# Cross-Correlation vs Convolution

## Cross-Correlation



$$q = a \times r + b \times s + c \times t + d \times u + e \times v + f \times w + g \times x + h \times y + i \times z$$

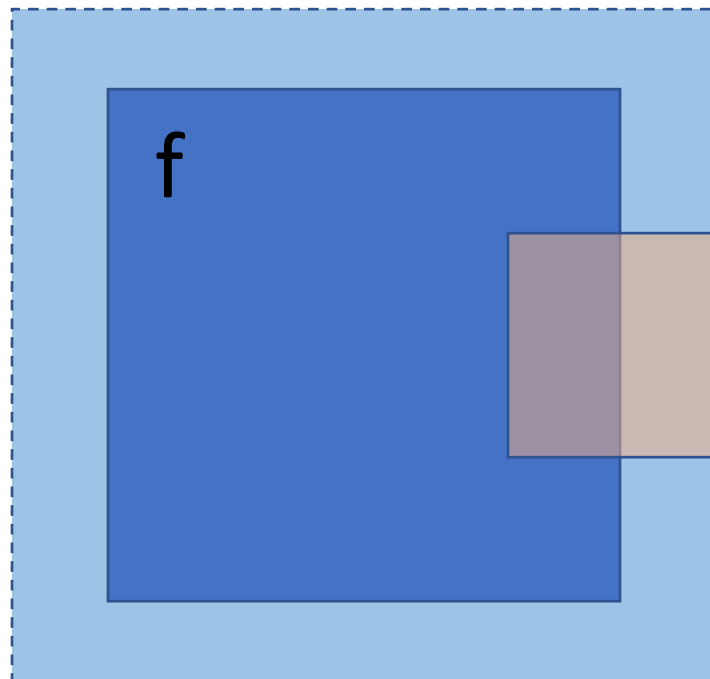
## Convolution



$$q = i \times r + h \times s + g \times t + f \times u + e \times v + d \times w + c \times x + b \times y + a \times z$$

# Image support and edge effect

- A computer will only convolve **finite support signals**
- What happens at the edge?

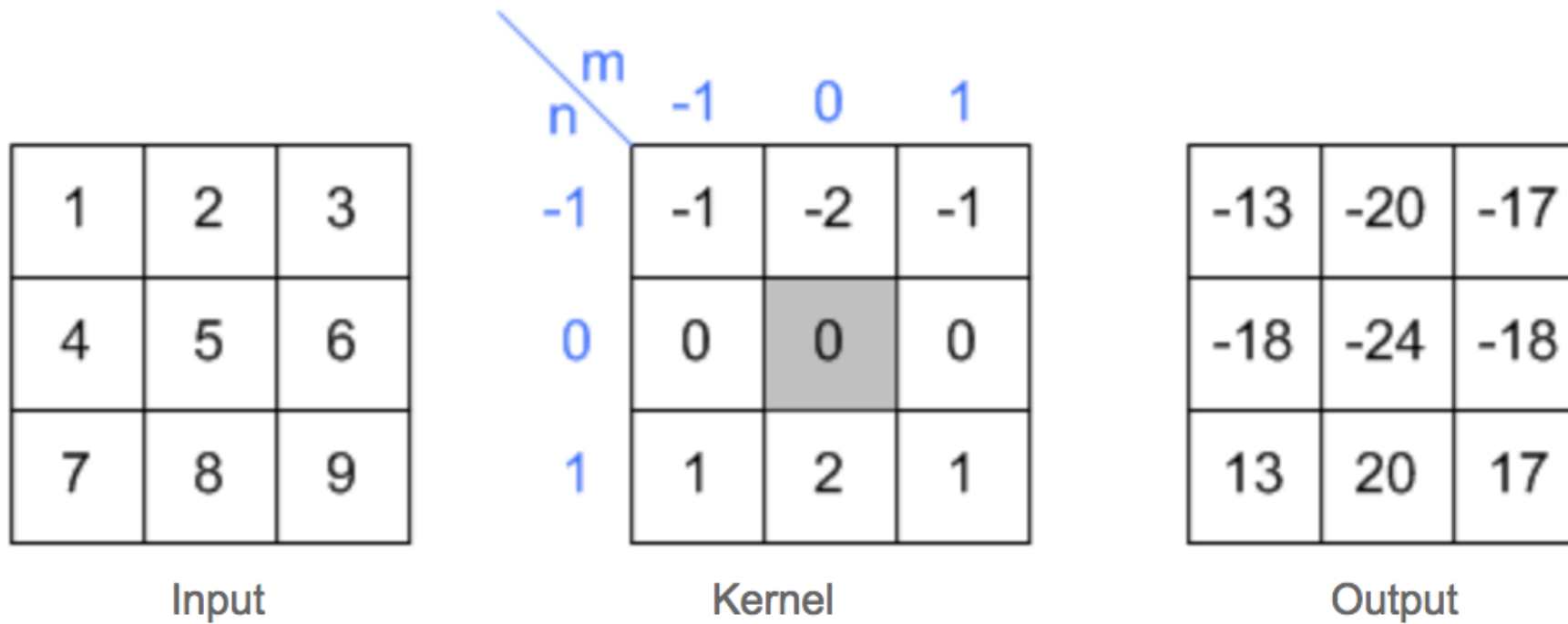


$h$

- zero “padding”
- edge value replication
- mirror extension
- ...

[Slide by Niebles]

## 2D convolution example



[Slide by Song Ho Ahn]



## 2D convolution example

1	2	1	
0	0	0	3
-1	-2	-1	6
	7	8	9

$$\begin{aligned}
 &= x[-1,-1] \cdot h[1,1] + x[0,-1] \cdot h[0,1] + x[1,-1] \cdot h[-1,1] \\
 &\quad + x[-1,0] \cdot h[1,0] + x[0,0] \cdot h[0,0] + x[1,0] \cdot h[-1,0] \\
 &\quad + x[-1,1] \cdot h[1,-1] + x[0,1] \cdot h[0,-1] + x[1,1] \cdot h[-1,-1] \\
 &= 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 + 2 \cdot 0 + 0 \cdot (-1) + 4 \cdot (-2) + 5 \cdot (-1) = -13
 \end{aligned}$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]



## 2D convolution example

	1	2	1
0	1	2	3
-1	4	5	6
	7	8	9

$$\begin{aligned}
 &= x[0,-1] \cdot h[1,1] + x[1,-1] \cdot h[0,1] + x[2,-1] \cdot h[-1,1] \\
 &\quad + x[0,0] \cdot h[1,0] + x[1,0] \cdot h[0,0] + x[2,0] \cdot h[-1,0] \\
 &\quad + x[0,1] \cdot h[1,-1] + x[1,1] \cdot h[0,-1] + x[2,1] \cdot h[-1,-1] \\
 &= 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 + 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 0 + 4 \cdot (-1) + 5 \cdot (-2) + 6 \cdot (-1) = -20
 \end{aligned}$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]



## 2D convolution example

		1	2	1
1	0	2	0	3
4	-1	5	-2	6
7	8	9		

$$\begin{aligned}
 &= x[1,-1] \cdot h[1,1] + x[2,-1] \cdot h[0,1] + x[3,-1] \cdot h[-1,1] \\
 &+ x[1,0] \cdot h[1,0] + x[2,0] \cdot h[0,0] + x[3,0] \cdot h[-1,0] \\
 &+ x[1,1] \cdot h[1,-1] + x[2,1] \cdot h[0,-1] + x[3,1] \cdot h[-1,-1] \\
 &= 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 + 2 \cdot 0 + 3 \cdot 0 + 0 \cdot 0 + 5 \cdot (-1) + 6 \cdot (-2) + 0 \cdot (-1) = -17
 \end{aligned}$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]



## 2D convolution example

1	2	1	3
0	0	0	6
-1	-2	-1	9

$$\begin{aligned}
 &= x[-1,0] \cdot h[1,1] + x[0,0] \cdot h[0,1] + x[1,0] \cdot h[-1,1] \\
 &\quad + x[-1,1] \cdot h[1,0] + x[0,1] \cdot h[0,0] + x[1,1] \cdot h[-1,0] \\
 &\quad + x[-1,2] \cdot h[1,-1] + x[0,2] \cdot h[0,-1] + x[1,2] \cdot h[-1,-1] \\
 &= 0 \cdot 1 + 1 \cdot 2 + 2 \cdot 1 + 0 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 + 0 \cdot (-1) + 7 \cdot (-2) + 8 \cdot (-1) = -18
 \end{aligned}$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]

## 2D convolution example

1	2	1
1	2	3
0	0	0
4	5	6
-1	-2	-1
7	8	9

$$\begin{aligned}
 &= x[0,0] \cdot h[1,1] + x[1,0] \cdot h[0,1] + x[2,0] \cdot h[-1,1] \\
 &\quad + x[0,1] \cdot h[1,0] + x[1,1] \cdot h[0,0] + x[2,1] \cdot h[-1,0] \\
 &\quad + x[0,2] \cdot h[1,-1] + x[1,2] \cdot h[0,-1] + x[2,2] \cdot h[-1,-1] \\
 &= 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 + 4 \cdot 0 + 5 \cdot 0 + 6 \cdot 0 + 7 \cdot (-1) + 8 \cdot (-2) + 9 \cdot (-1) = -24
 \end{aligned}$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]



## 2D convolution example

1	1	2	3	1
4	0	5	0	0
7	-1	8	-2	-1

$$\begin{aligned}
 &= x[1,0] \cdot h[1,1] + x[2,0] \cdot h[0,1] + x[3,0] \cdot h[-1,1] \\
 &\quad + x[1,1] \cdot h[1,0] + x[2,1] \cdot h[0,0] + x[3,1] \cdot h[-1,0] \\
 &\quad + x[1,2] \cdot h[1,-1] + x[2,2] \cdot h[0,-1] + x[3,2] \cdot h[-1,-1] \\
 &= 2 \cdot 1 + 3 \cdot 2 + 0 \cdot 1 + 5 \cdot 0 + 6 \cdot 0 + 0 \cdot 0 + 8 \cdot (-1) + 9 \cdot (-2) + 0 \cdot (-1) = -18
 \end{aligned}$$

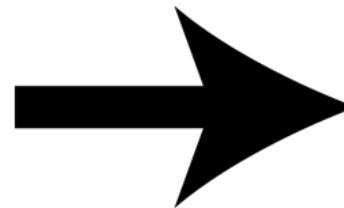
-13	-20	-17
-18	-24	-18
13	20	17

Output

[Slide by Song Ho Ahn]

## Calculate it!

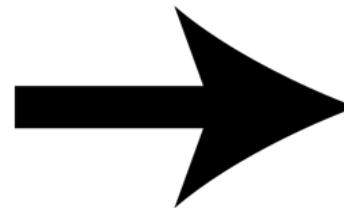
<b>2</b>	<b>-1</b>	<b>-1</b>				
<b>-1</b>	<b>2</b>	<b>-1</b>	...			
<b>-1</b>	<b>-1</b>	<b>2</b>				
			<b>45</b>	<b>51</b>	<b>57</b>	
...			<b>46</b>	<b>46</b>	<b>67</b>	...
			<b>26</b>	<b>19</b>	<b>64</b>	
			...			



			...			
...			<b>9</b>			...
			...			

## Calculate it!

-1	-1	-1				
-1	8	-1	...			
-1	-1	-1				
			16	105	153	
...			15	104	113	...
			18	111	136	
			...			

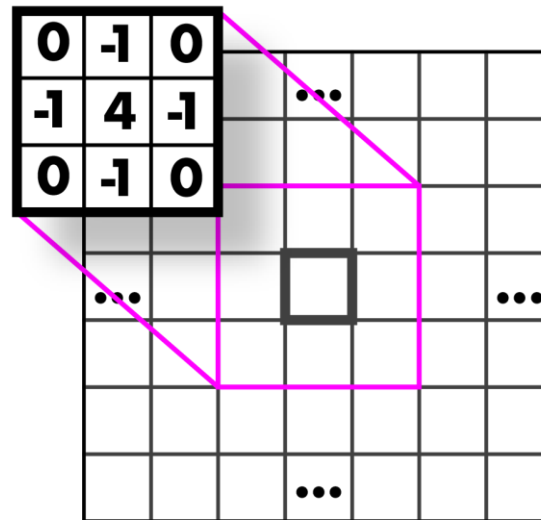


			...			
...			9			...
			...			

# Today's Agenda

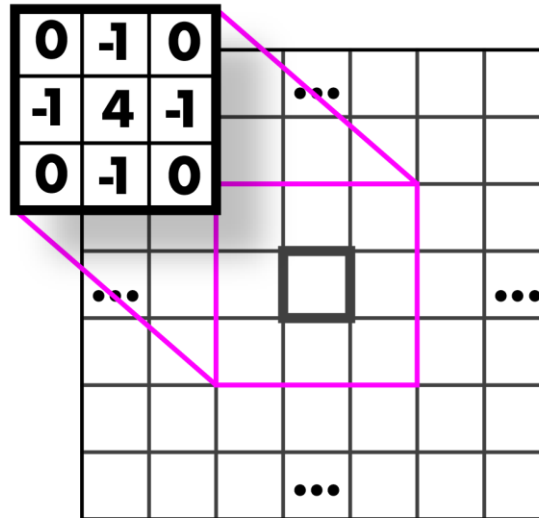
- Averaging vs Interpolation
- Systems - filters
- Convolution
  - Box Filter
  - Gaussian
  - Cross correlation vs Convolution
- Examples of filters

## Guess that kernel!

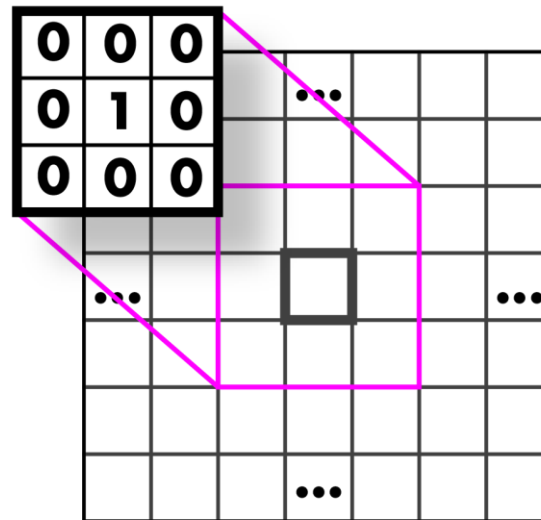


## Highpass Kernel: finds edges

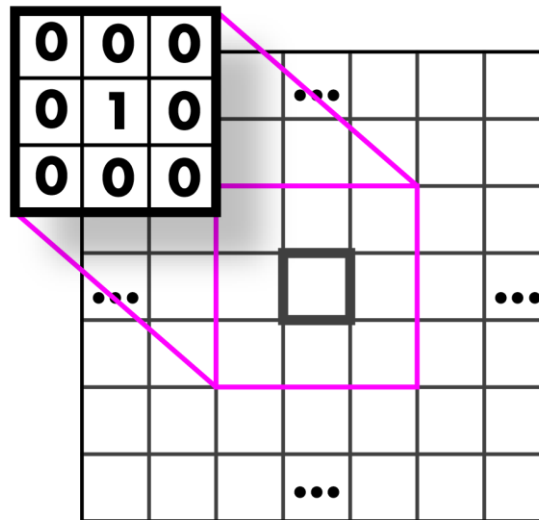
applied to grayscale



## Guess that kernel!

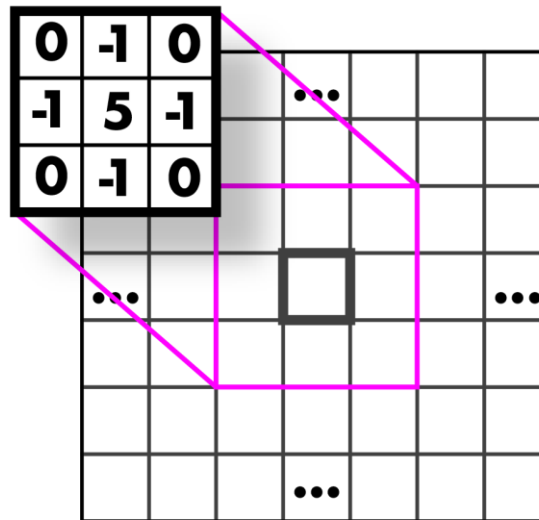


# Identity Kernel: Does nothing!



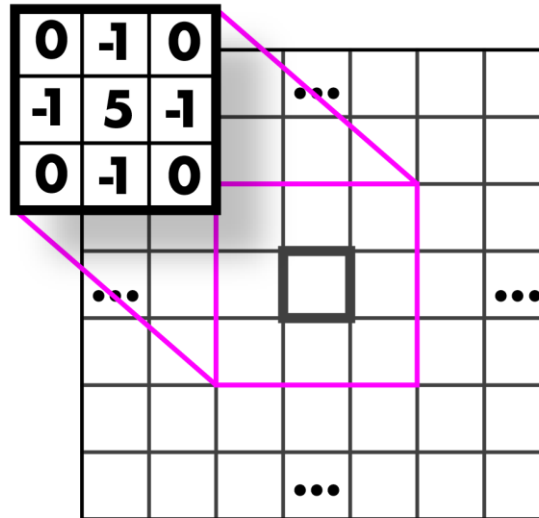


## Guess that kernel!



# Sharpen Kernel: sharpens!

applied to all three channels



Note: sharpen = highpass + identity! Why ?

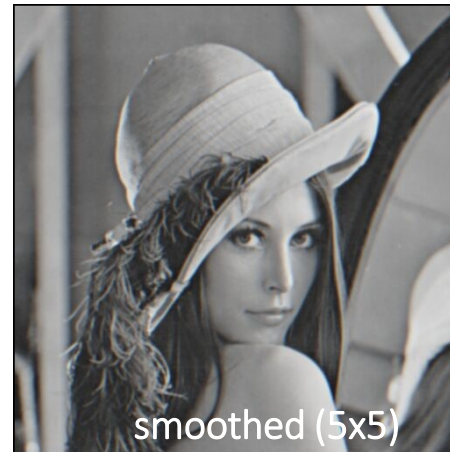
# Sharpen Kernel: sharpens!

What does blurring take away?

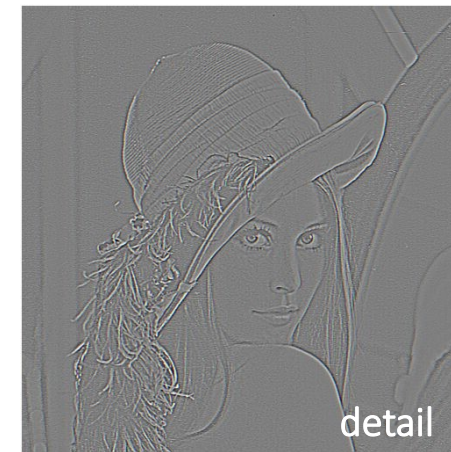
Highpass



-

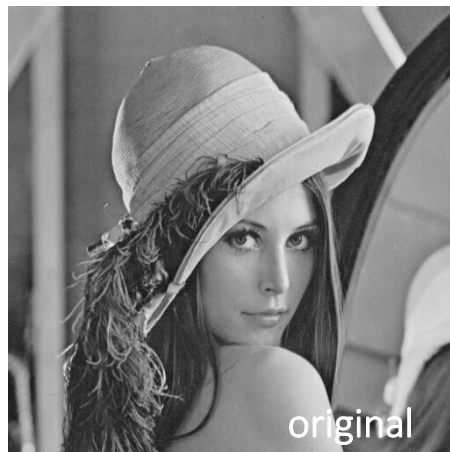


=



Let's add it back:

Identity + Highpass



+

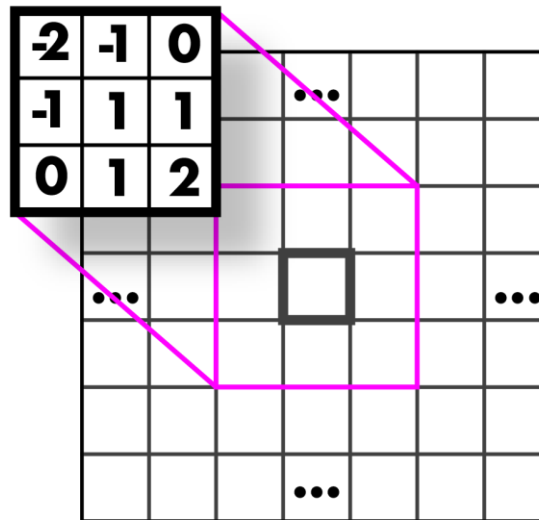


=



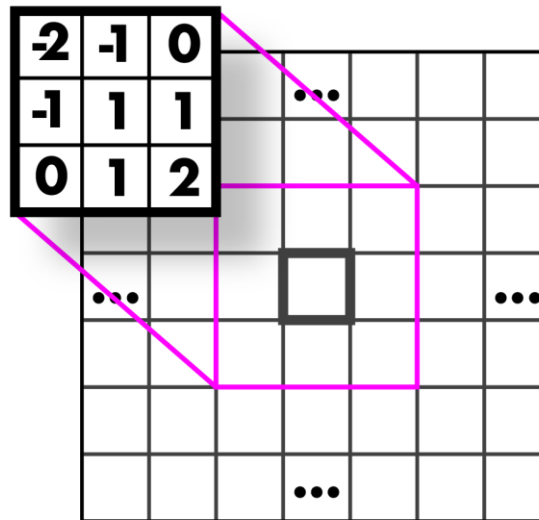
[Slide by D. Lowe]

## Guess that kernel!

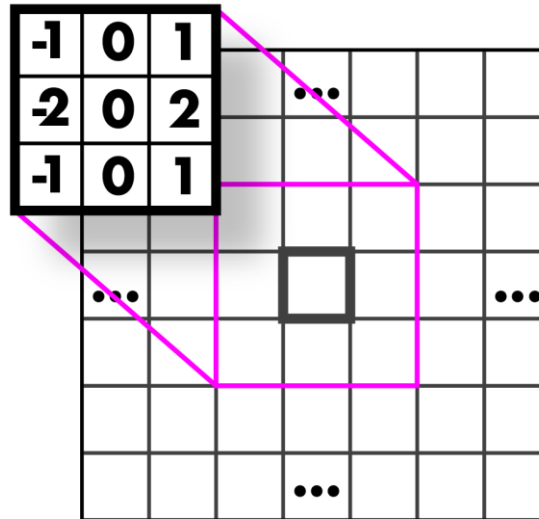
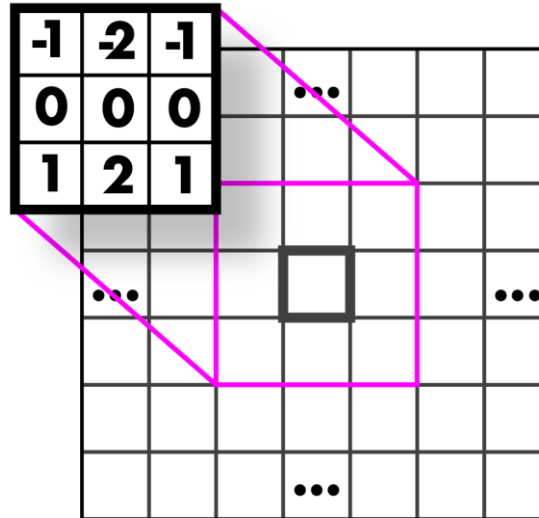


## Emboss Kernel: styling

applied to all three channels

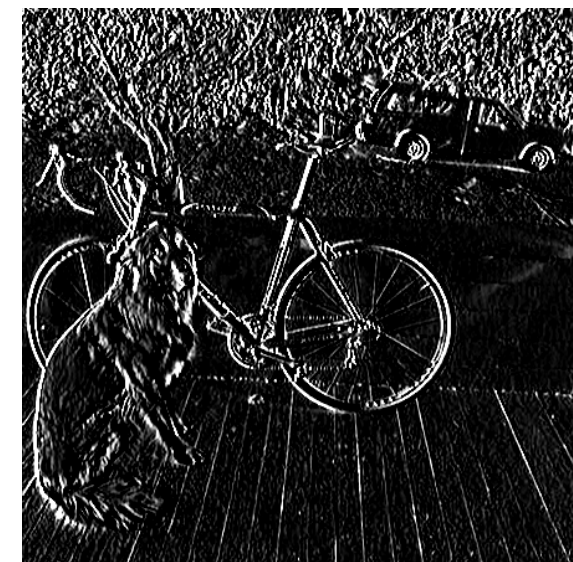
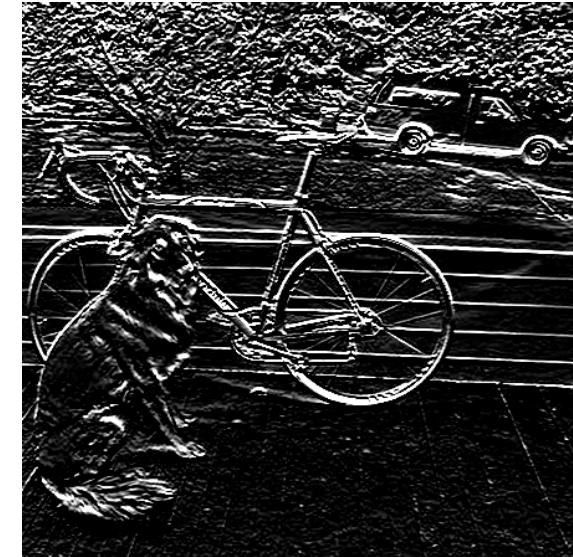
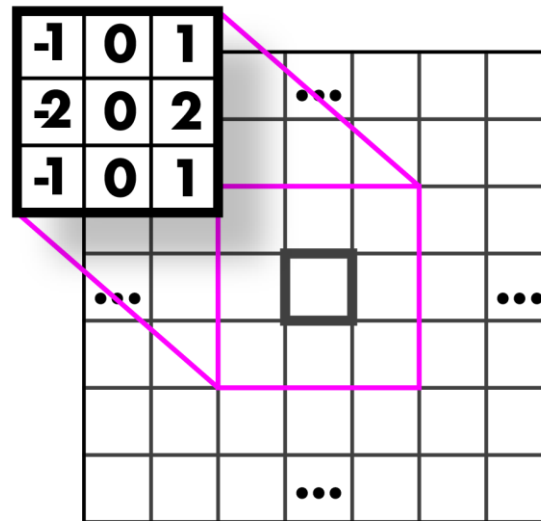
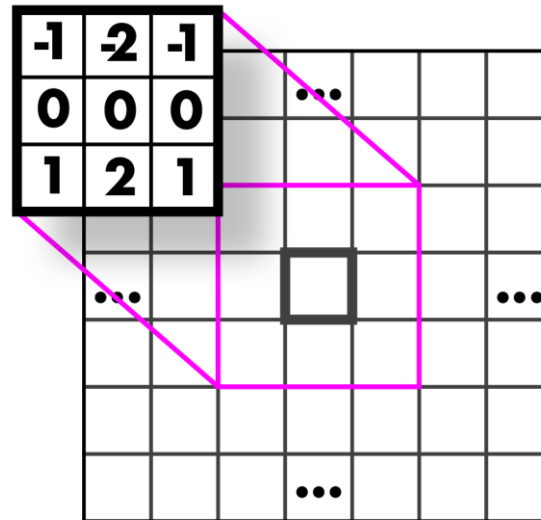


## Guess those kernels!

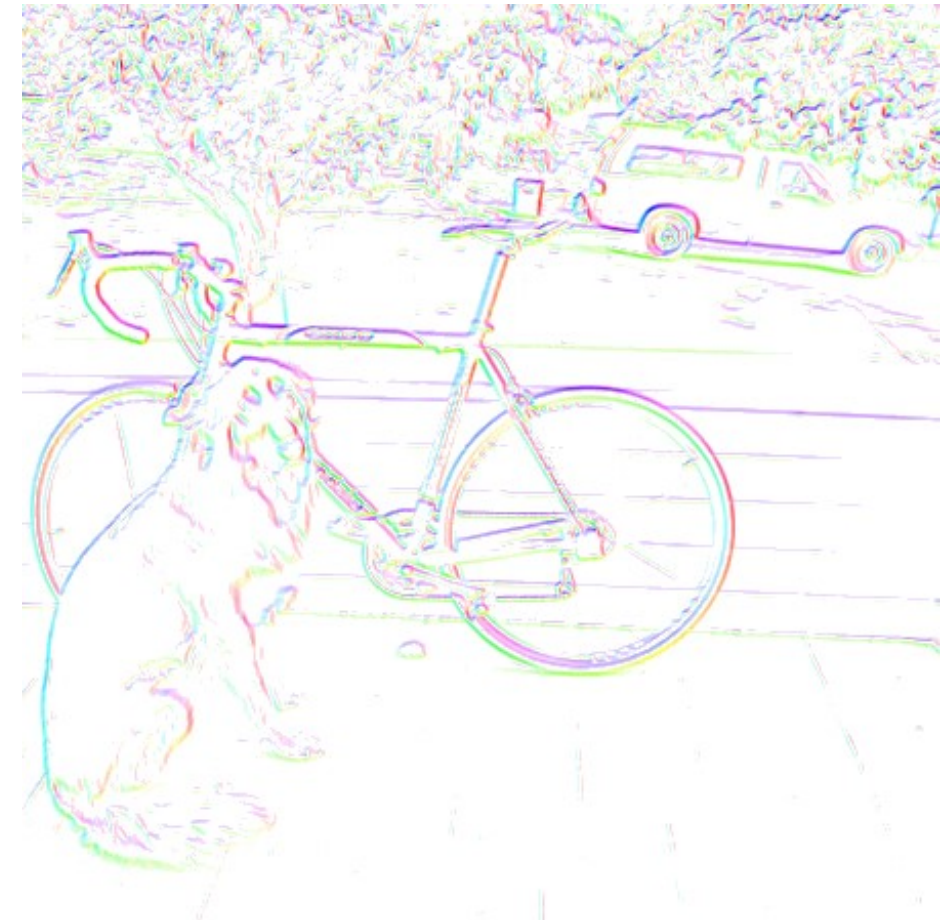
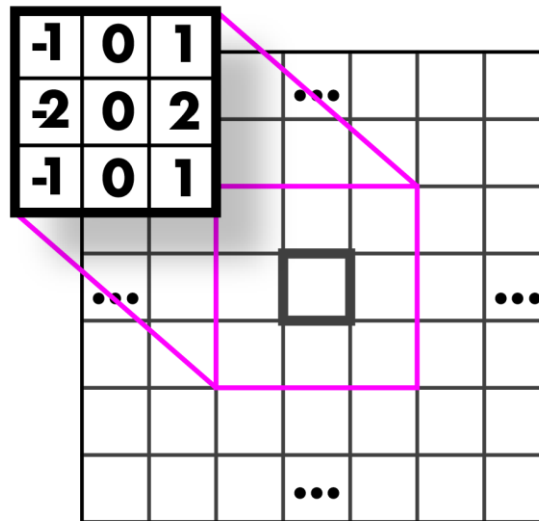
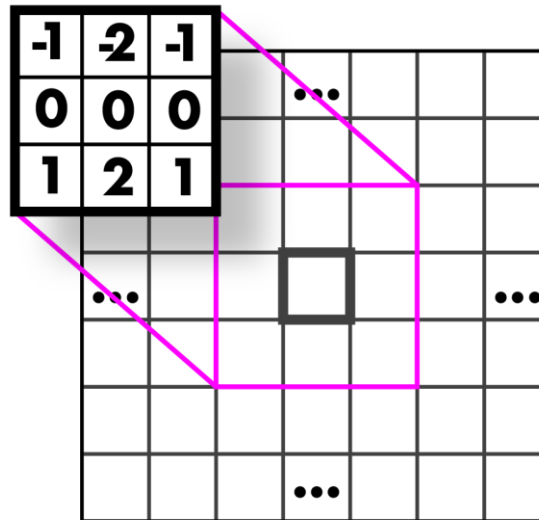


## Sobel Kernels: edges and...

applied to grayscale and thresholded



## Sobel Kernels: edges and gradient!





# Sobel Kernels: edges and gradient!



This visualization is showing the magnitude and direction of the gradient

# And so much more!!

- Next time
  - Edges
  - Features

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# Thank you.

