



University of Cyprus – MSc Artificial Intelligence

# MAI644 – COMPUTER VISION

## Lecture 6: Edges

**Melinos Averkiou**

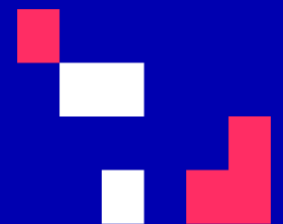
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# Last time

- Averaging vs Interpolation
- Systems - filters
- Convolution
  - Box Filter
  - Gaussian
  - Cross correlation vs Convolution
- Examples of filters

# Today's Agenda

- What can we do with convolutions
- What is an edge – image derivatives
- Sobel filters
- Laplacian filters
- Difference of Gaussian filters
- Canny edge detection

**[material based on Joseph Redmon's course]**

# Today's Agenda

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# What can we do with convolutions

Mathematically: nice linear properties

- Commutative
  - $A*B = B*A$
- Associative
  - $A*(B*C) = (A*B)*C$
- Distributes over addition
  - $A*(B+C) = A*B + A*C$
- Plays well with scalars
  - $x(A*B) = (xA)*B = A*(xB)$

# What can we do with convolutions

This means some convolutions decompose:

- 2d gaussian is just composition of 1d gaussians
  - Faster to run 2 1d convolutions

# What can we do with convolutions

- Blurring
- Sharpening
- Edges
- Features
- Derivatives
- Super-resolution
- Classification
- Detection
- Image captioning
- ...

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A big part of computer vision  
is **convolutions**



So what can we do with these convolutions anyway?

- Blurring
- Sharpening
- Edges
- Features
- Derivatives
- Super-resolution
- Classification
- Detection
- Image captioning
- ...

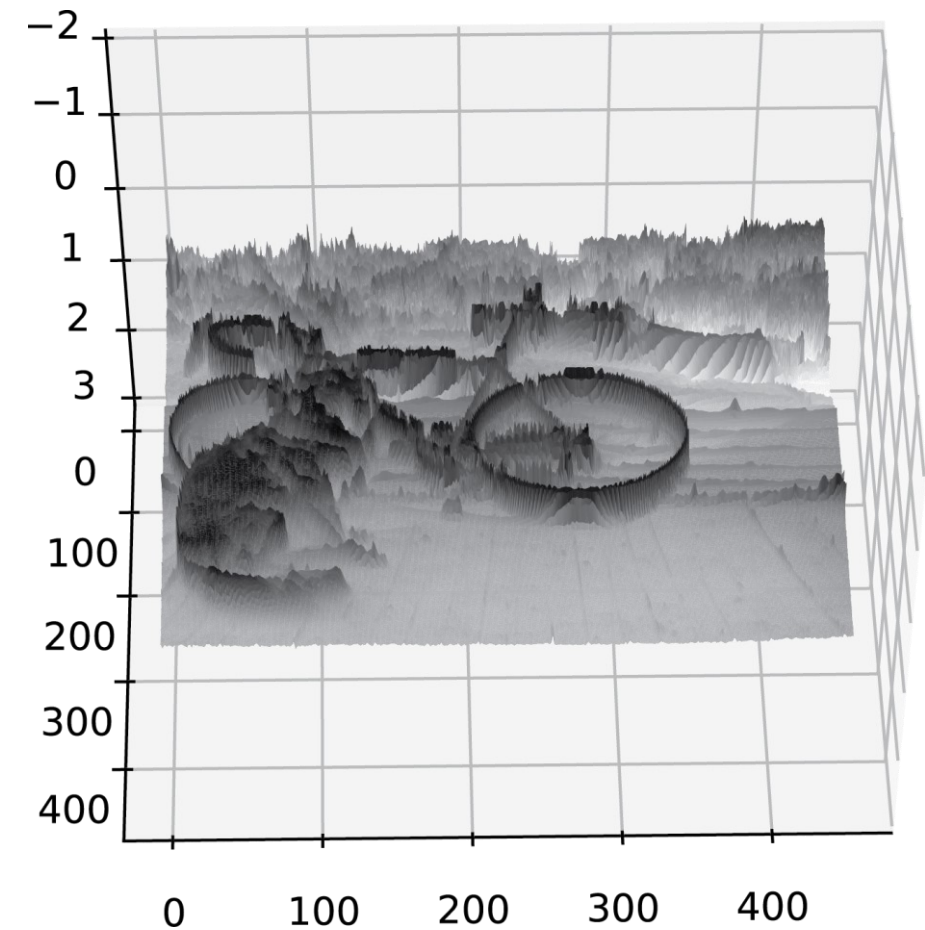
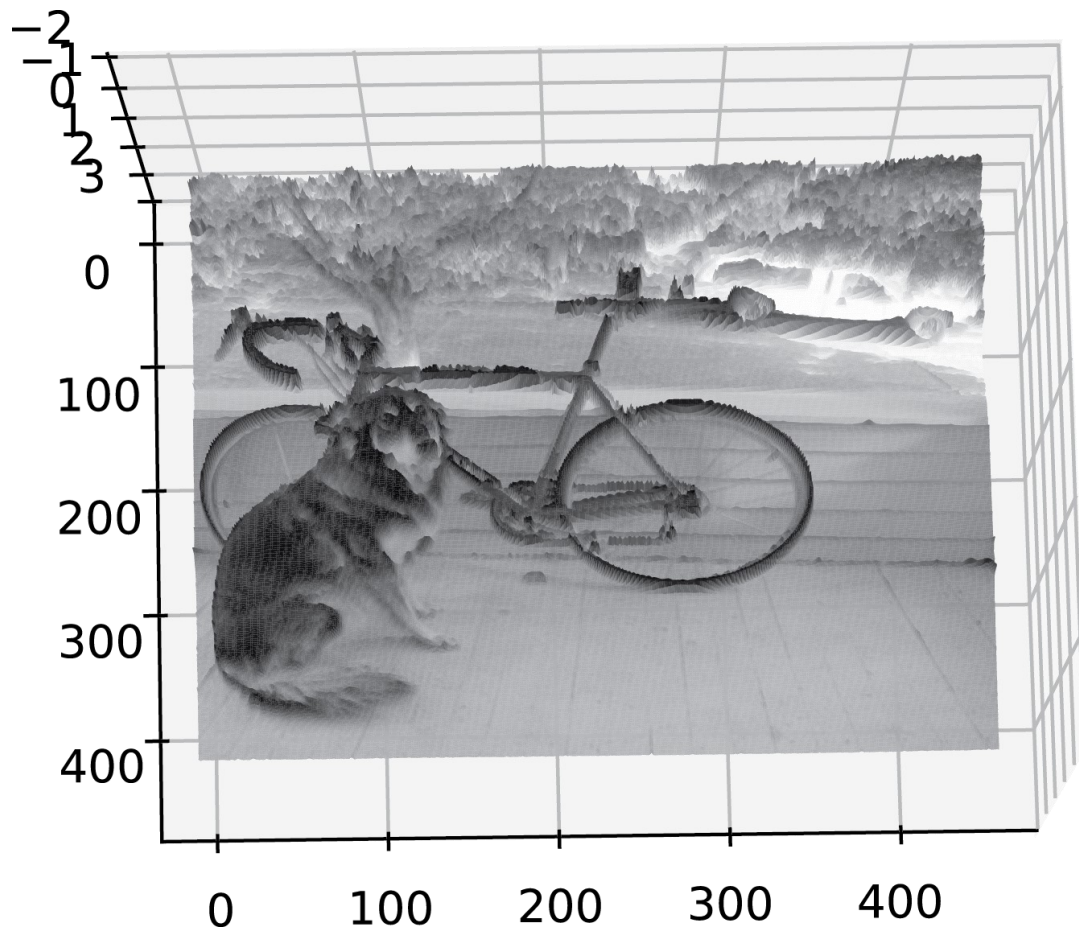
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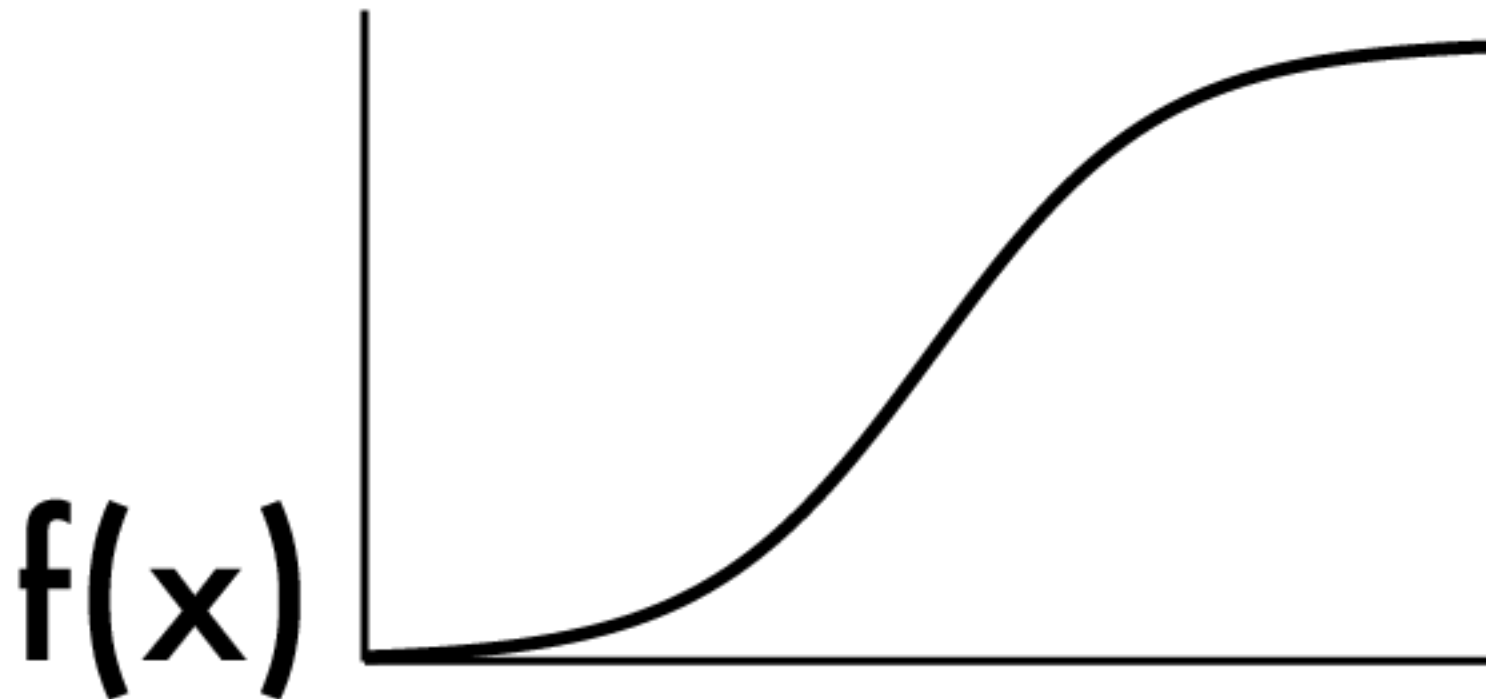
# What's an edge?

- Image is a function
- Edges are rapid changes in this function



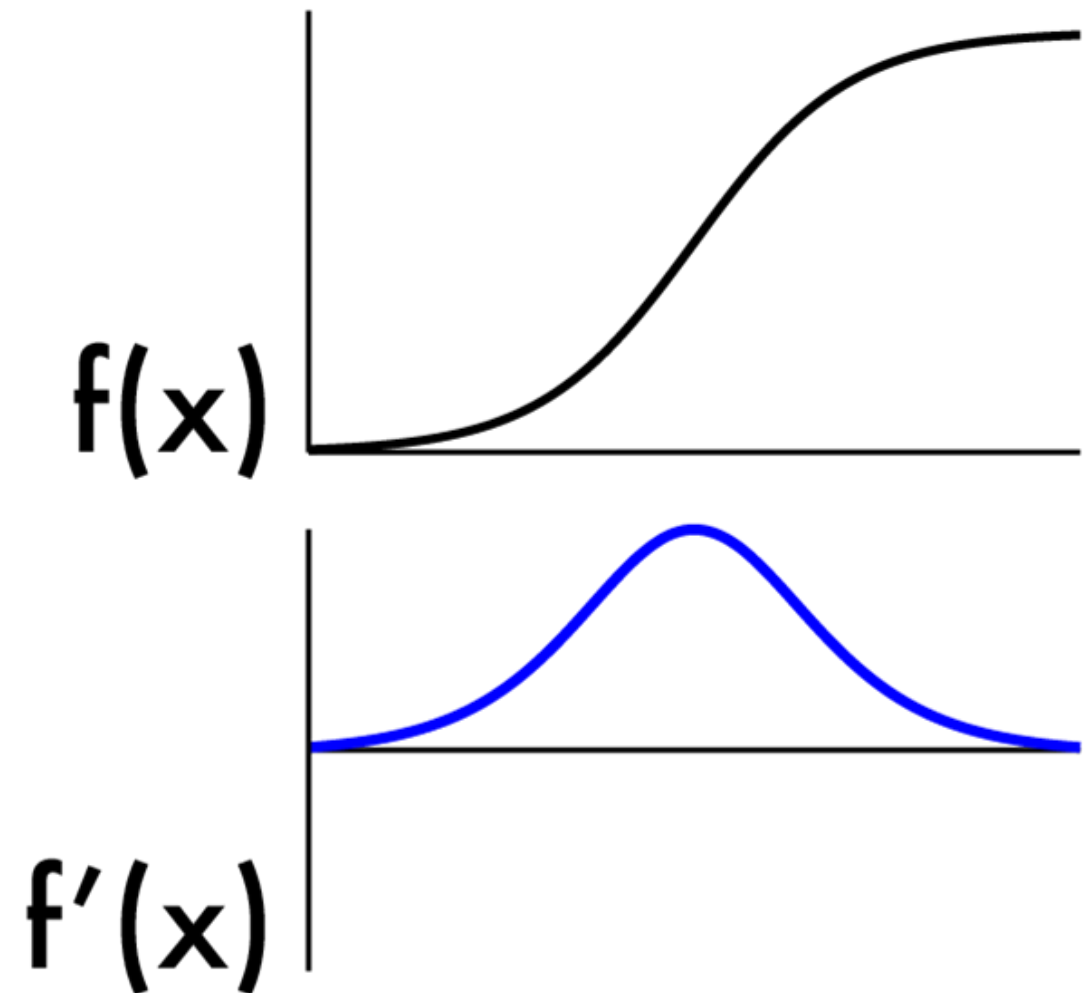
# What's an edge?

- Image is a function
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# Finding edges

- Could take derivative
- Edges = high response

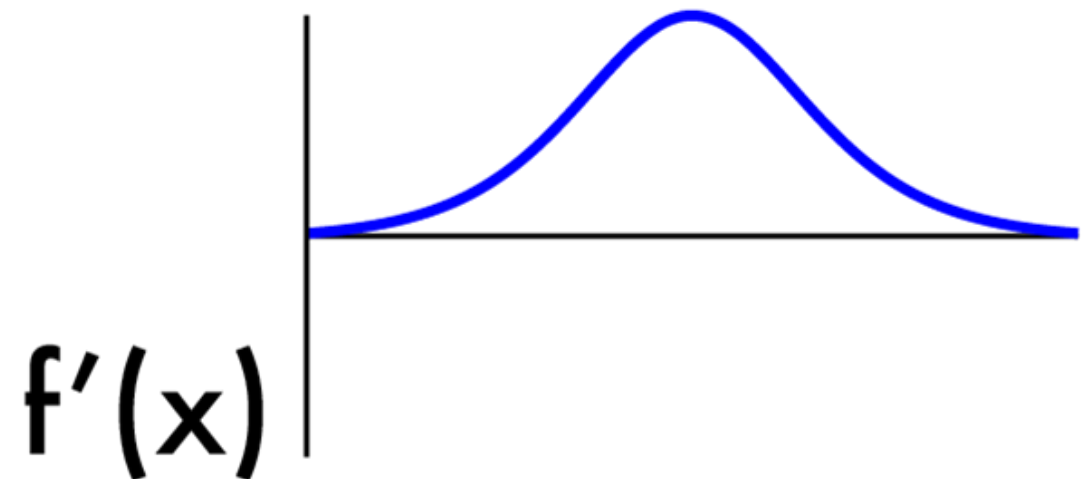
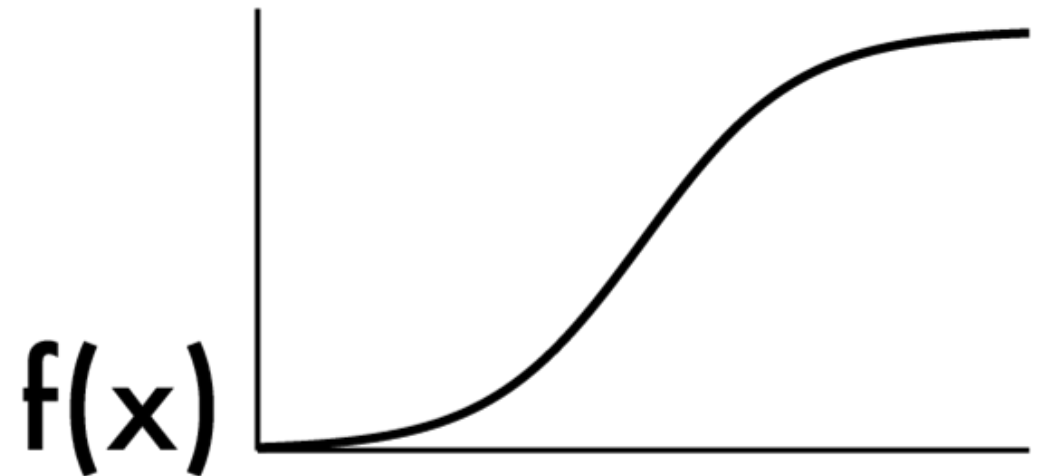


# Image derivatives

- Recall:

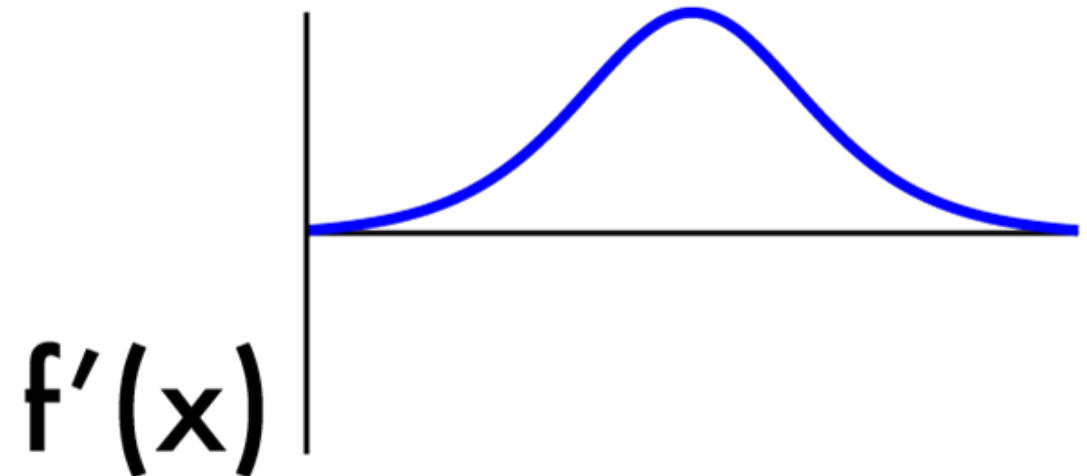
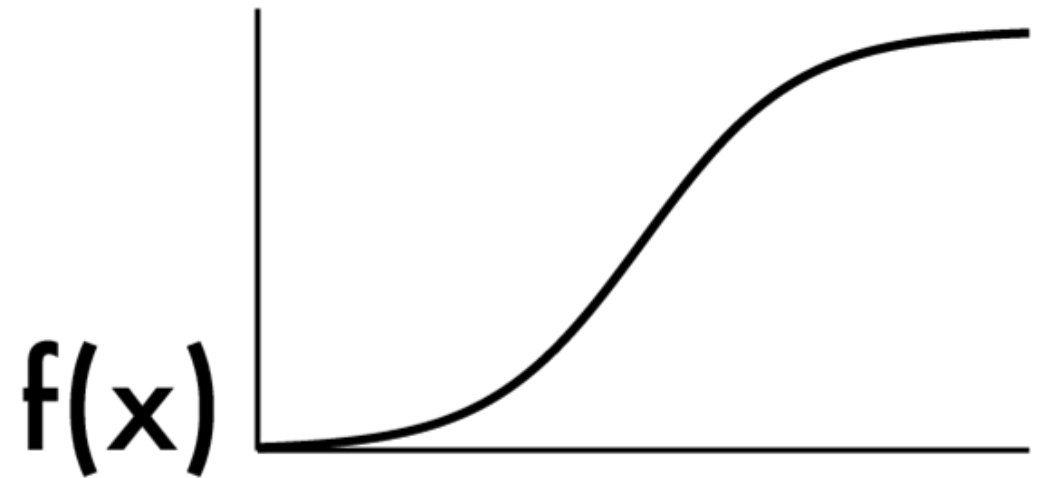
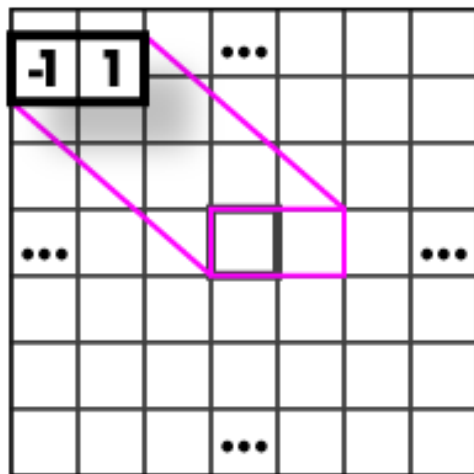
$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- We don't have an "actual" function, must estimate
- Possibility: set  $h = 1$
- What will that look like?



## Image derivatives

- Recall:
  - $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ .
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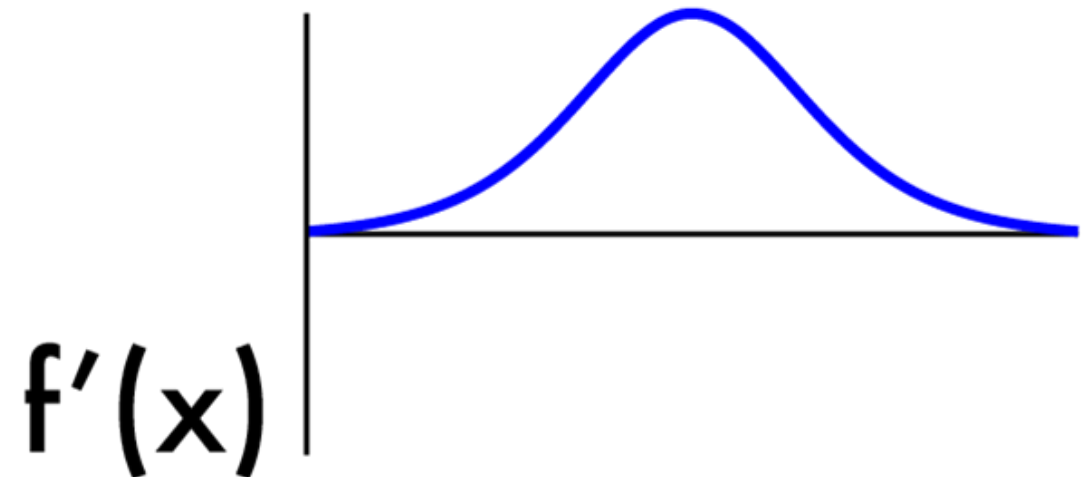
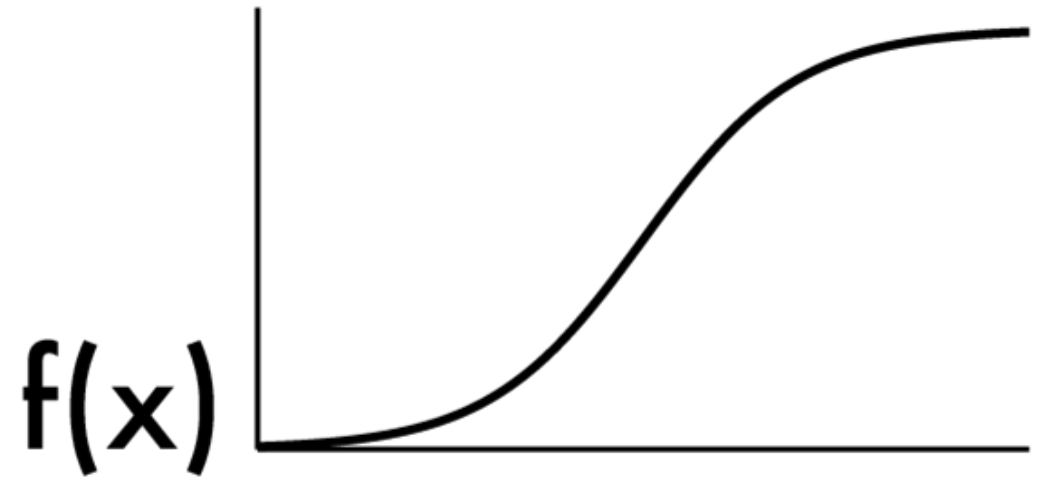


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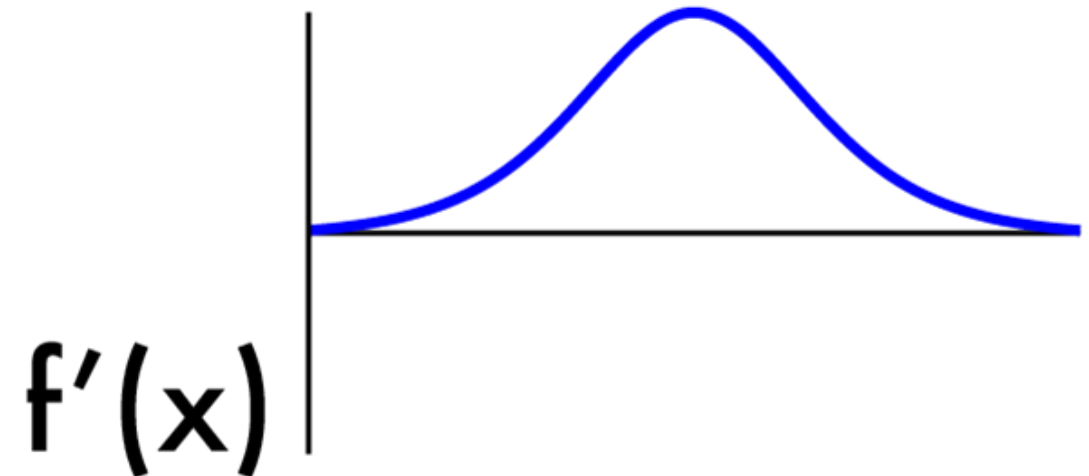
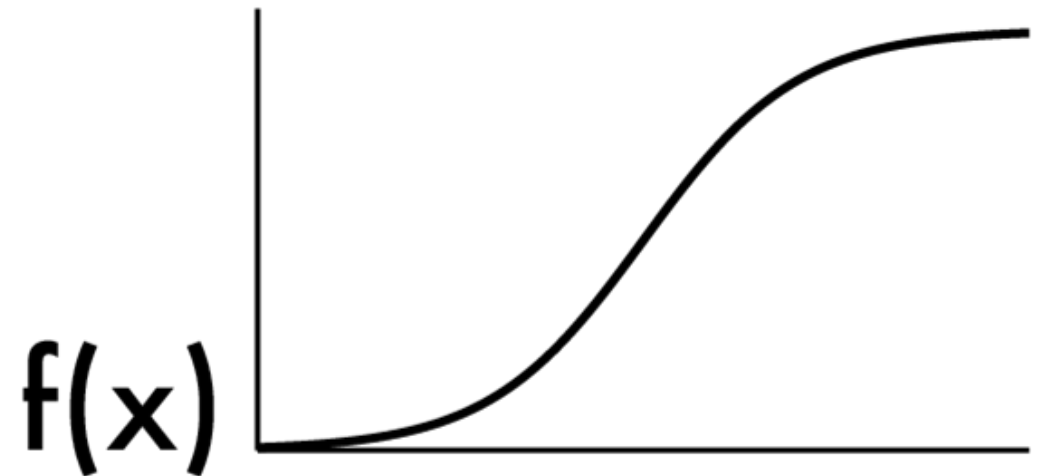
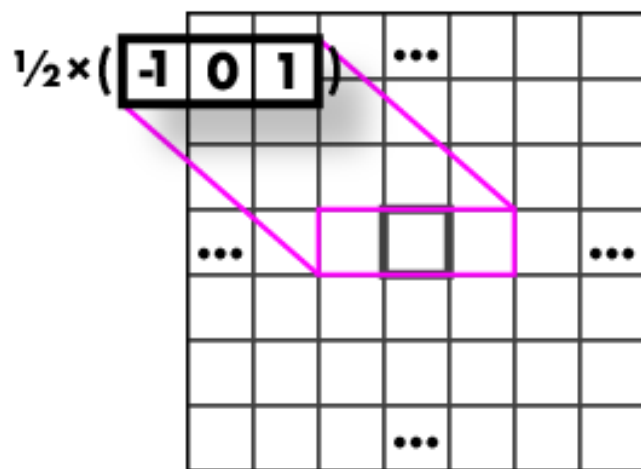
- We don't have an "actual" function, must estimate
- Possibility: set  $h = 2$
- What will that look like?





# Image derivatives

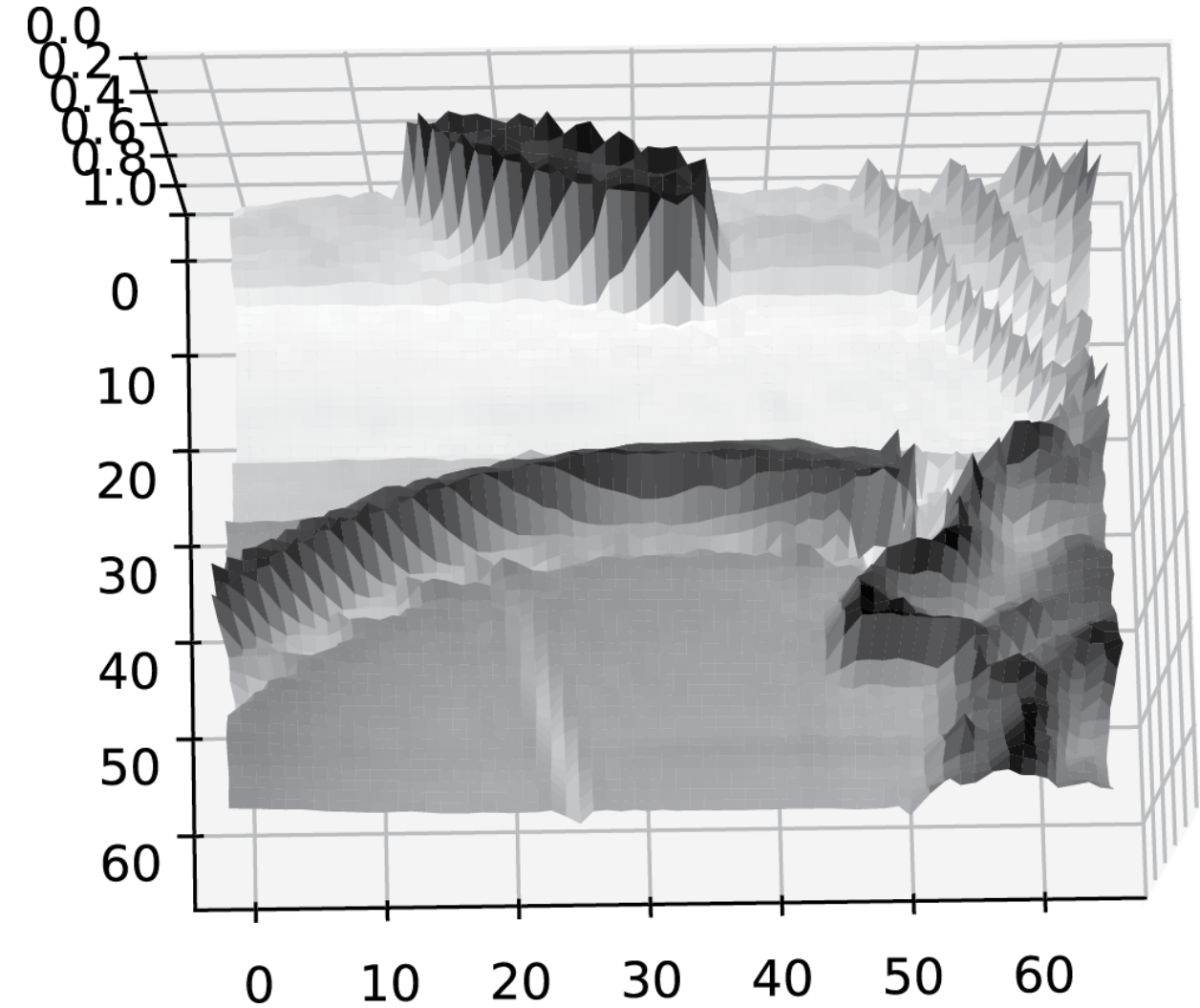
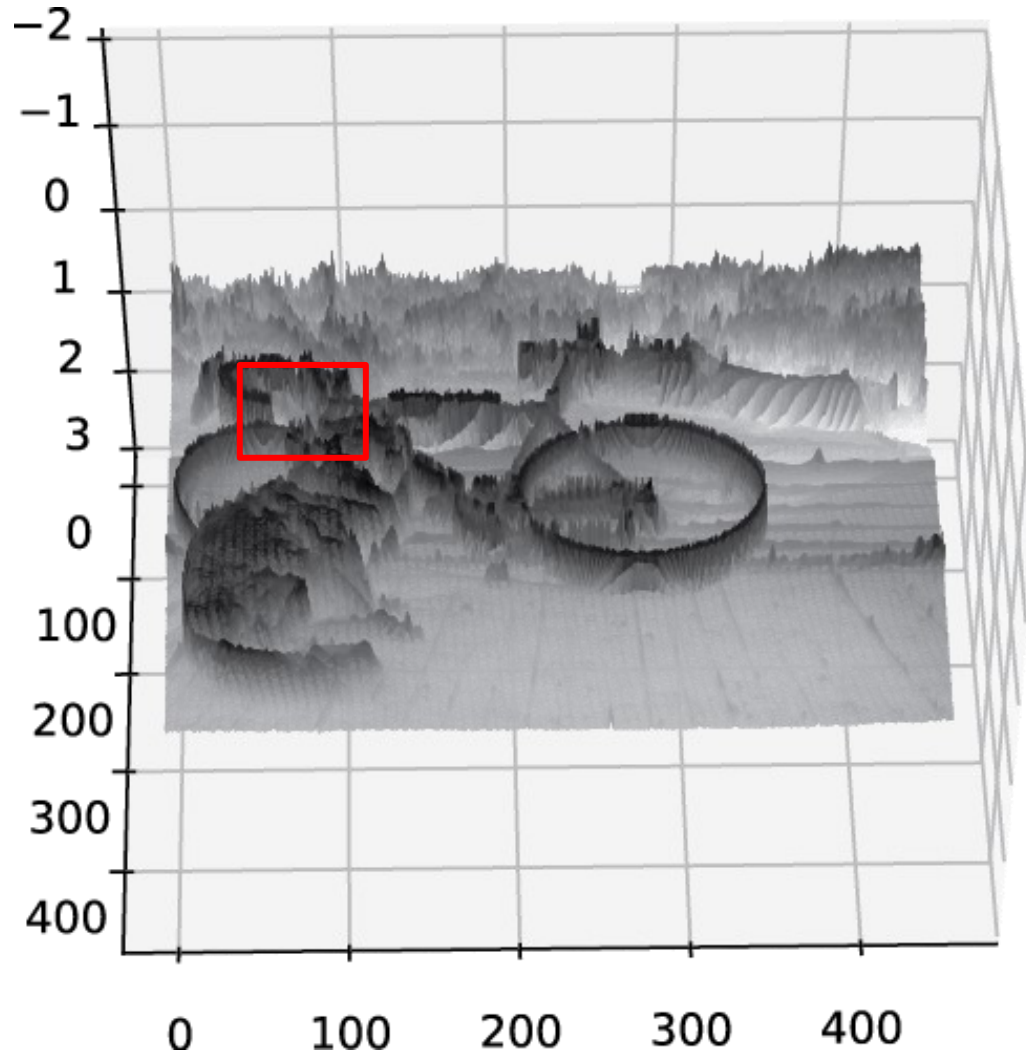
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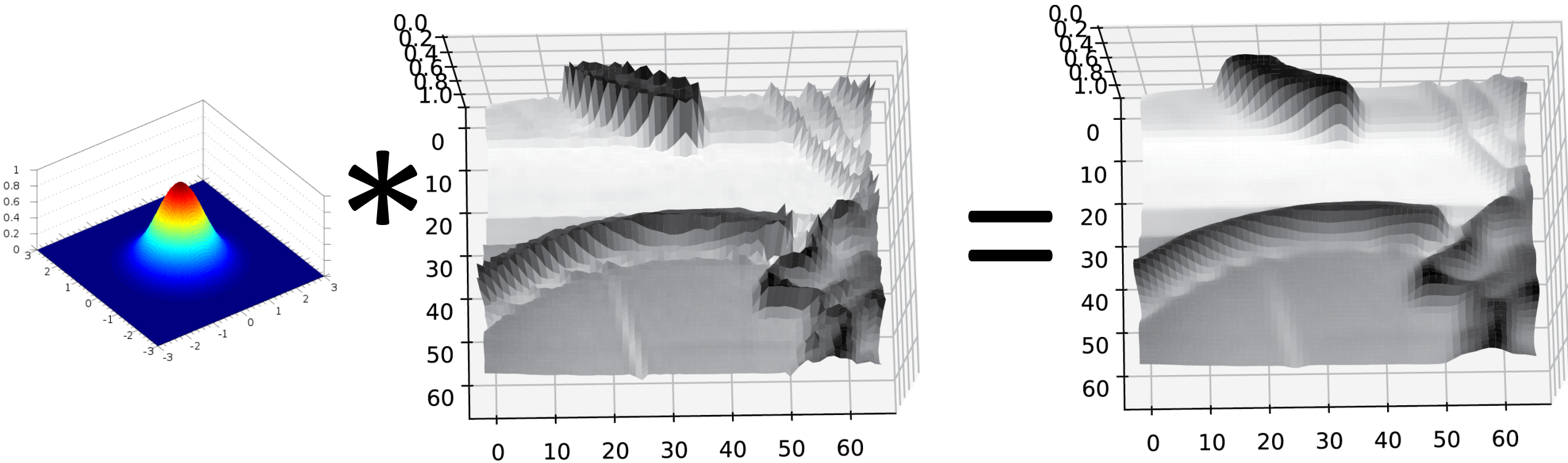
# Today's Agenda

- What can we do with convolutions
- What is an edge – image derivatives
- Sobel filters
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## Images are noisy!



## But we already know how to smooth



# Smooth first, then derivative

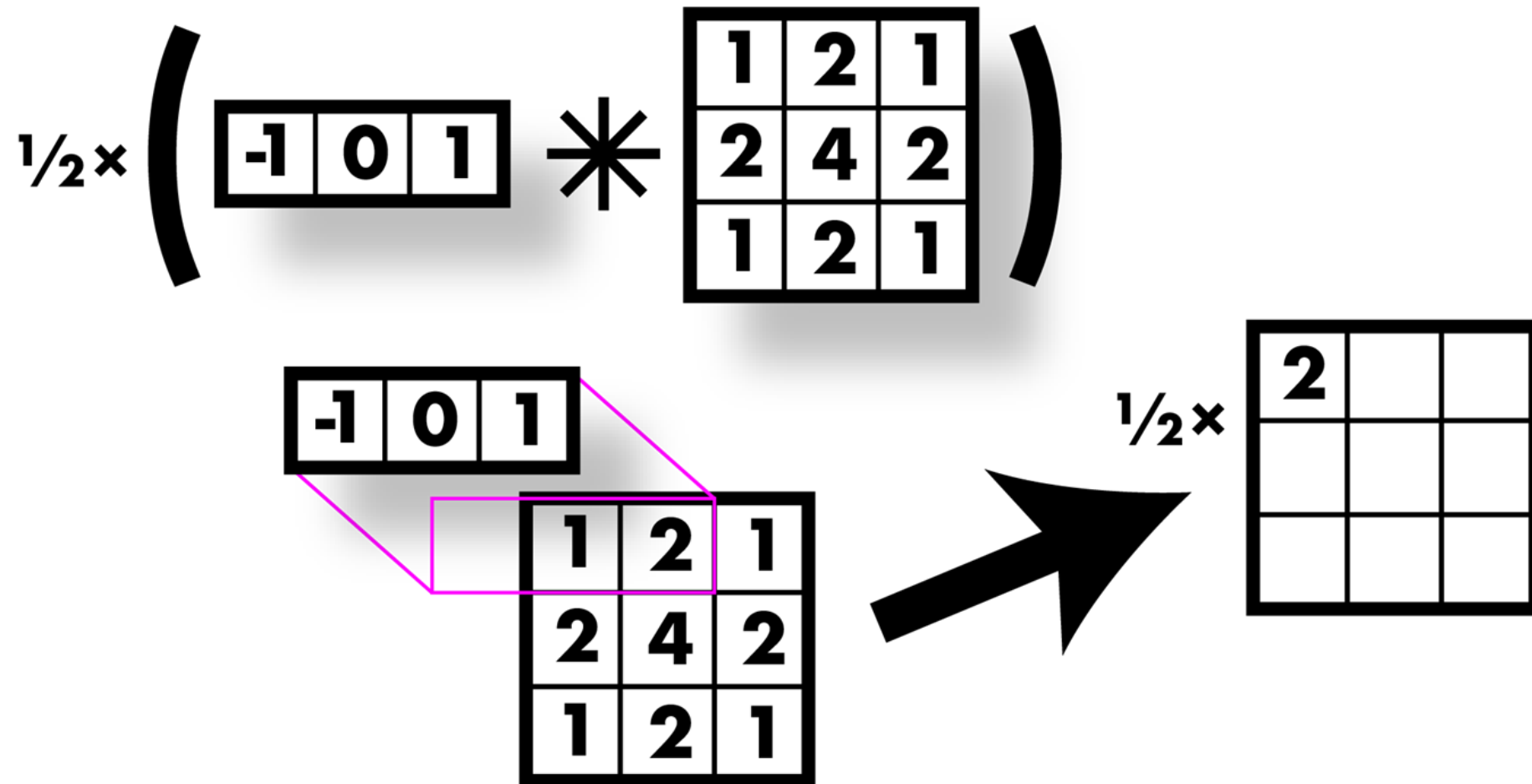
$$\frac{1}{2} \times \begin{pmatrix} -1 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * \begin{pmatrix} \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} & \phantom{0} \end{pmatrix}$$

# Smooth first, then derivative

$$\frac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right) *$$

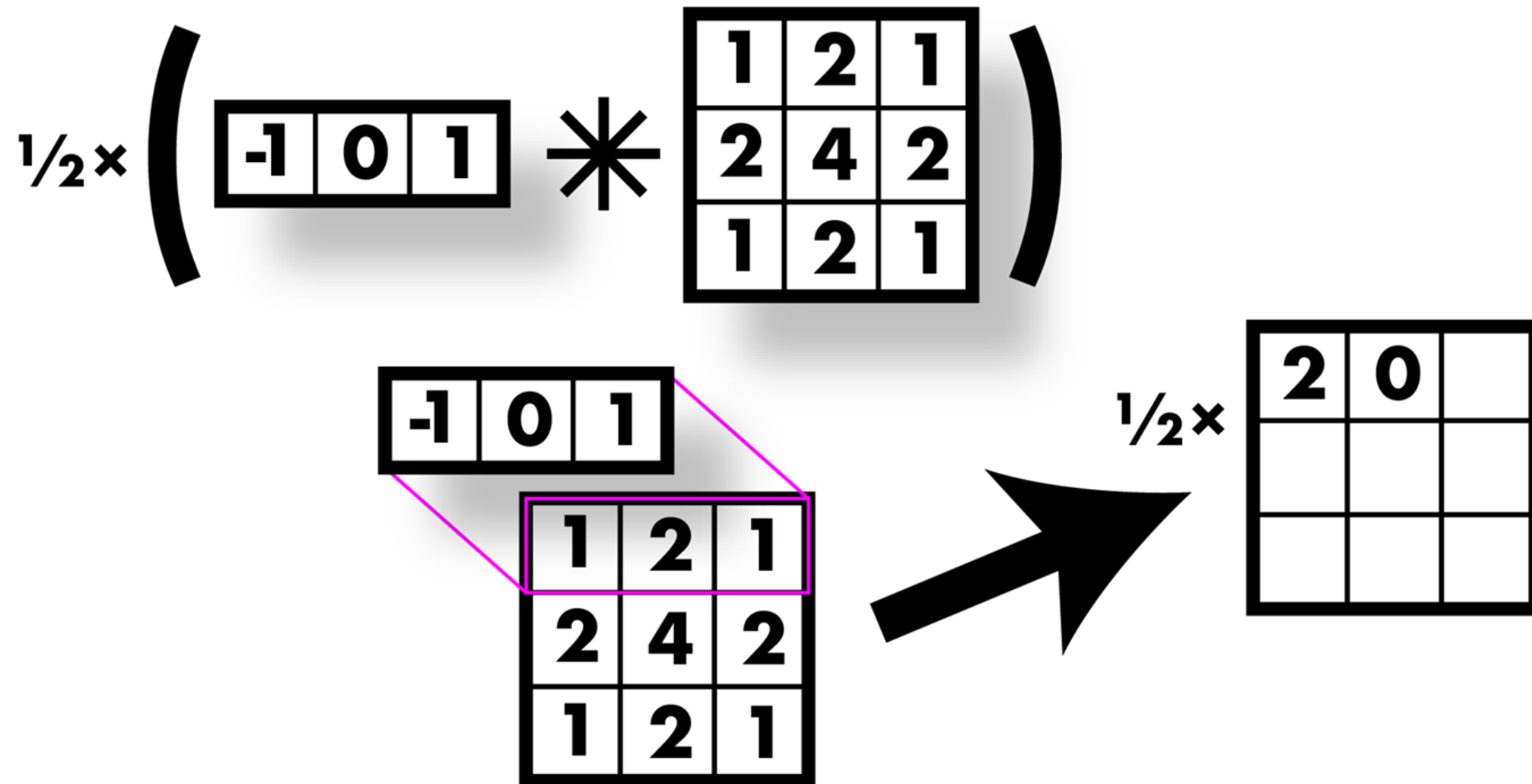



# Smooth first, then derivative



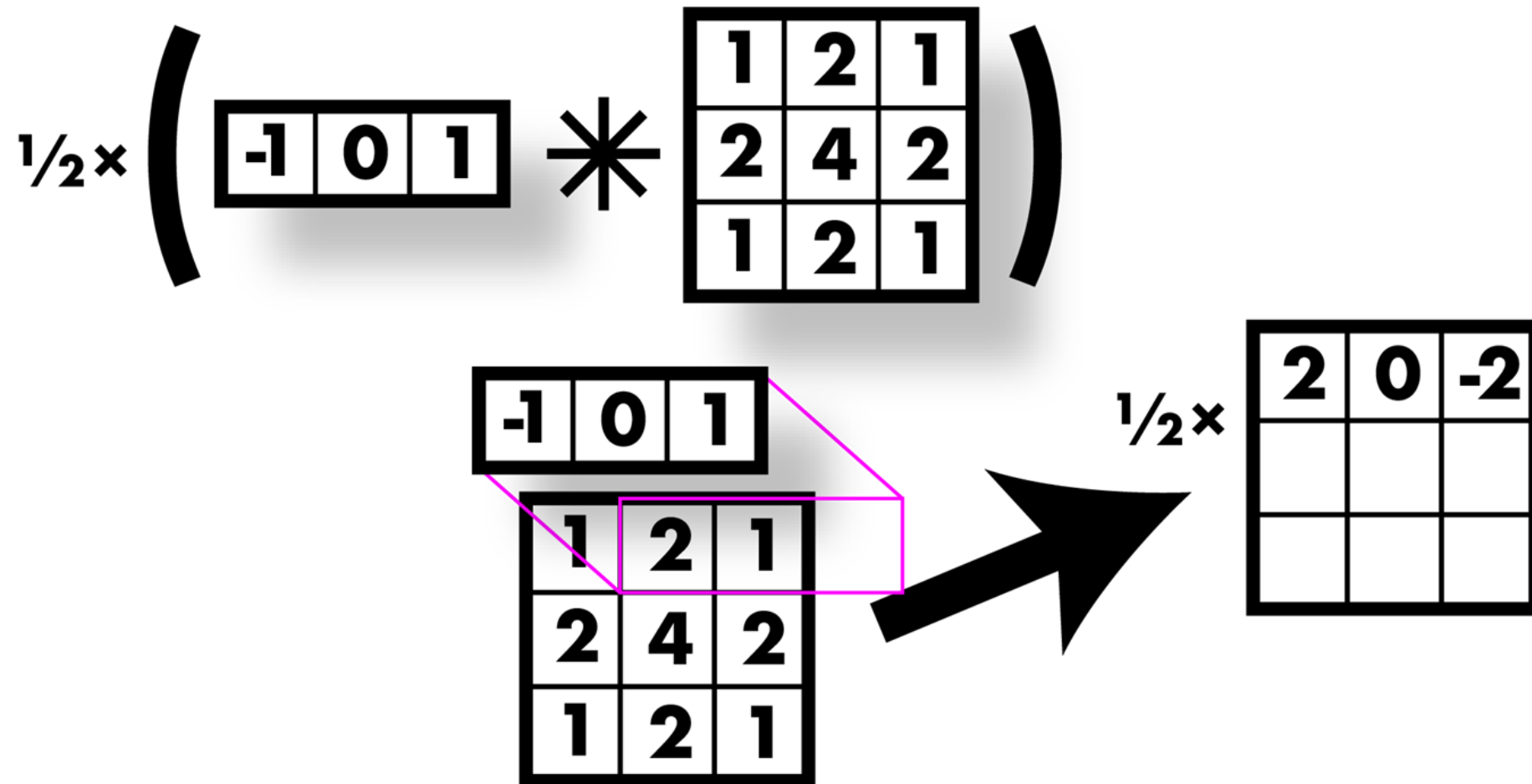


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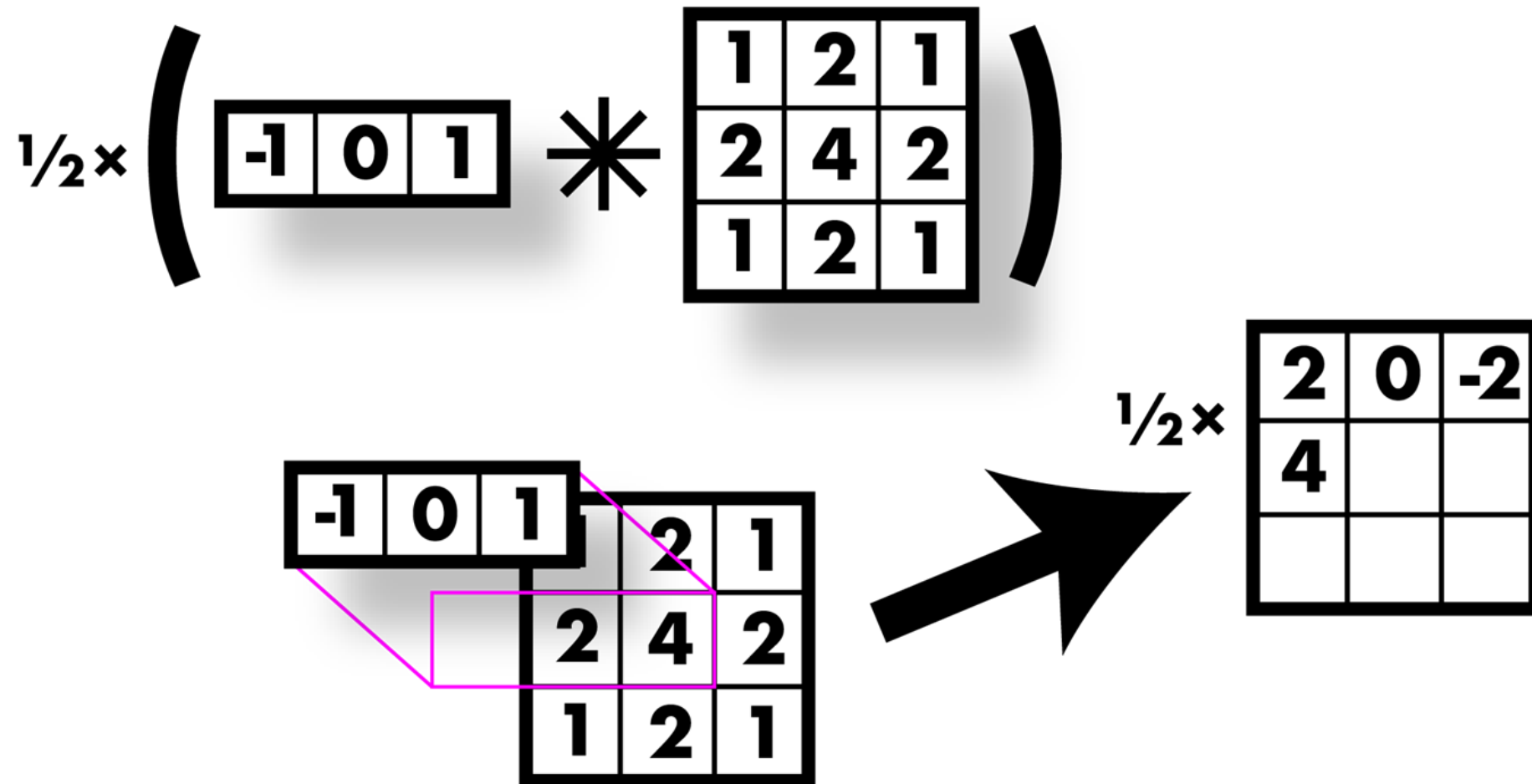




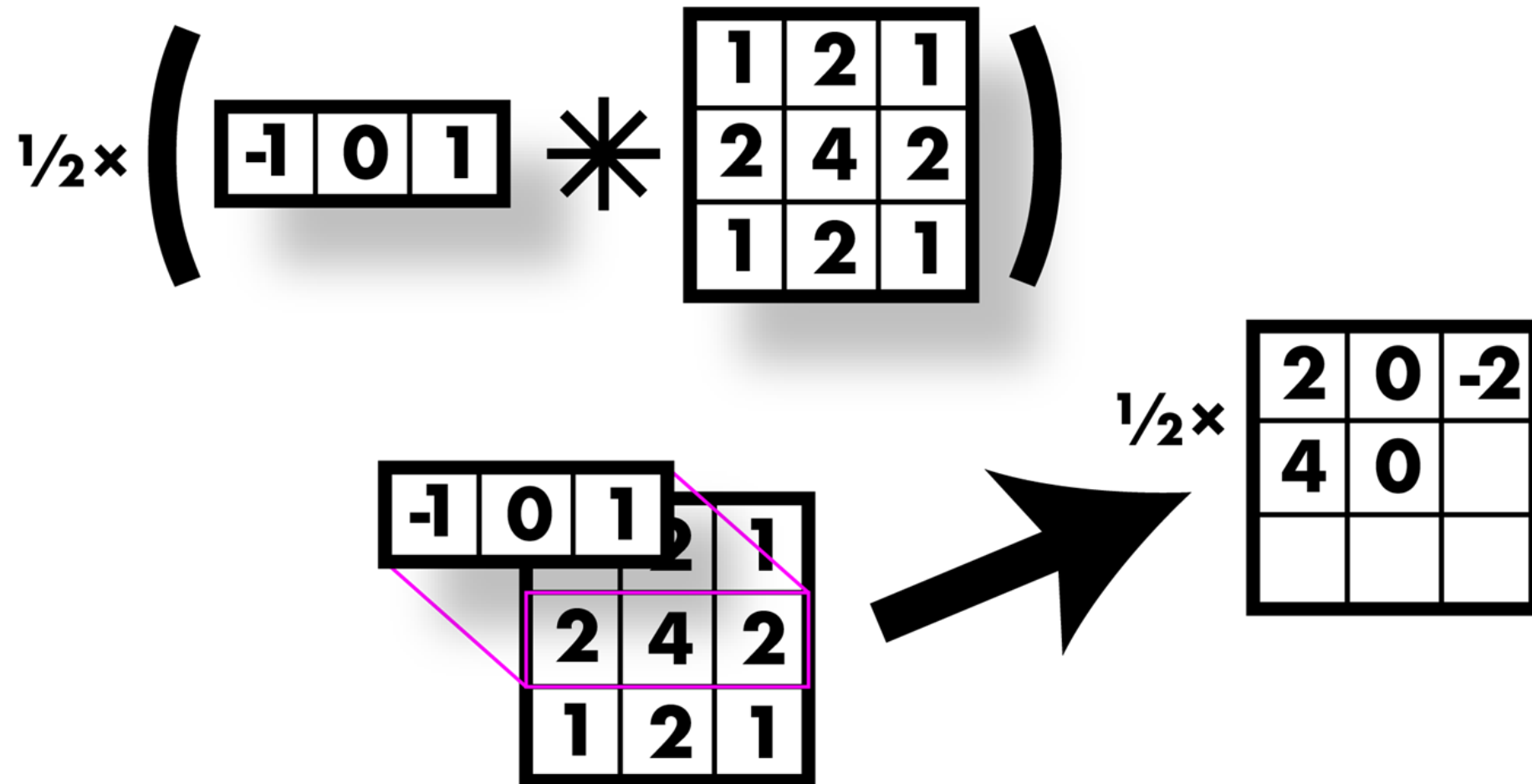
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## Smooth first, then derivative

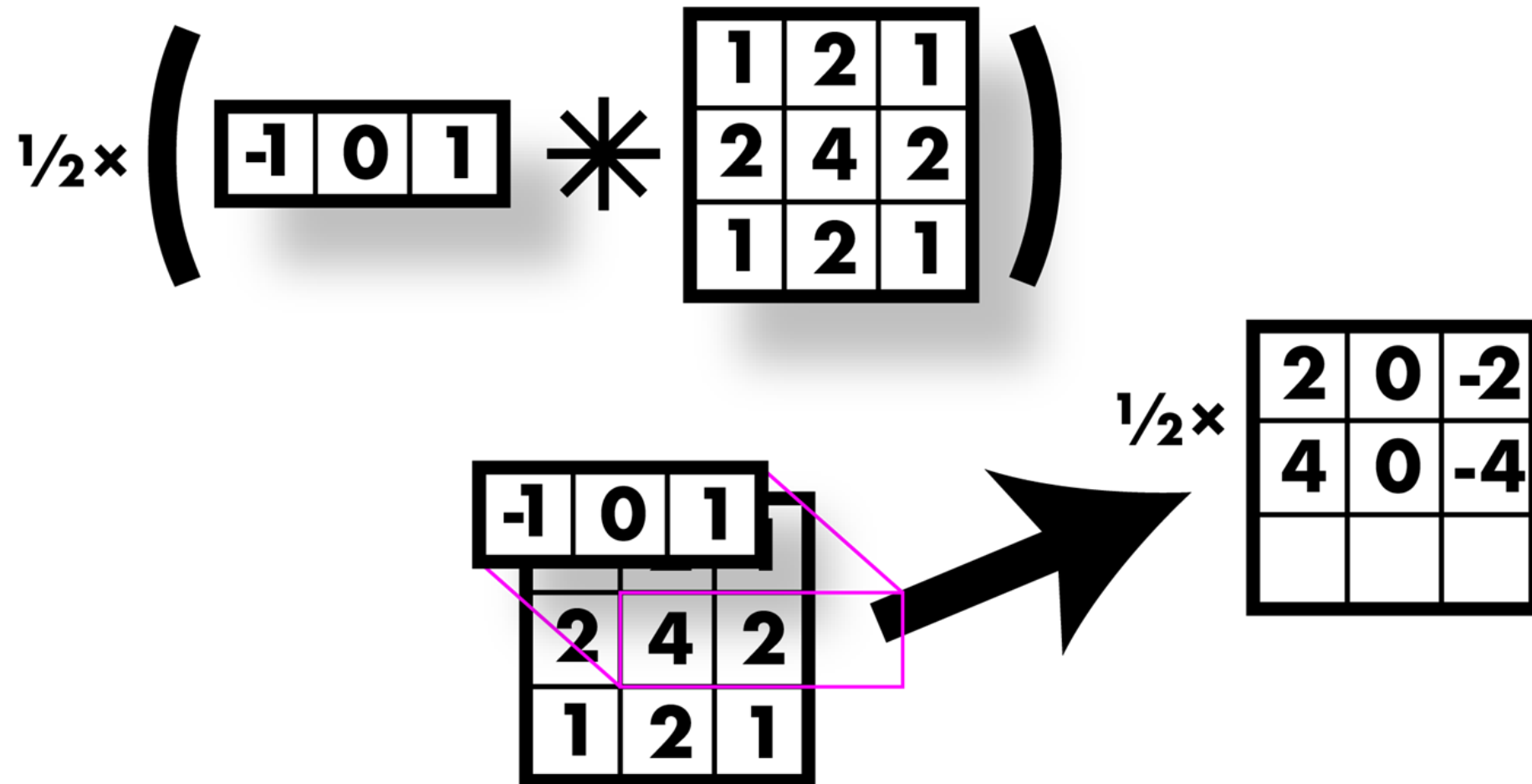


# Smooth first, then derivative



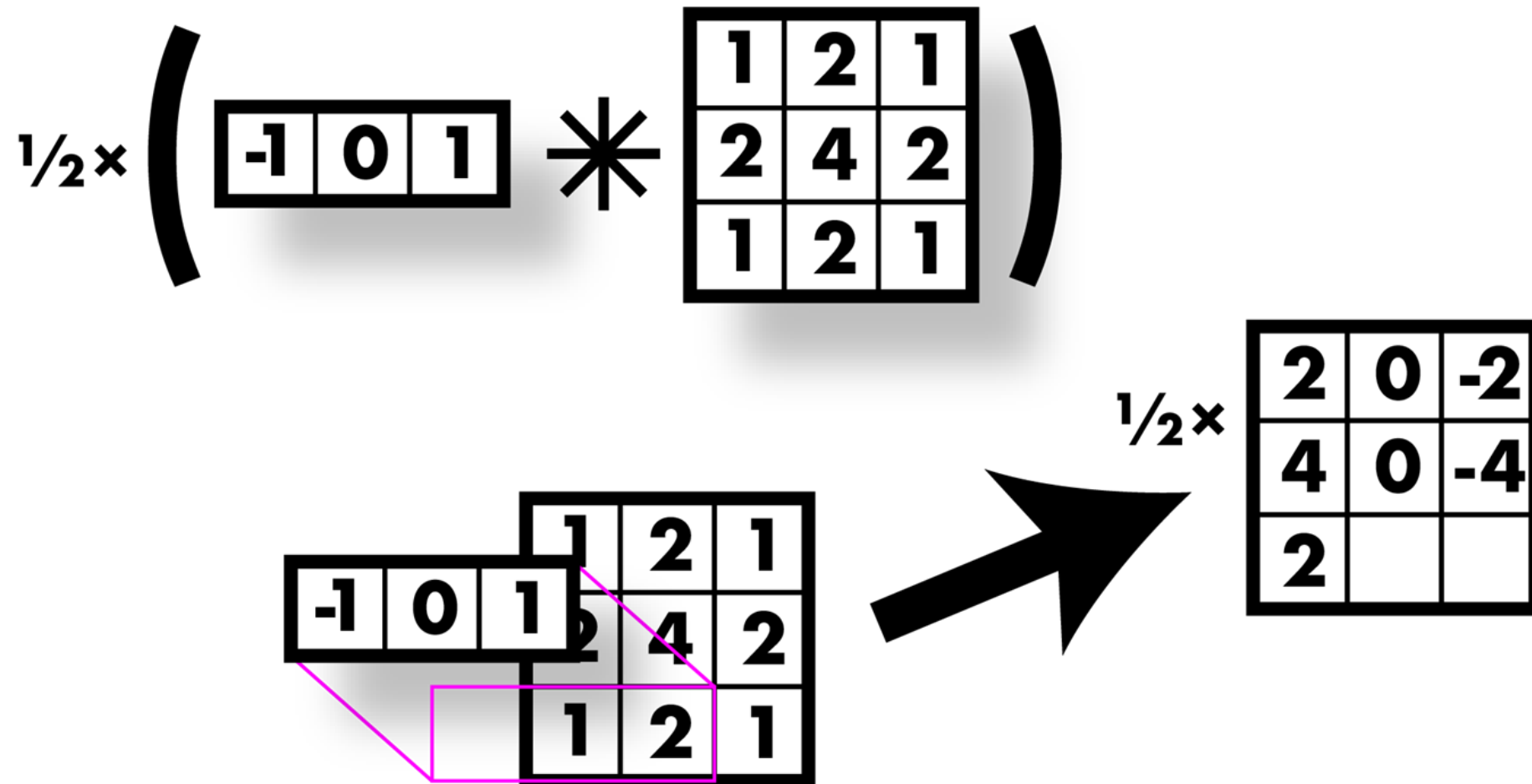


# Smooth first, then derivative



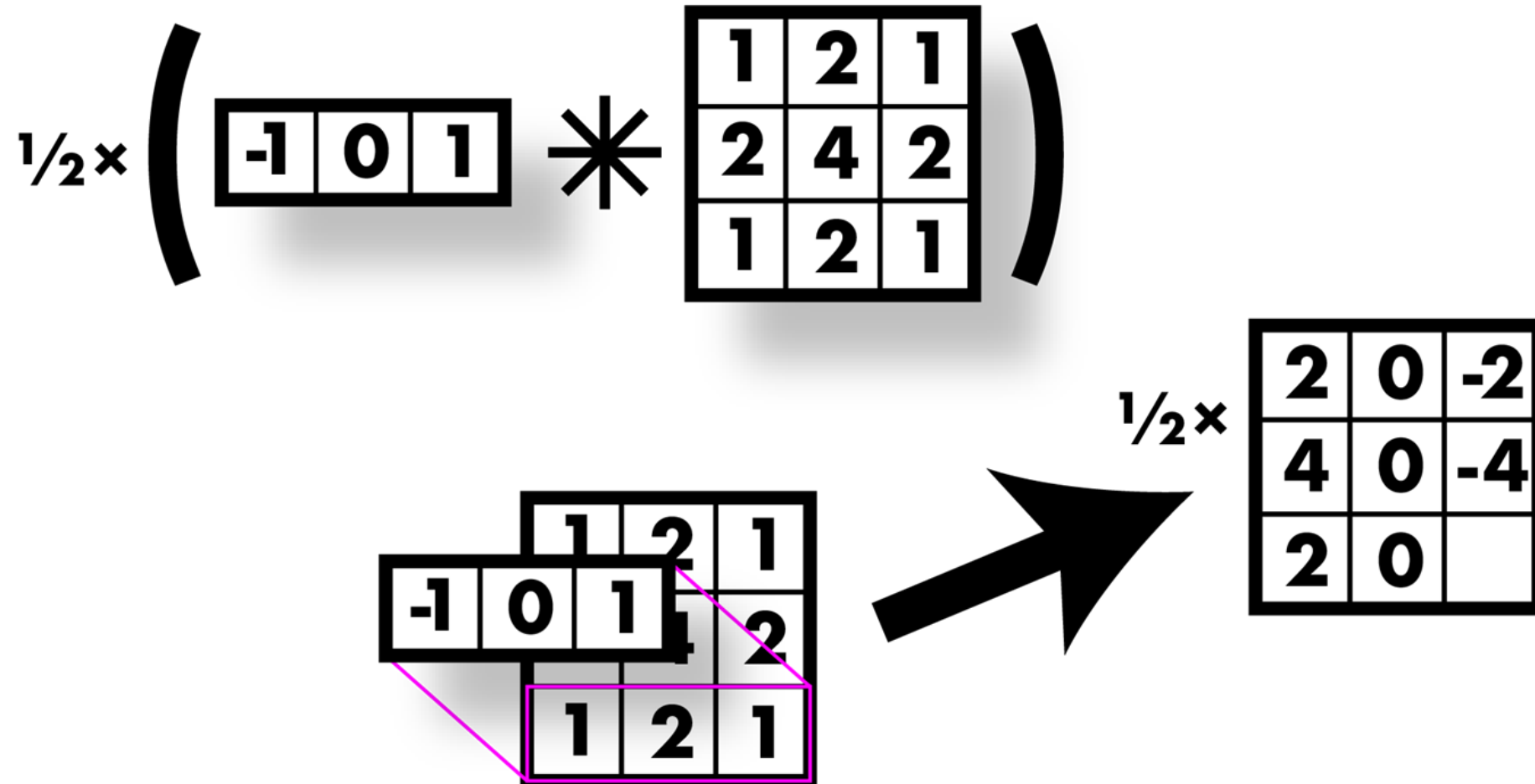


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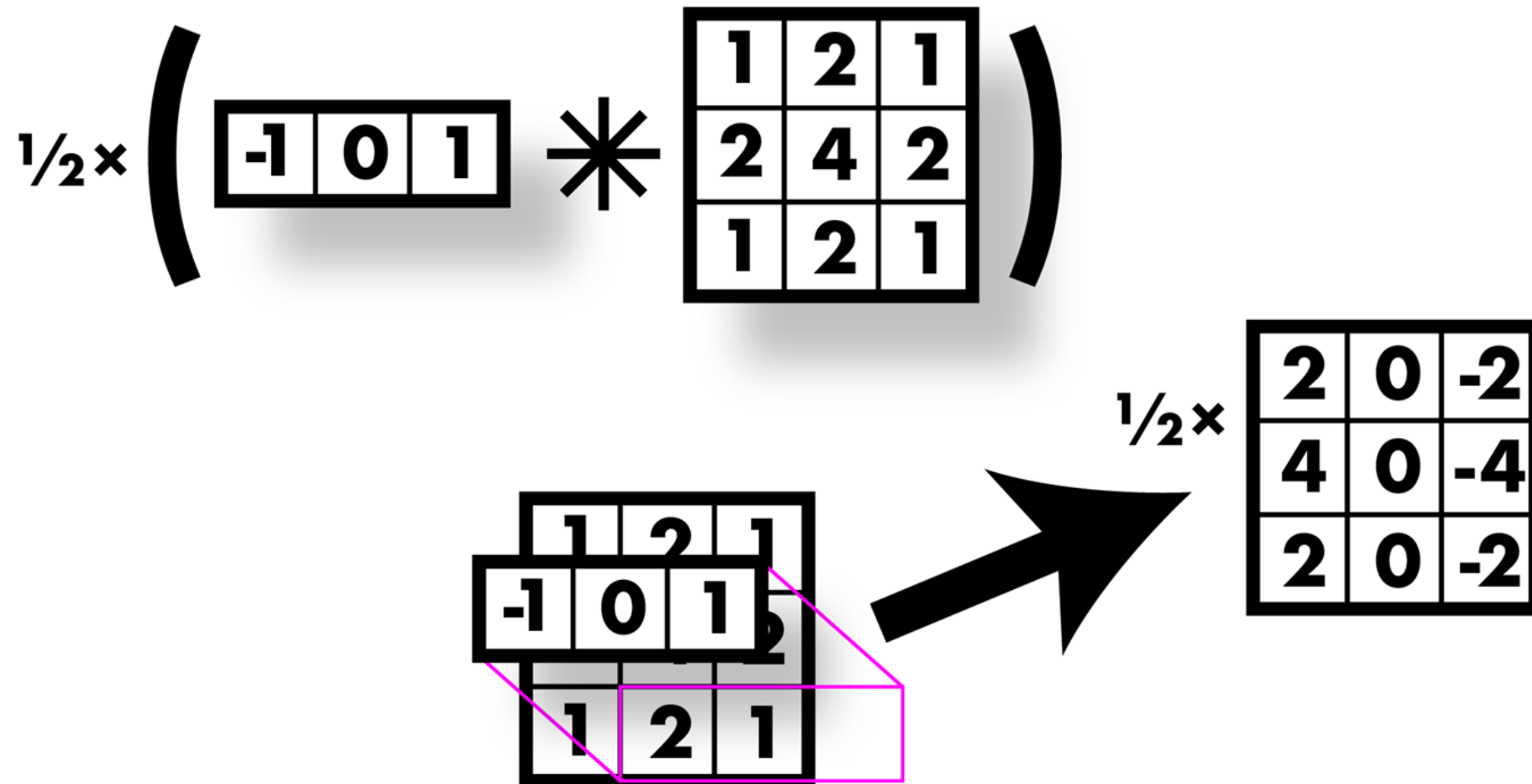


# Smooth first, then derivative





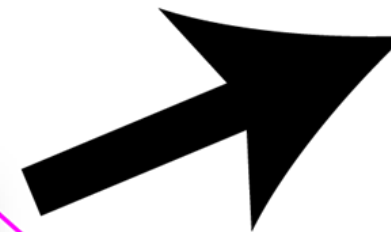
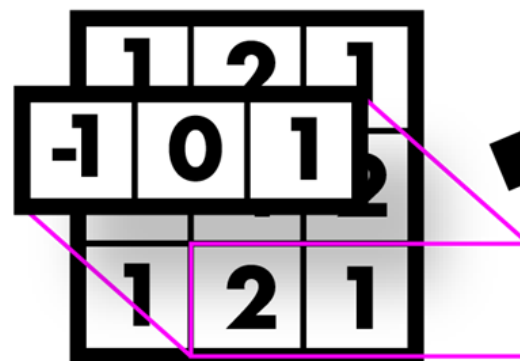
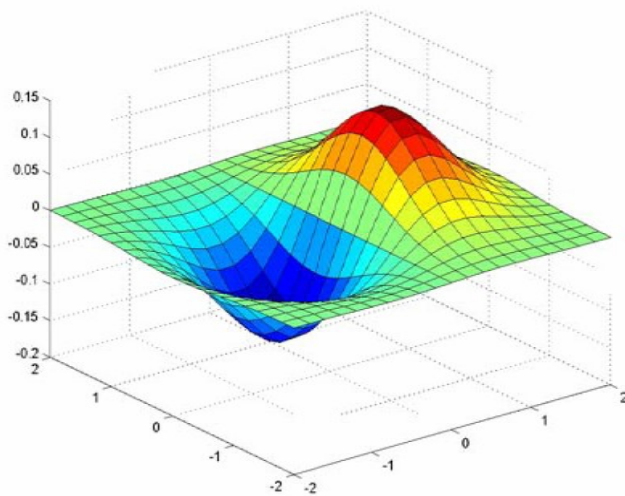
## Smooth first, then derivative





## Sobel filter! Smooth & derivative

$$\frac{1}{2} \times \left( \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$



1	0	-1
2	0	-2
1	0	-1



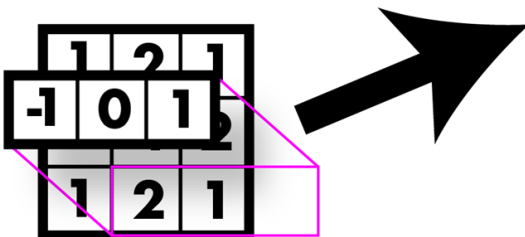


## Image derivatives

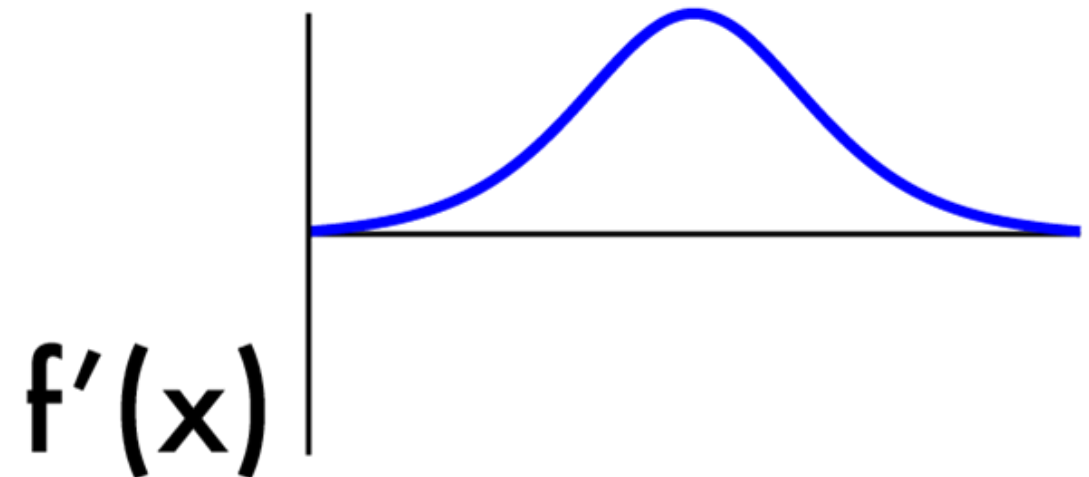
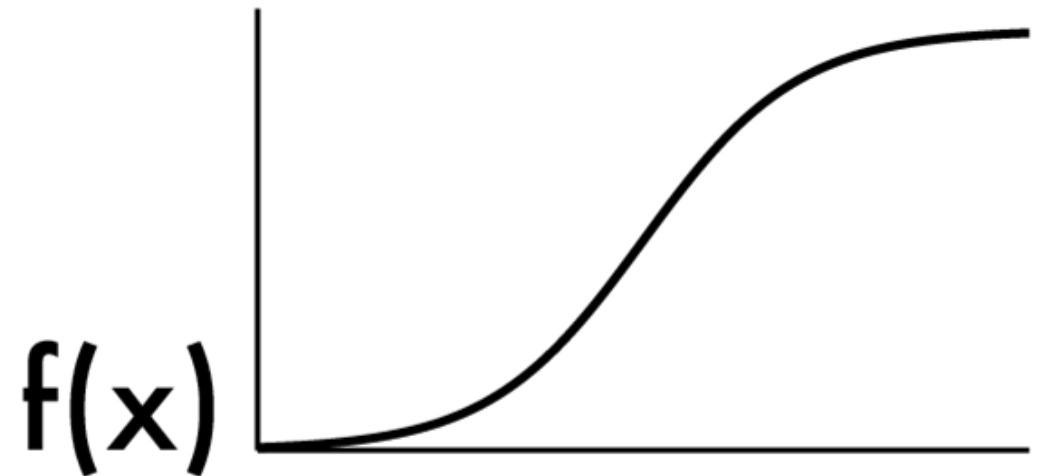
- Recall:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- Want smoothing too!

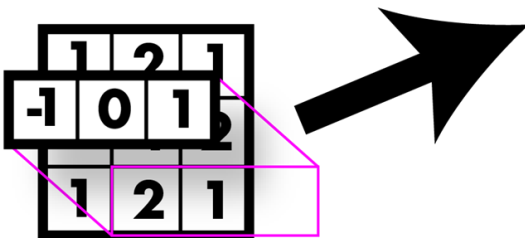
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$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

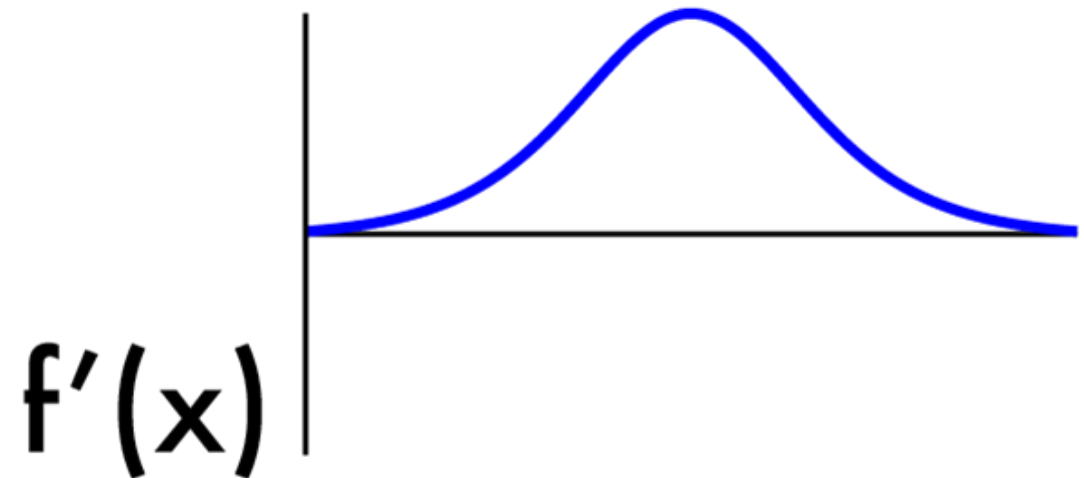
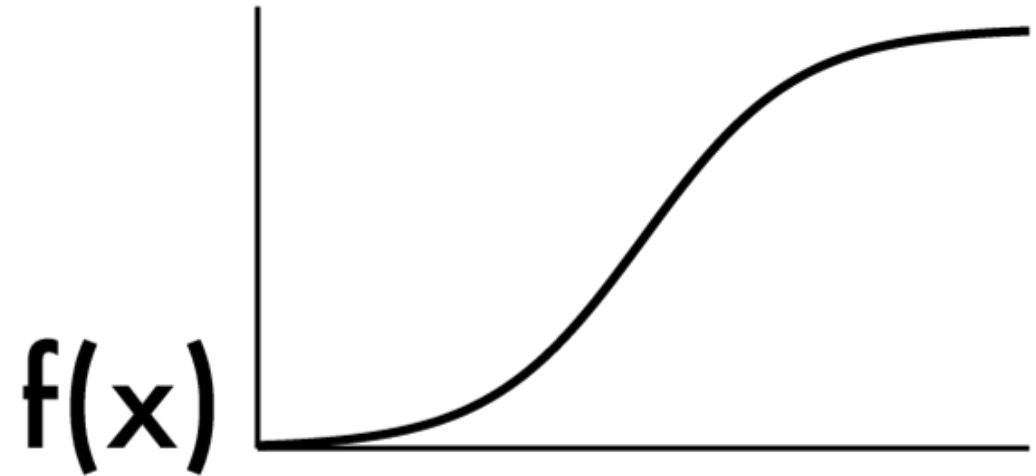


## Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- What about y direction ?

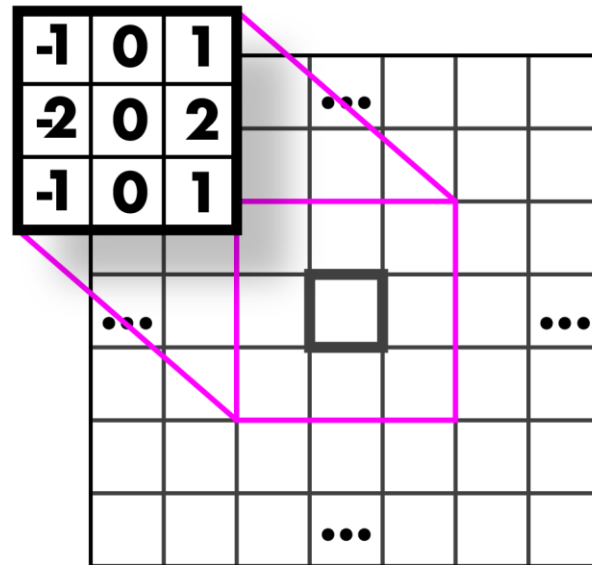
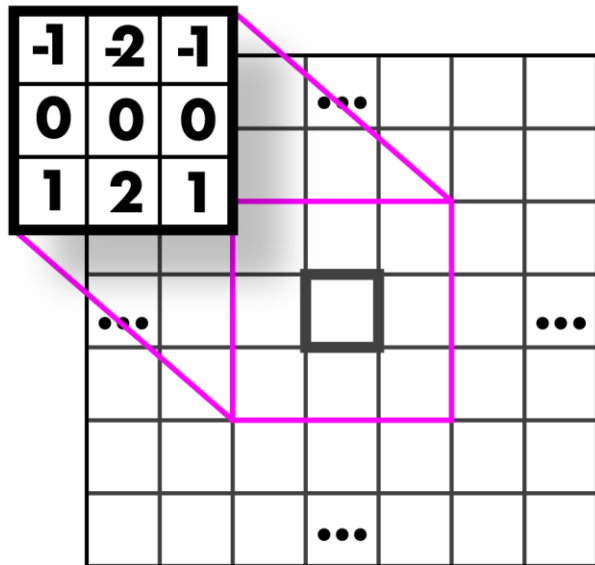
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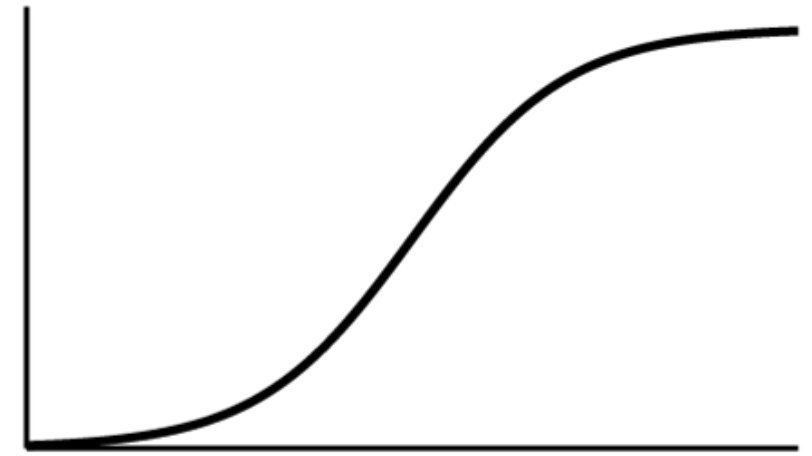


## Finding edges

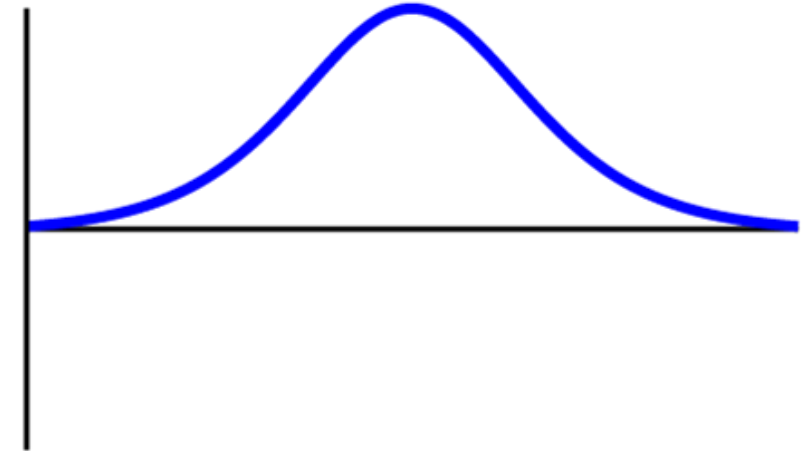
- Could take derivative
- Find high responses
- Sobel filters!
- Let's stop a moment and get some basics



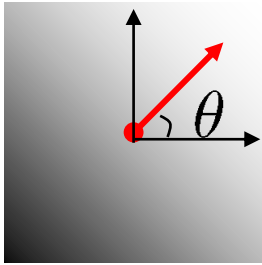
$f(x)$



$f'(x)$



# Simplest image gradient



$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

$$\frac{\partial f}{\partial x} = f(x + 1, y) - f(x, y)$$

Likewise for df/dy

The **gradient direction** is  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$

How does this relate to the direction of the edge? -Perpendicular

The *edge strength* is given by the **gradient magnitude**

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

## Sobel filters

$$g_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad g_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

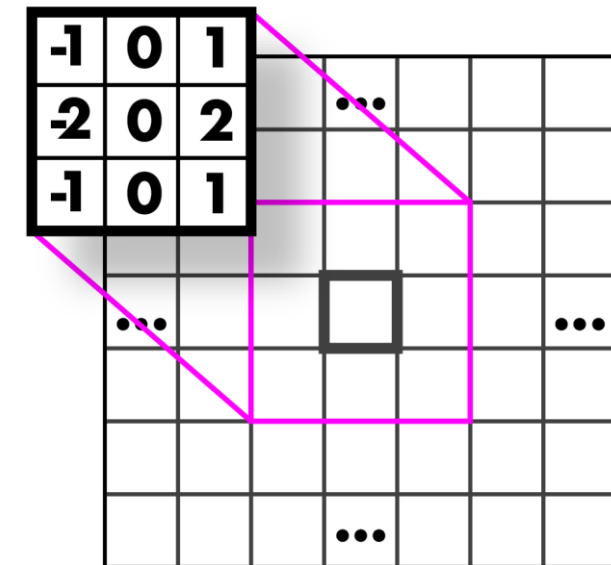
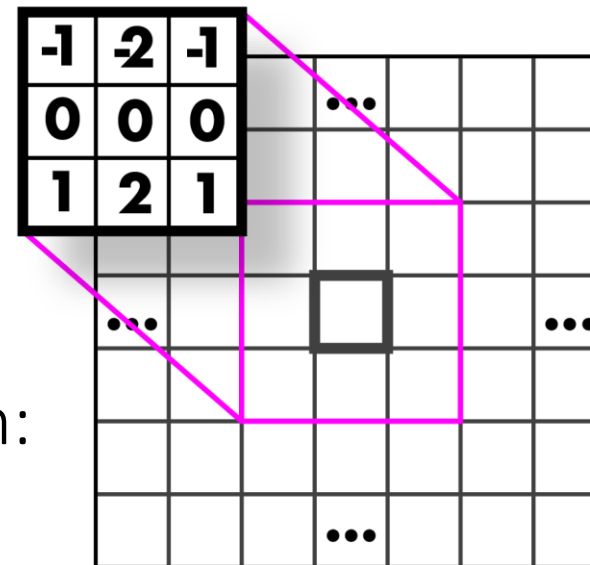
Magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$

Orientation:

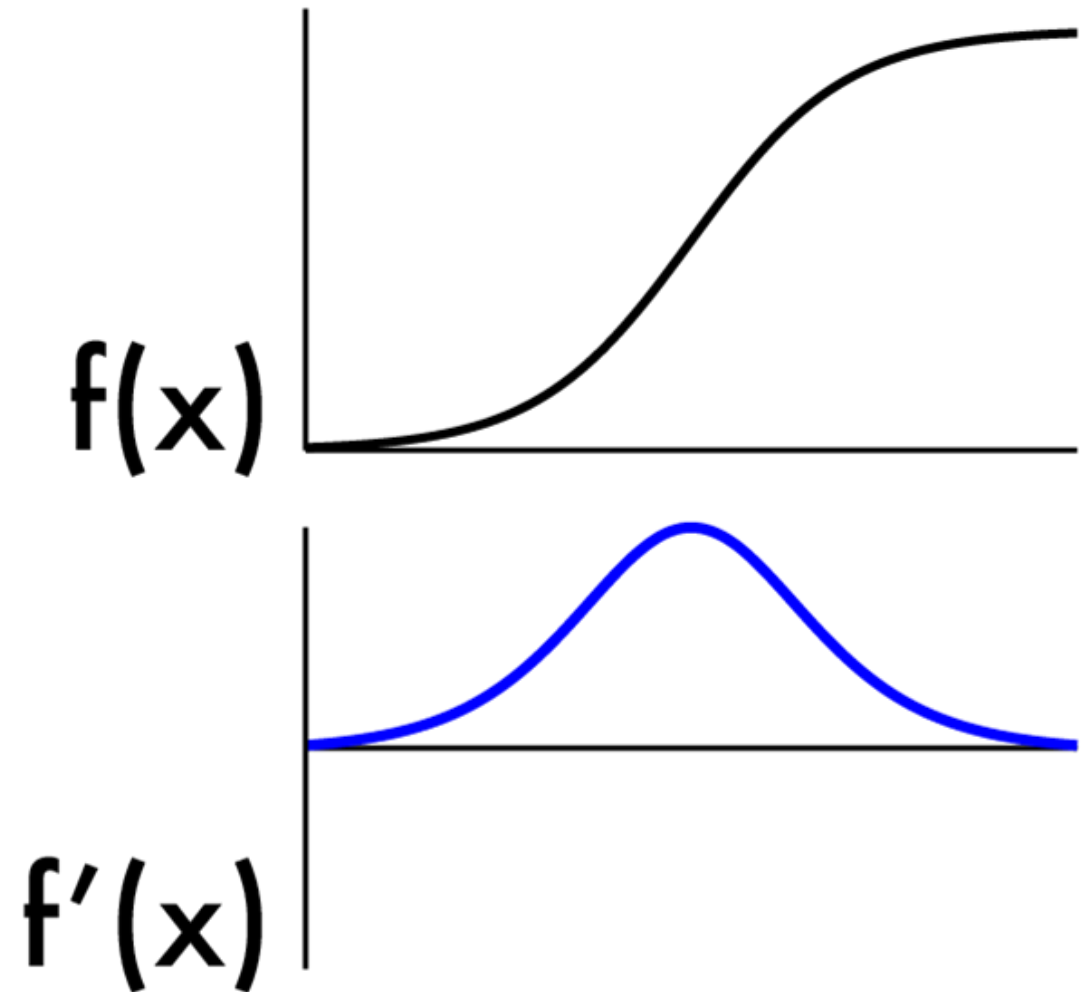
$$\Theta = \tan^{-1} \left( \frac{g_y}{g_x} \right)$$

We can change the sign:



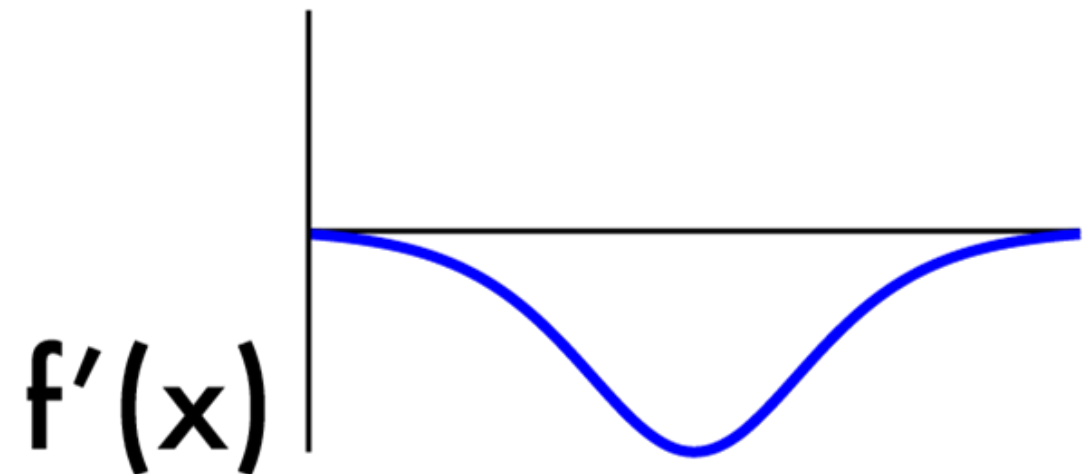
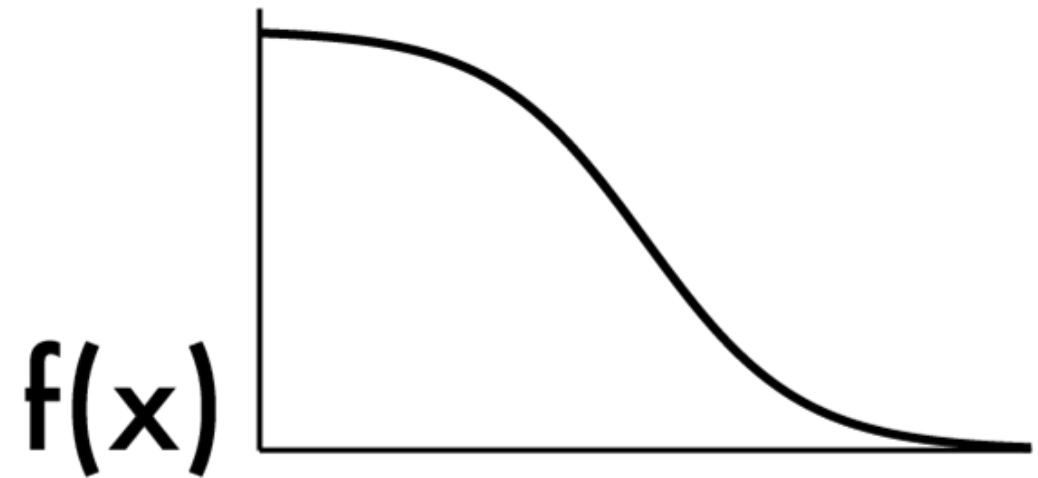
# Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But...



# Finding edges

- Could take derivative
- Find high responses
- Sobel filters!
- But...
- Edges go both ways
- Want to find extrema



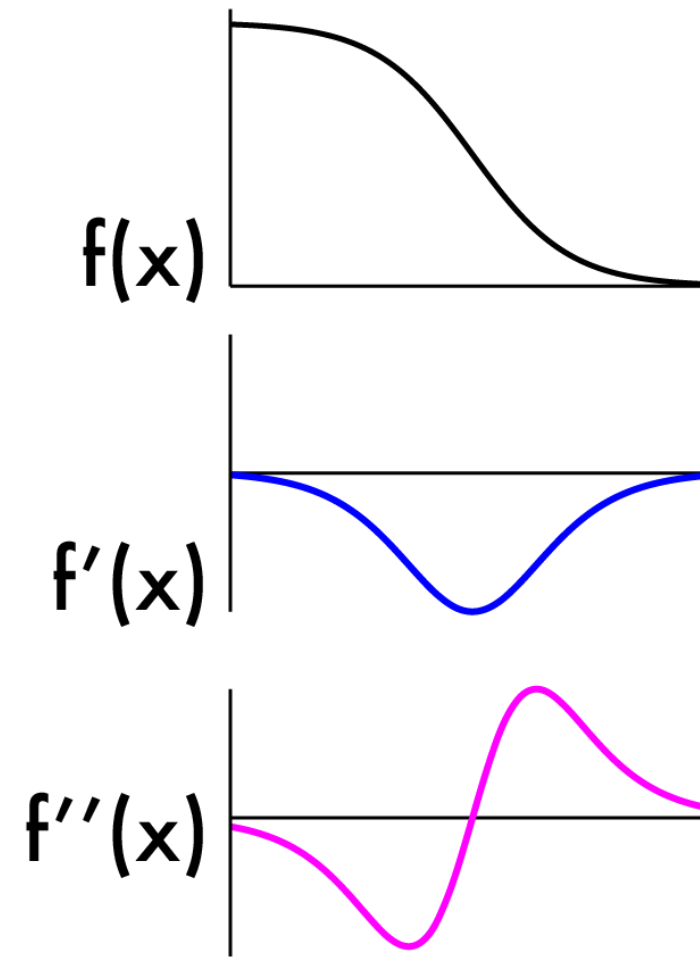
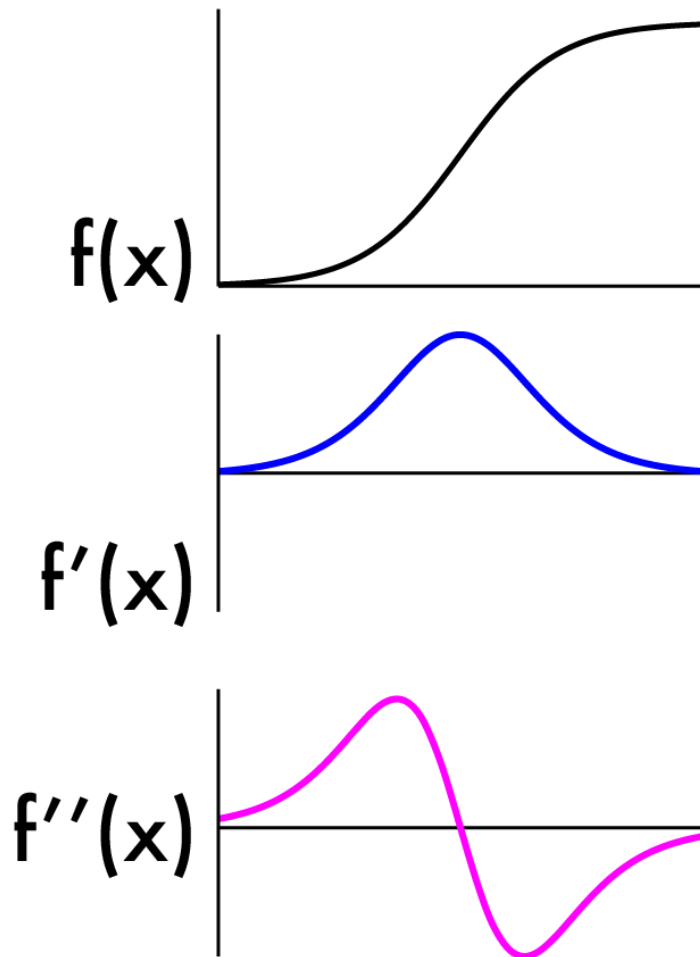
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# 2nd derivative!

- Crosses zero at extrema



## Laplacian (2nd derivative)!

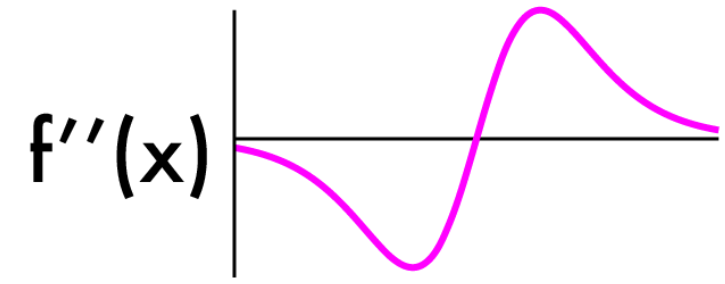
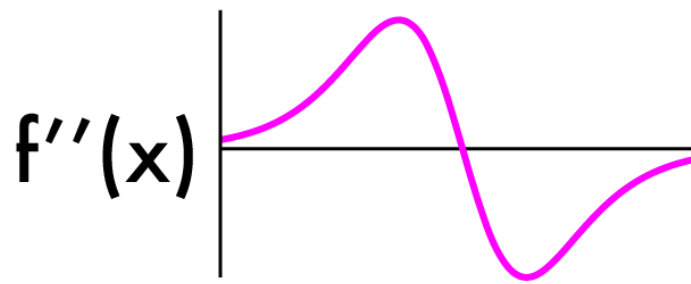
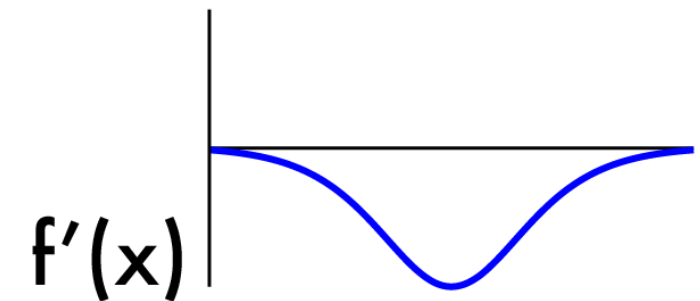
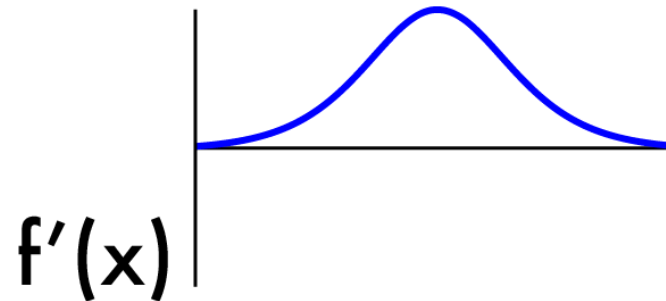
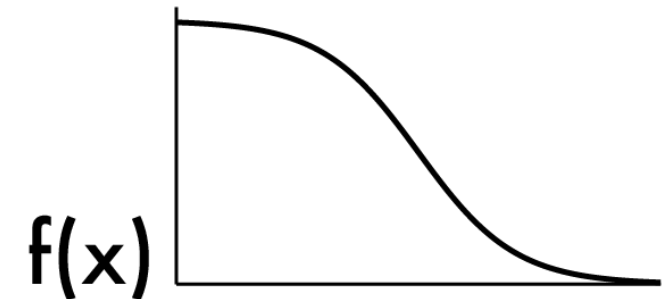
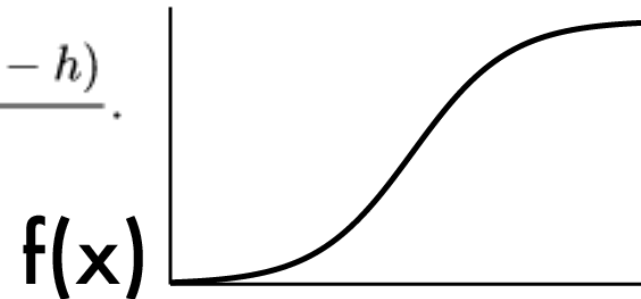
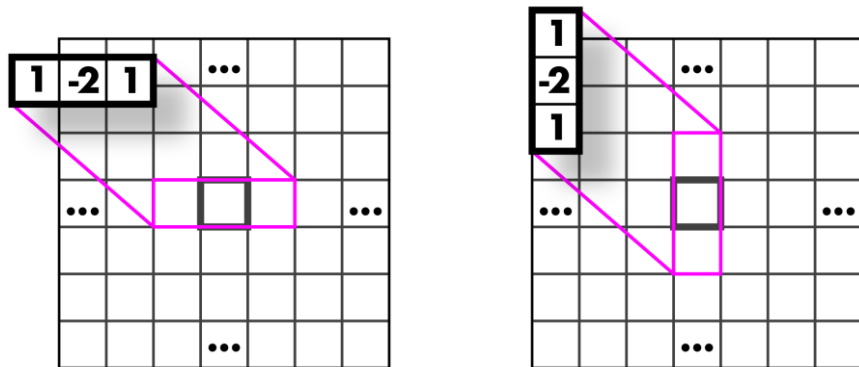
- Crosses zero at extrema
- Recall:

$$f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

- Laplacian:

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Again, have to estimate  $f''(x)$ :



# Laplacians

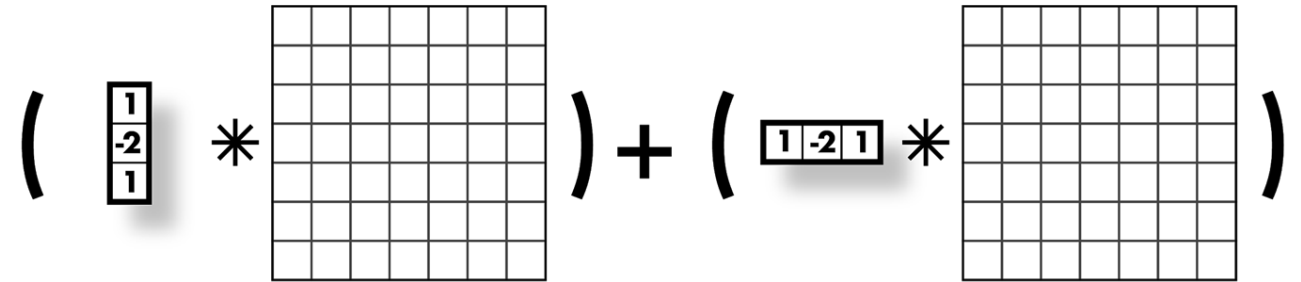
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# Laplacians

- Laplacian:

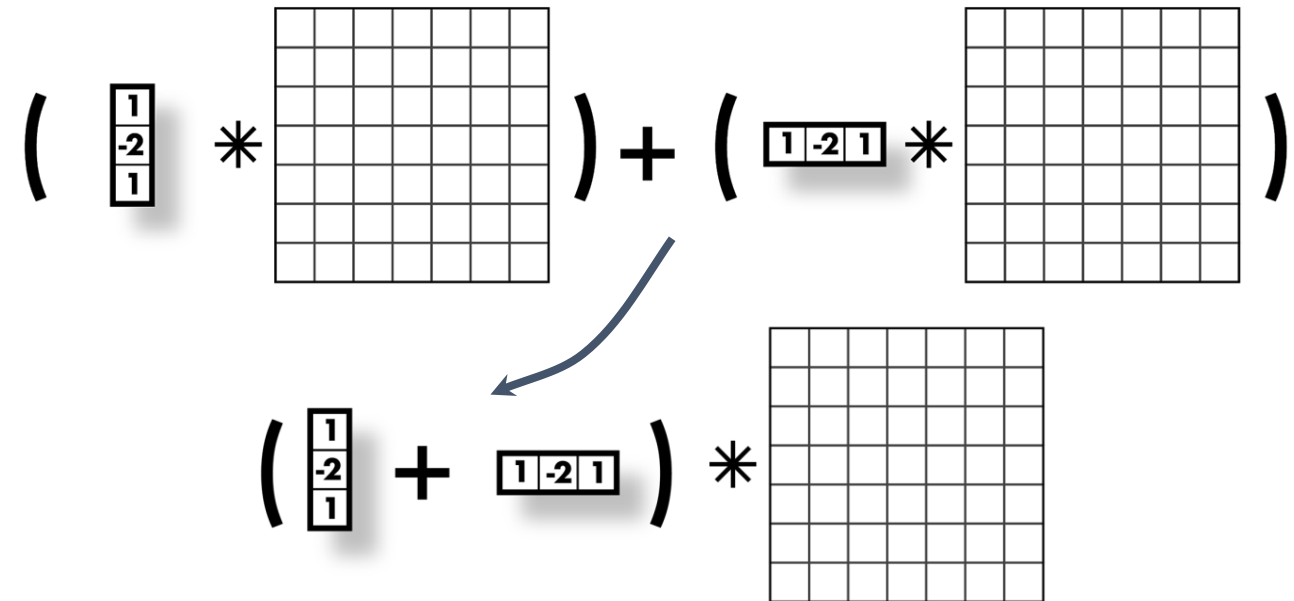
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# Laplacians

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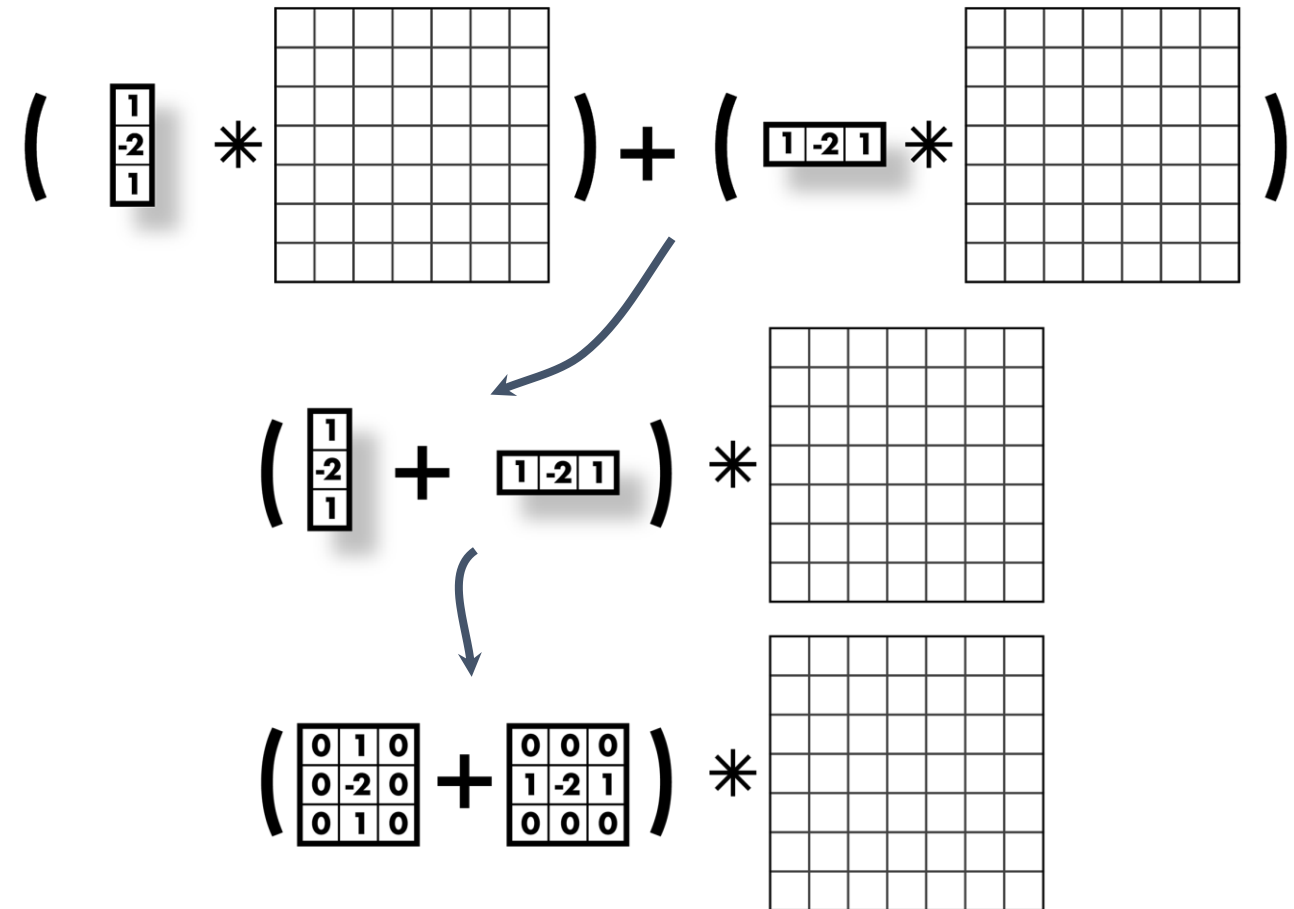
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# Laplacians

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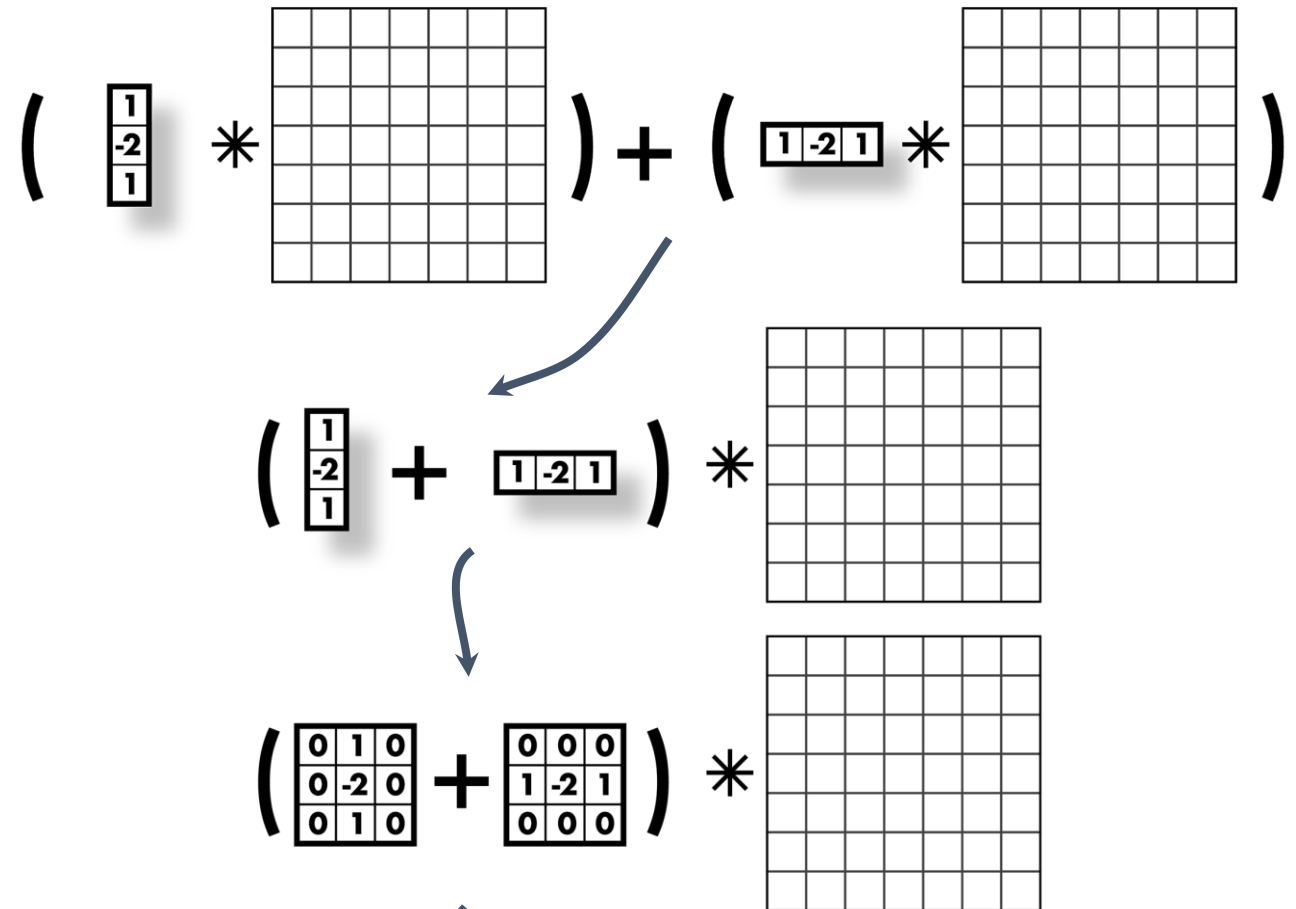
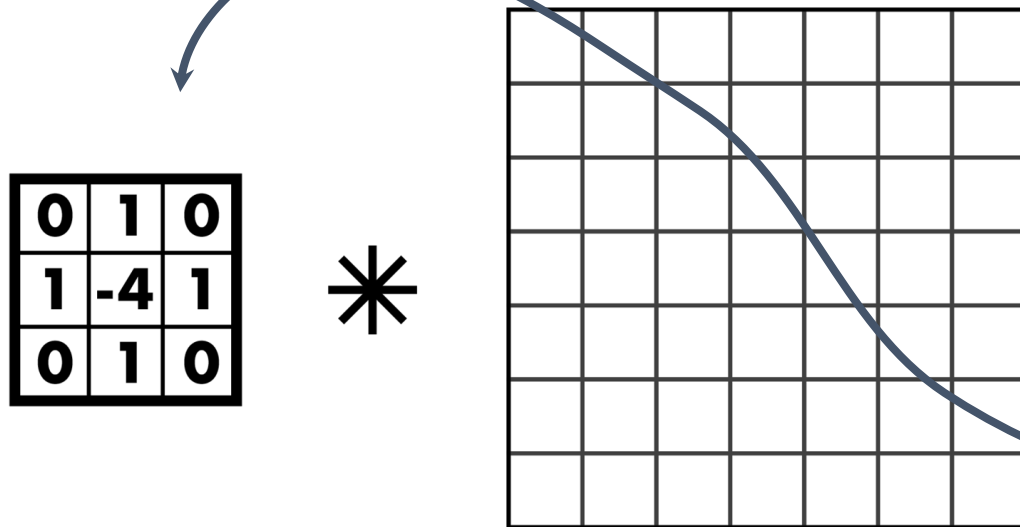
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# Laplacians

- Laplacian:

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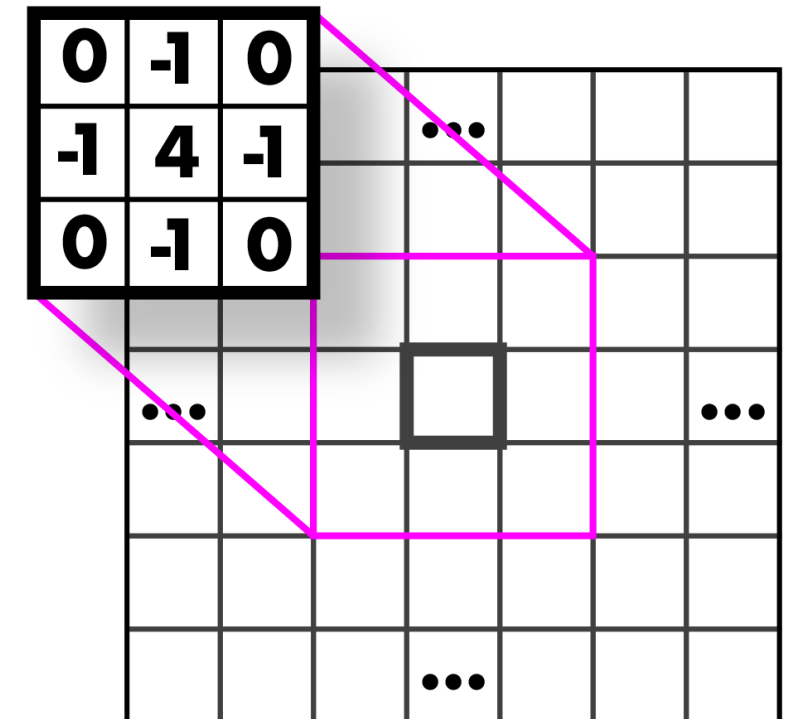
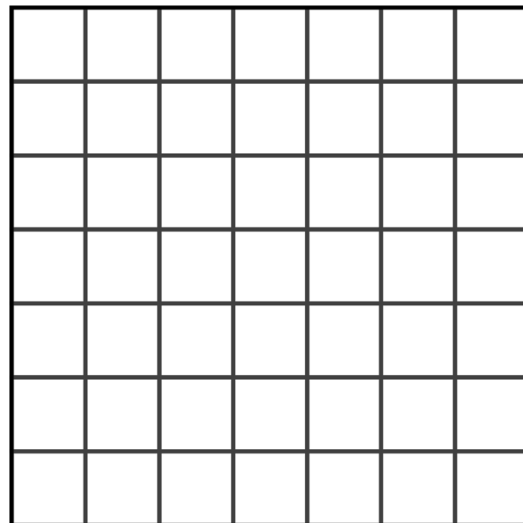


# Laplacians

- Laplacian:
  - $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
- Negative Laplacian, -4 in middle
- Positive Laplacian --->

0	1	0
1	-4	1
0	1	0

\*

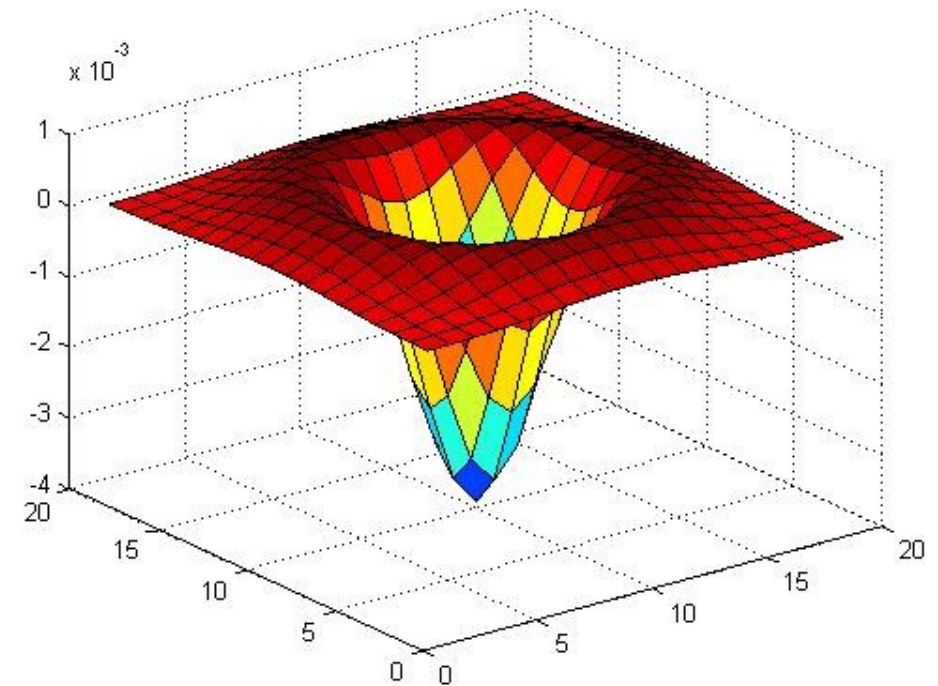






# Laplacians also sensitive to noise

- Again, use gaussian smoothing
- Can just use one kernel since convs commute
- Laplacian of Gaussian, LoG
- Can get good approx. with 5x5 - 9x9 kernels



# Today's Agenda

- What can we do with convolutions
- What is an edge – image derivatives
- Sobel filters
- Laplacian filters
- Difference of Gaussian filters
- Canny edge detection

# Another edge detector

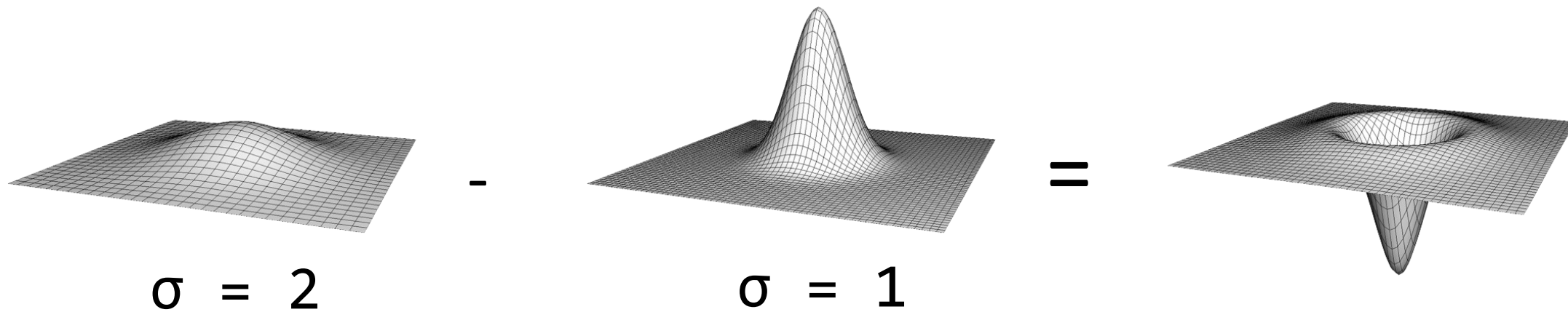
- Image is a function
  - Has high frequency and low frequency components
  - Think in terms of fourier transform
- Edges are high frequency changes
- Maybe we want to find edges of a specific size (i.e. specific frequency)

# Difference of Gaussian (DoG)

- Gaussian is a low pass filter
- Strongly reduce components with frequency  $f > \sigma$
- $(g * I)$  low frequency components
- $I - (g * I)$  high frequency components
- $g(\sigma_1) * I - g(\sigma_2) * I$ 
  - Components in between these frequencies
- $g(\sigma_1) * I - g(\sigma_2) * I = [g(\sigma_1) - g(\sigma_2)] * I$

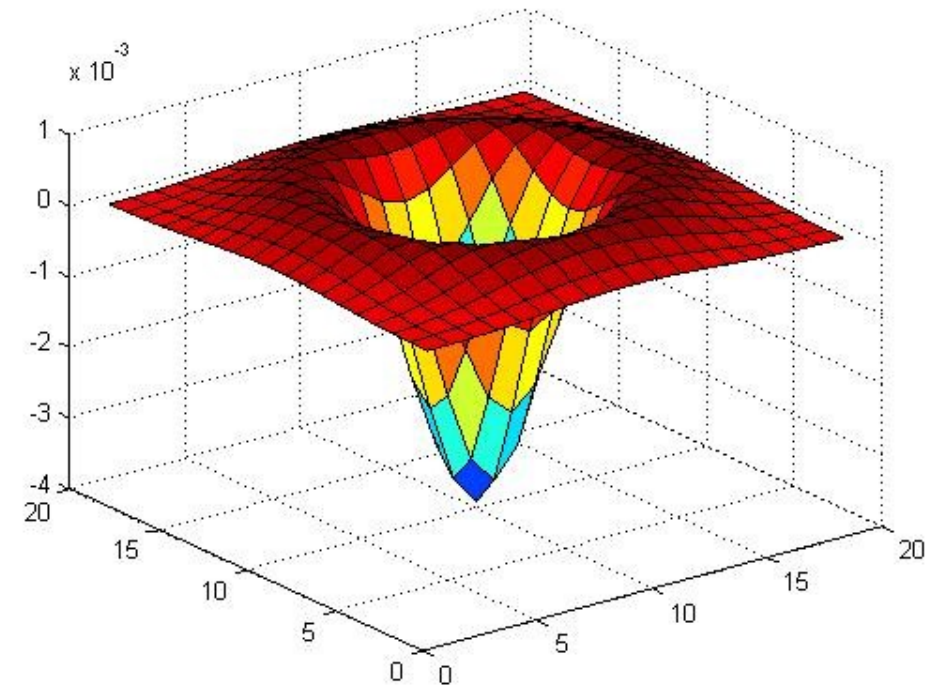
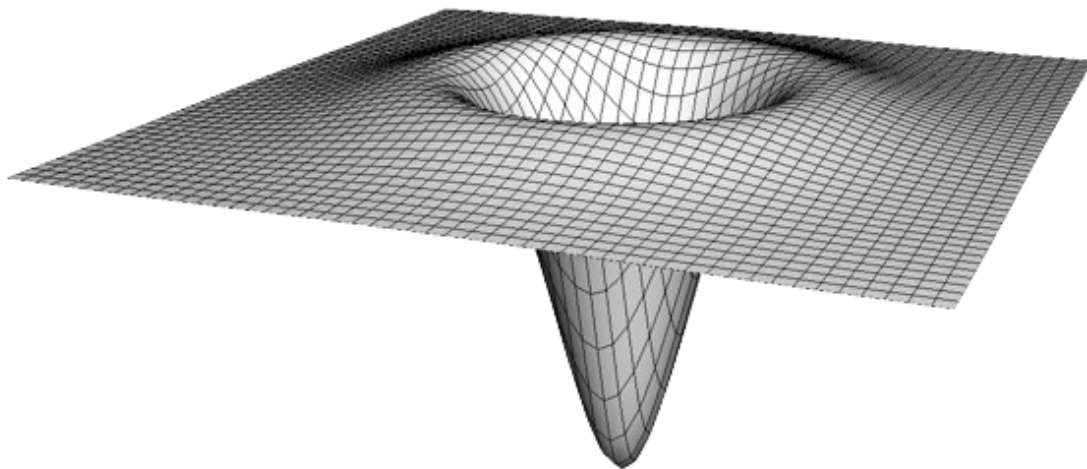
# Difference of Gaussian (DoG)

$$- g(\sigma_1) * I - g(\sigma_2) * I = [g(\sigma_1) - g(\sigma_2)] * I$$



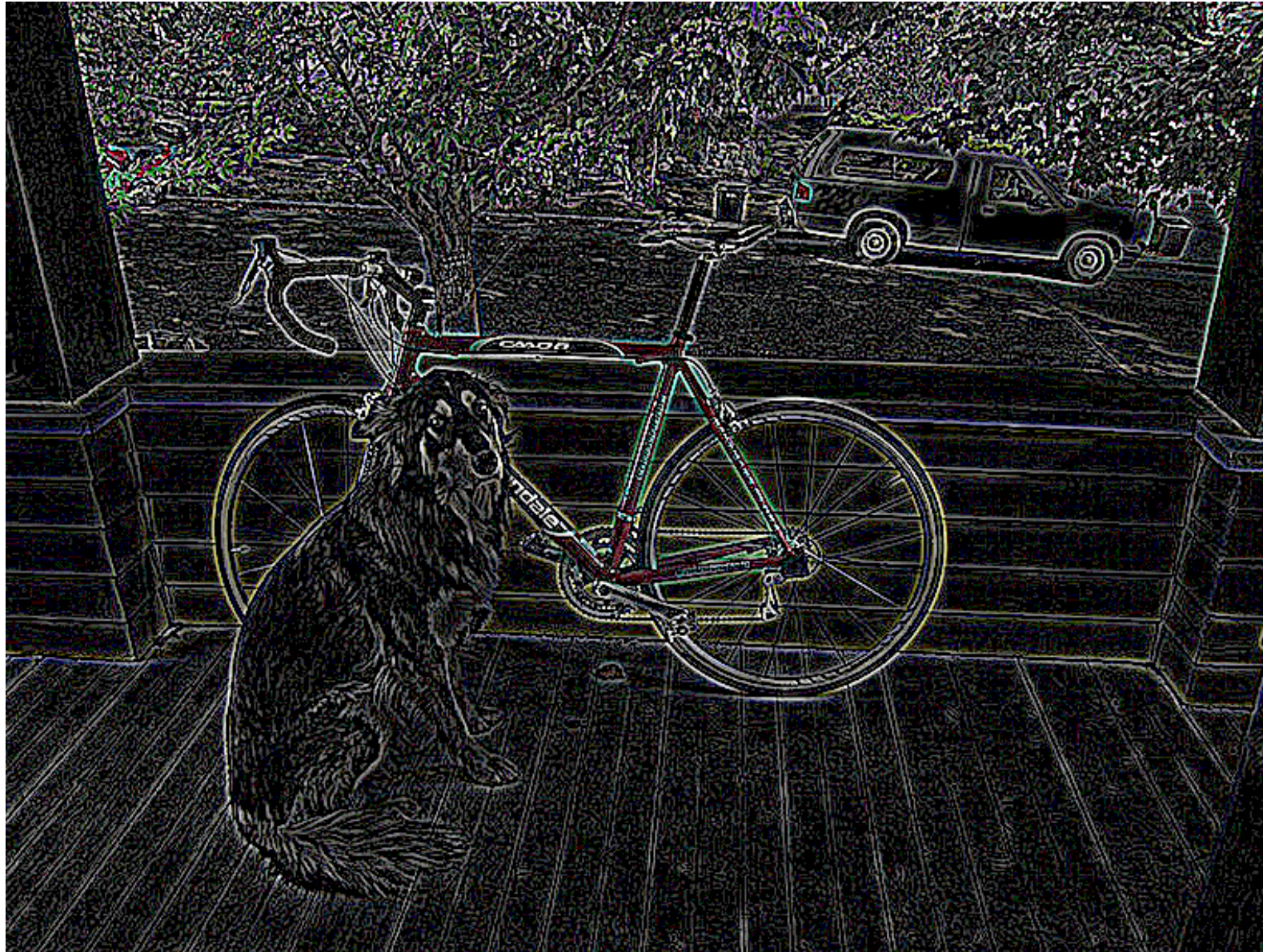
# Difference of Gaussian (DoG)

- $g(\sigma_1) * I - g(\sigma_2) * I = [g(\sigma_1) - g(\sigma_2)] * I$
- This looks a lot like our LoG!
- (not actually the same but similar)





## DoG (1 - 0)





## DoG (2 - 1)





## DoG (3 - 2)



## DoG (4 - 3)

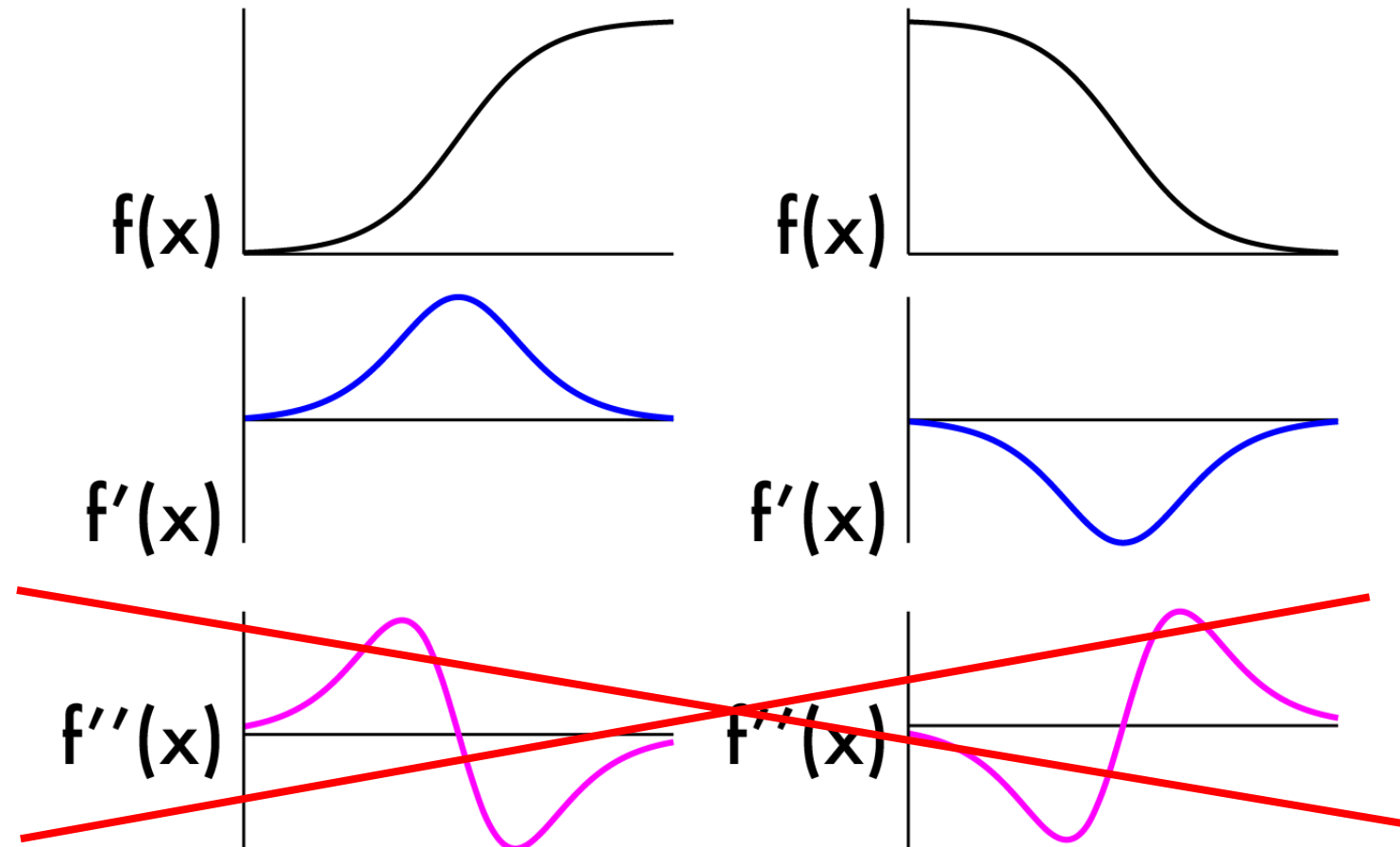


# Today's Agenda

- What can we do with convolutions
- What is an edge – image derivatives
- Sobel filters
- Laplacian filters
- Difference of Gaussian filters
- Canny edge detection

# Another approach: gradient magnitude

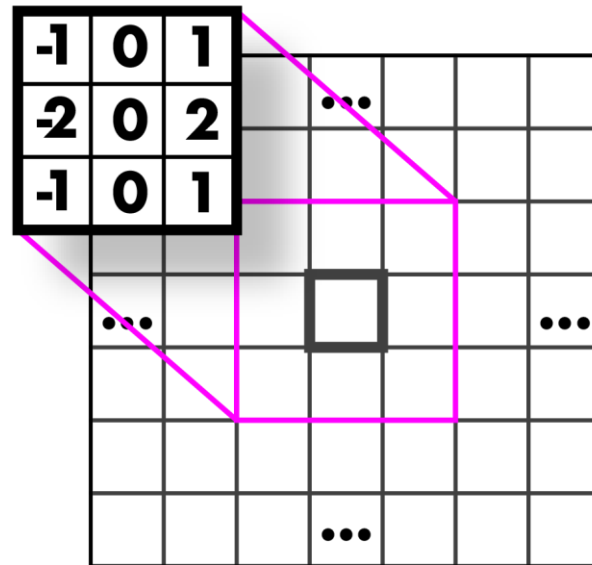
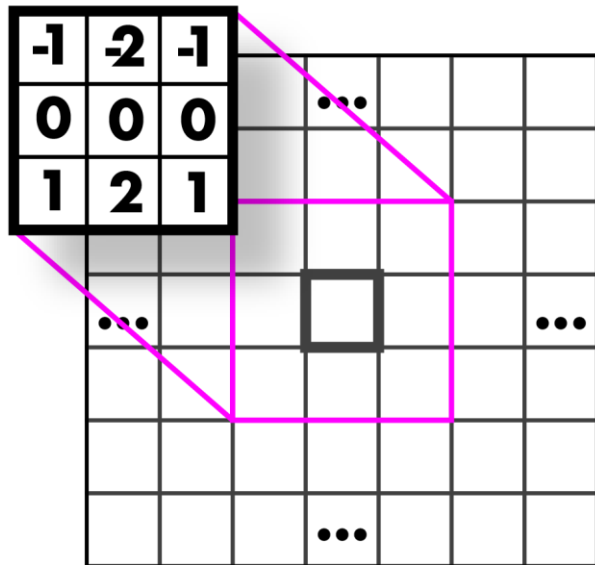
- Don't need 2nd derivatives
- Just use magnitude of gradient





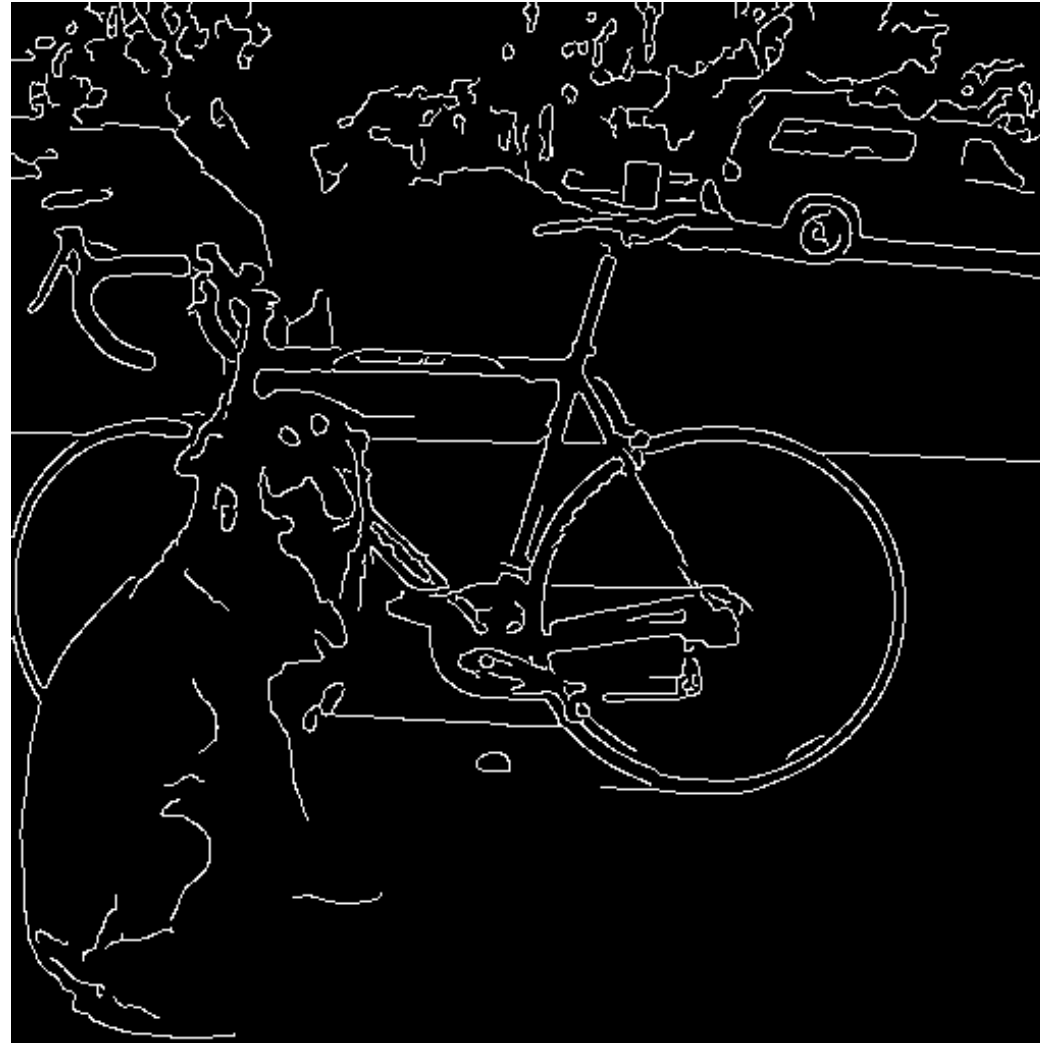
## Another approach: gradient magnitude

- Don't need 2nd derivatives
- Just use magnitude of gradient
- Are we done? No!





## What we really want: line drawing



# Canny Edge Detection

## Algorithm:

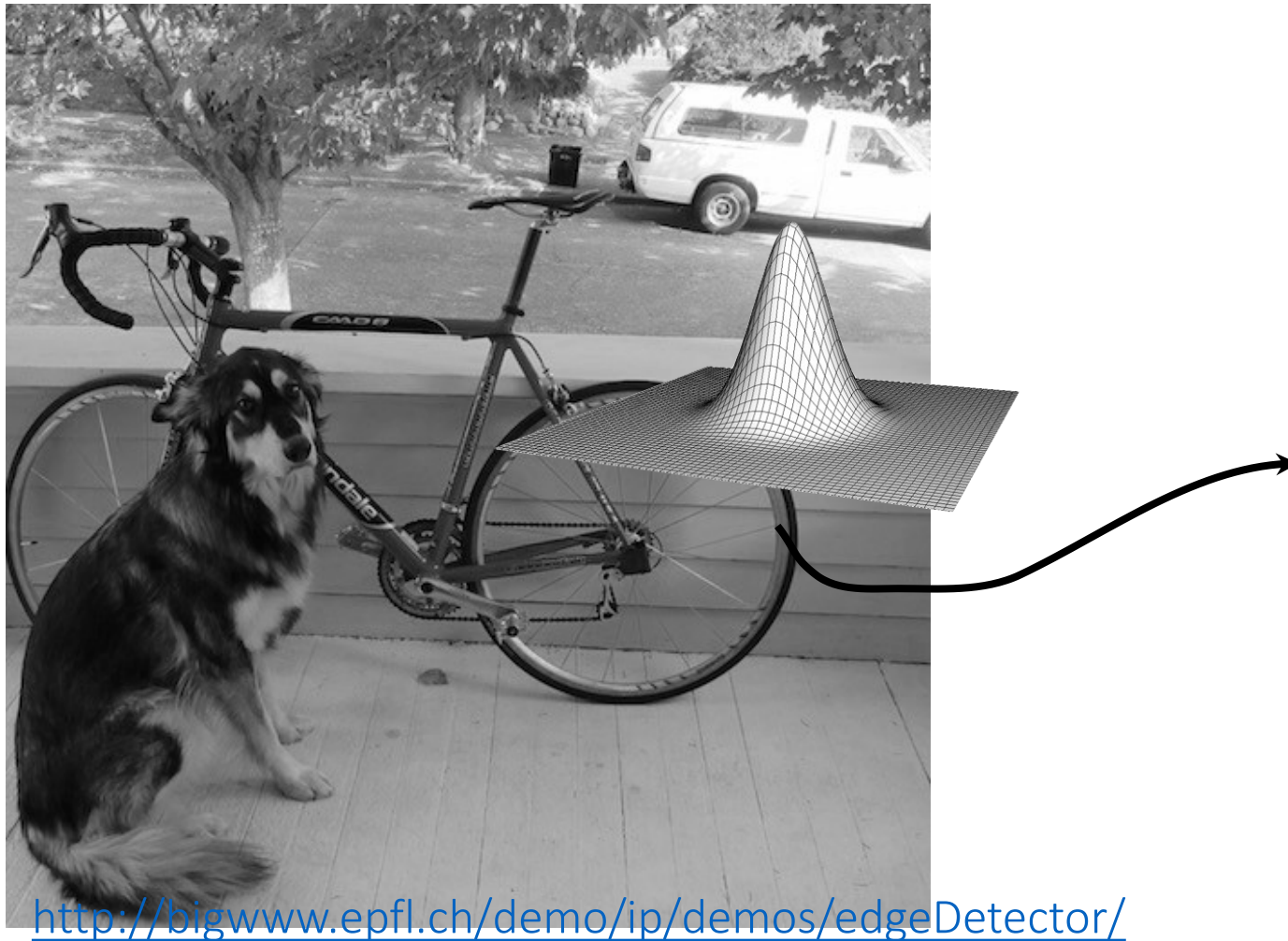
- Smooth image (only want “real” edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Connect together components

<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>



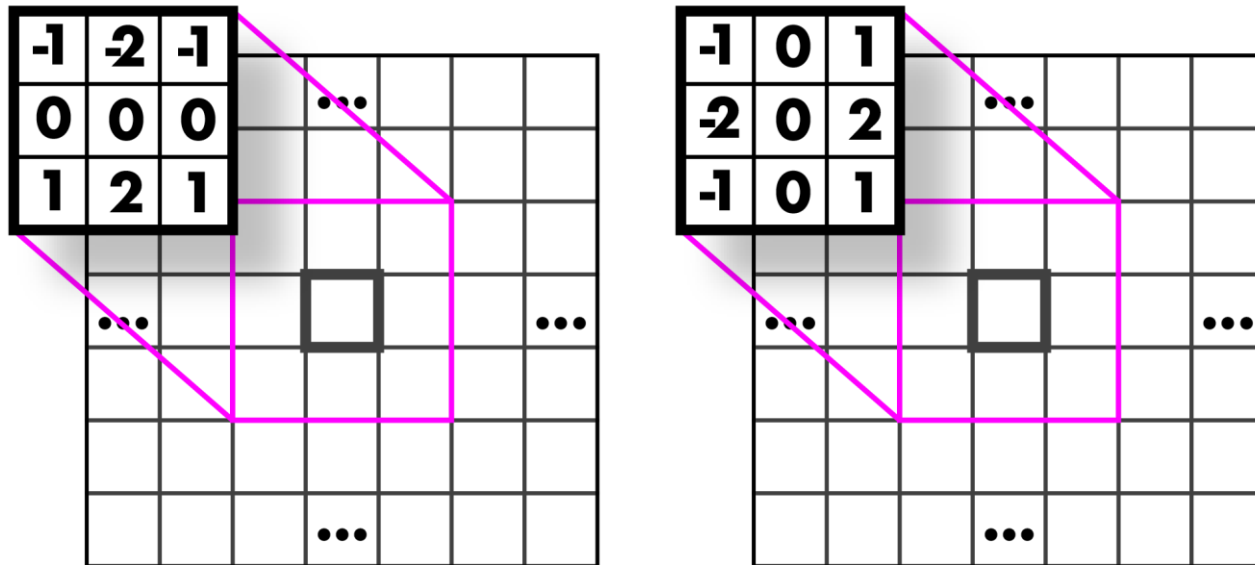
# Smooth image

- You know how to do this, gaussians!



# Gradient magnitude and direction

- Sobel filter



<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

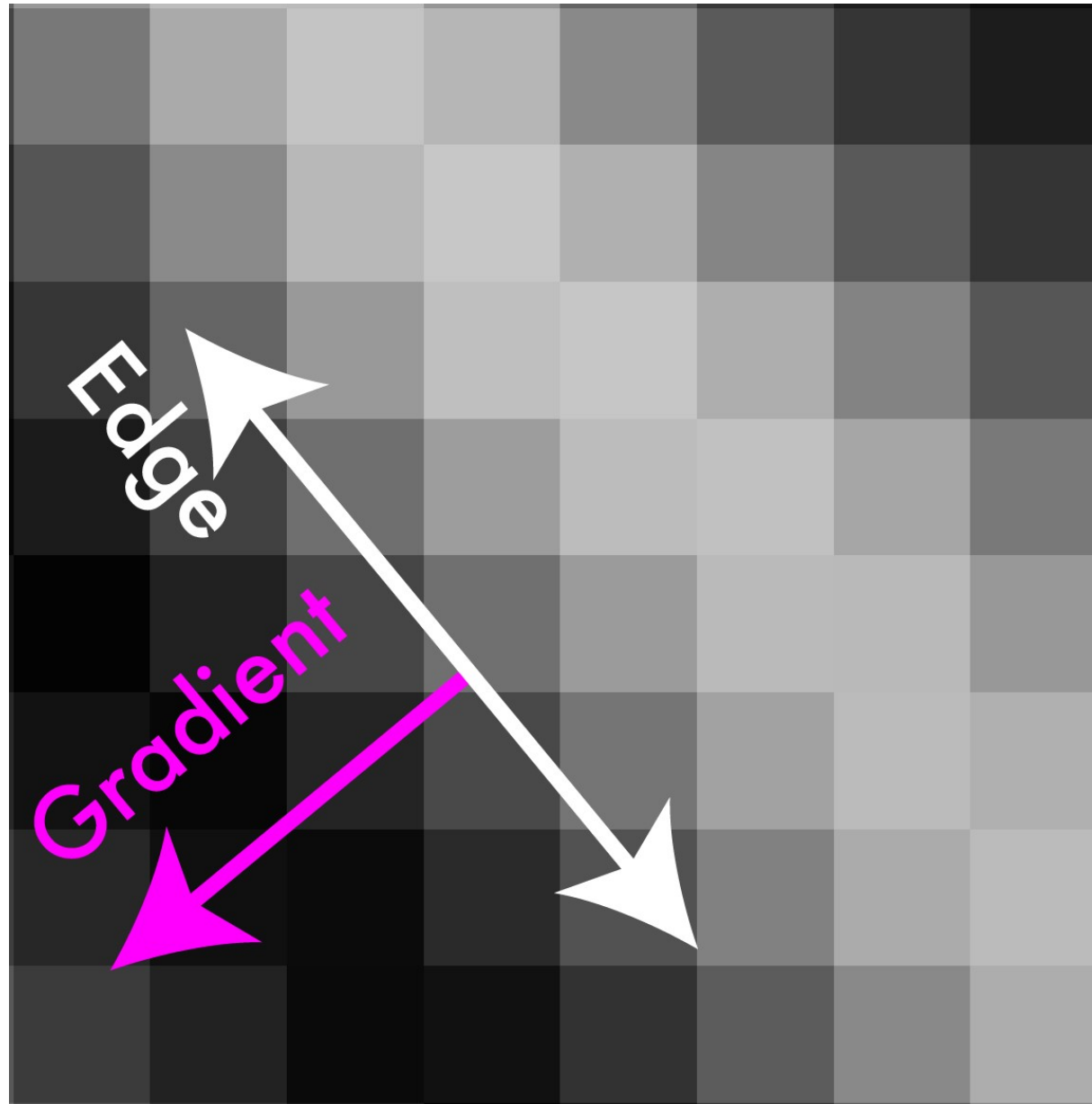
# Non-maximum suppression

- Want single pixel edges, not thick blurry lines
- Need to check nearby pixels
- See if response is highest

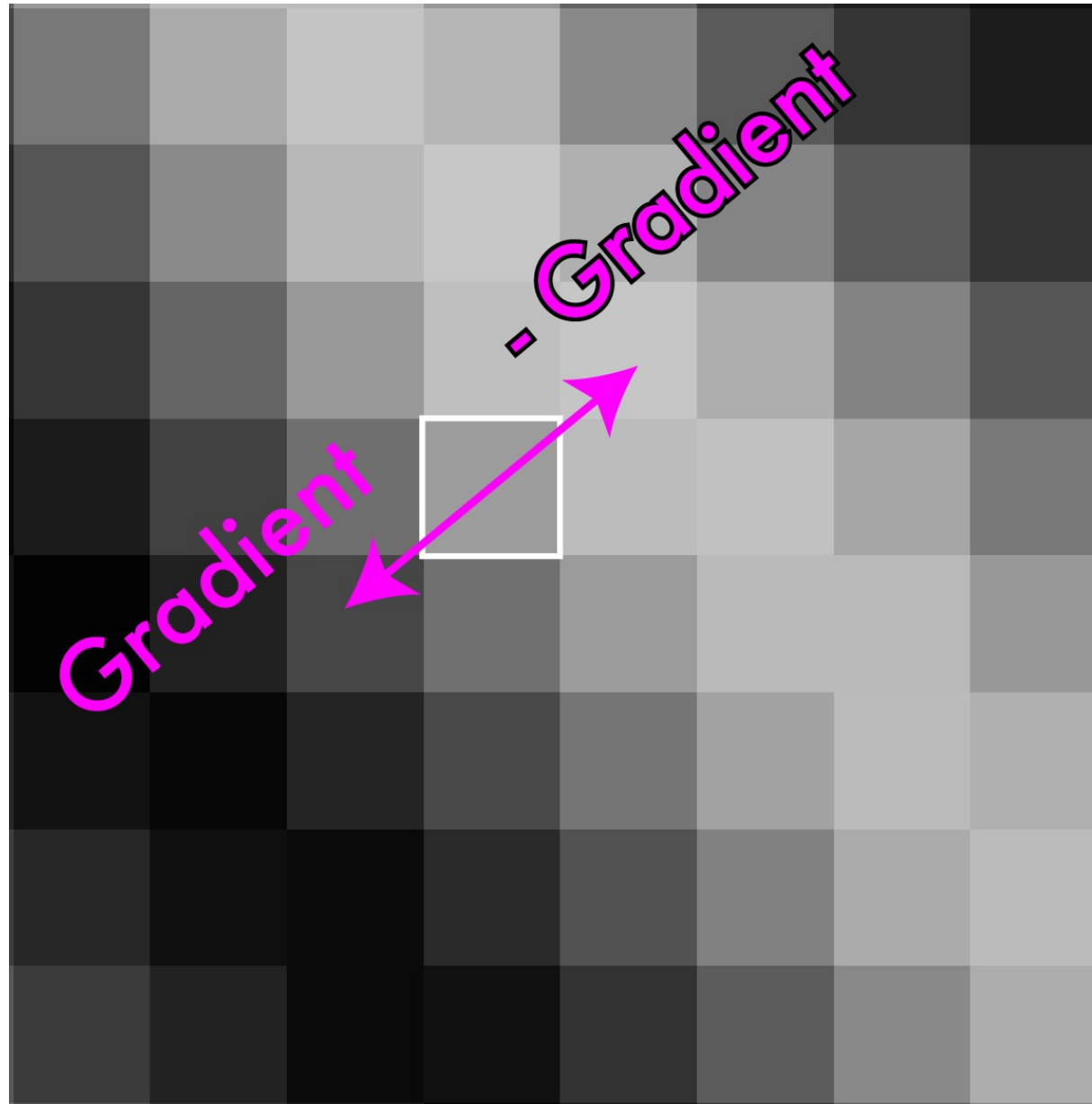


<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

# Non-maximum suppression

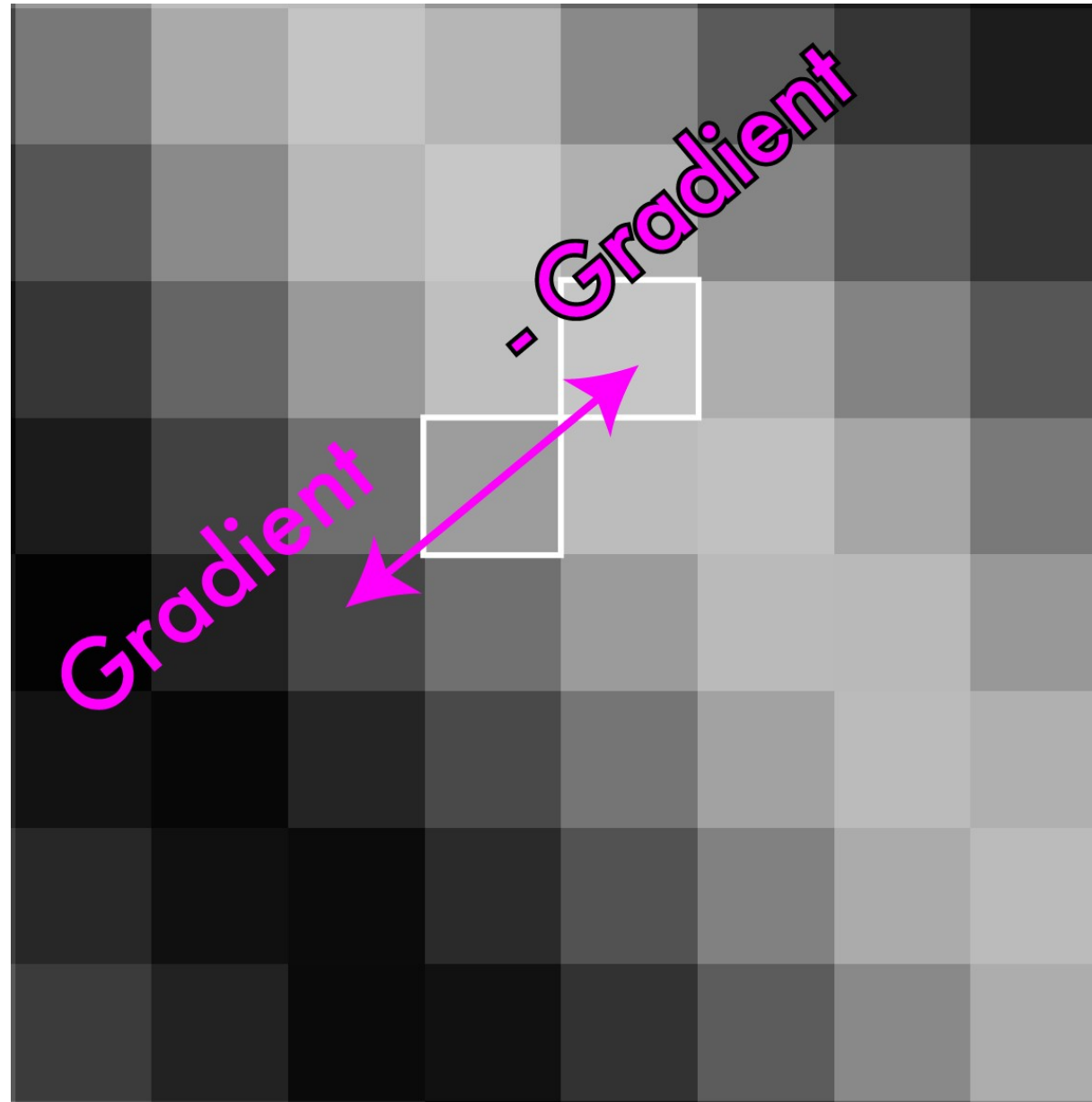


# Non-maximum suppression

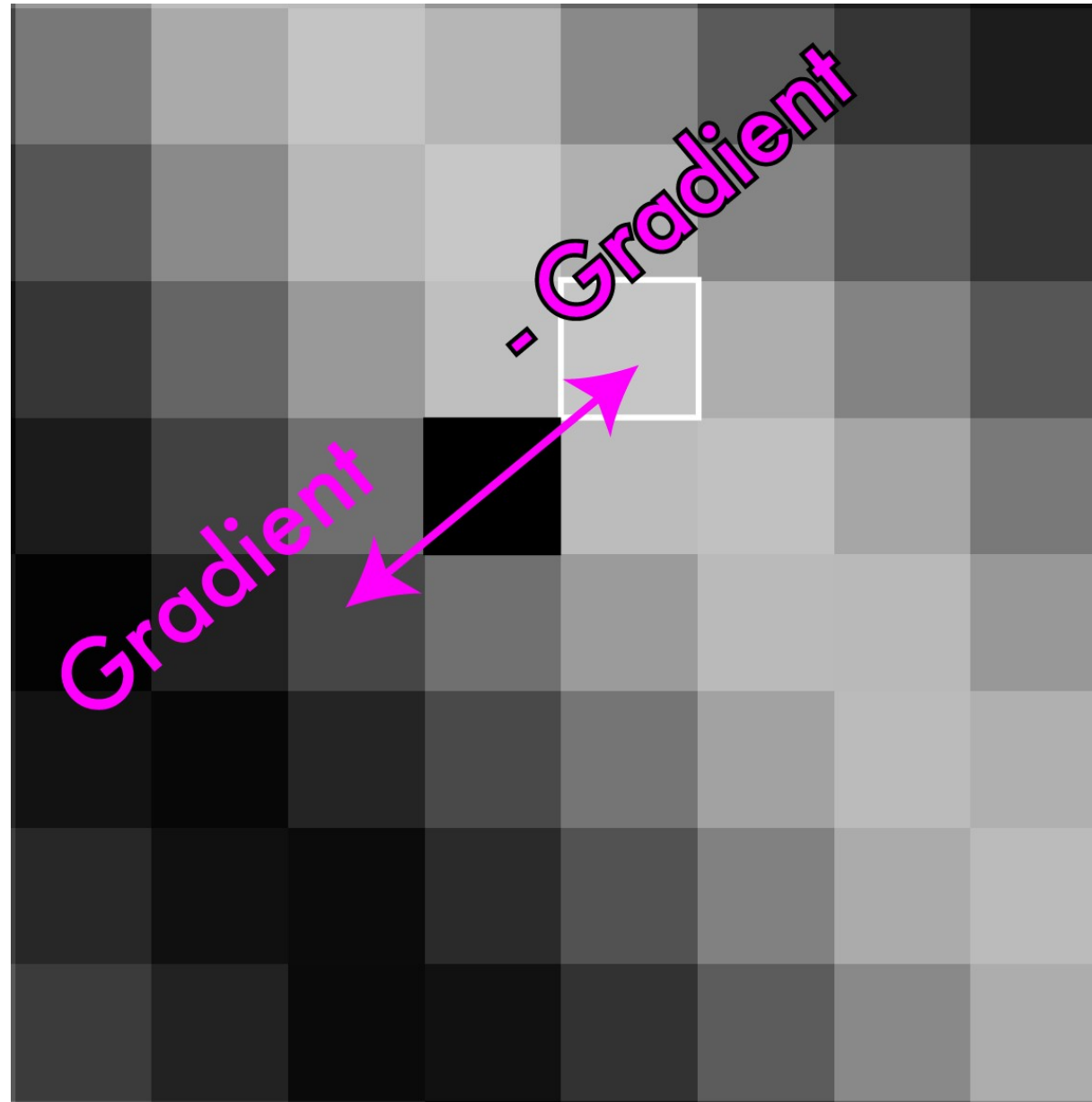




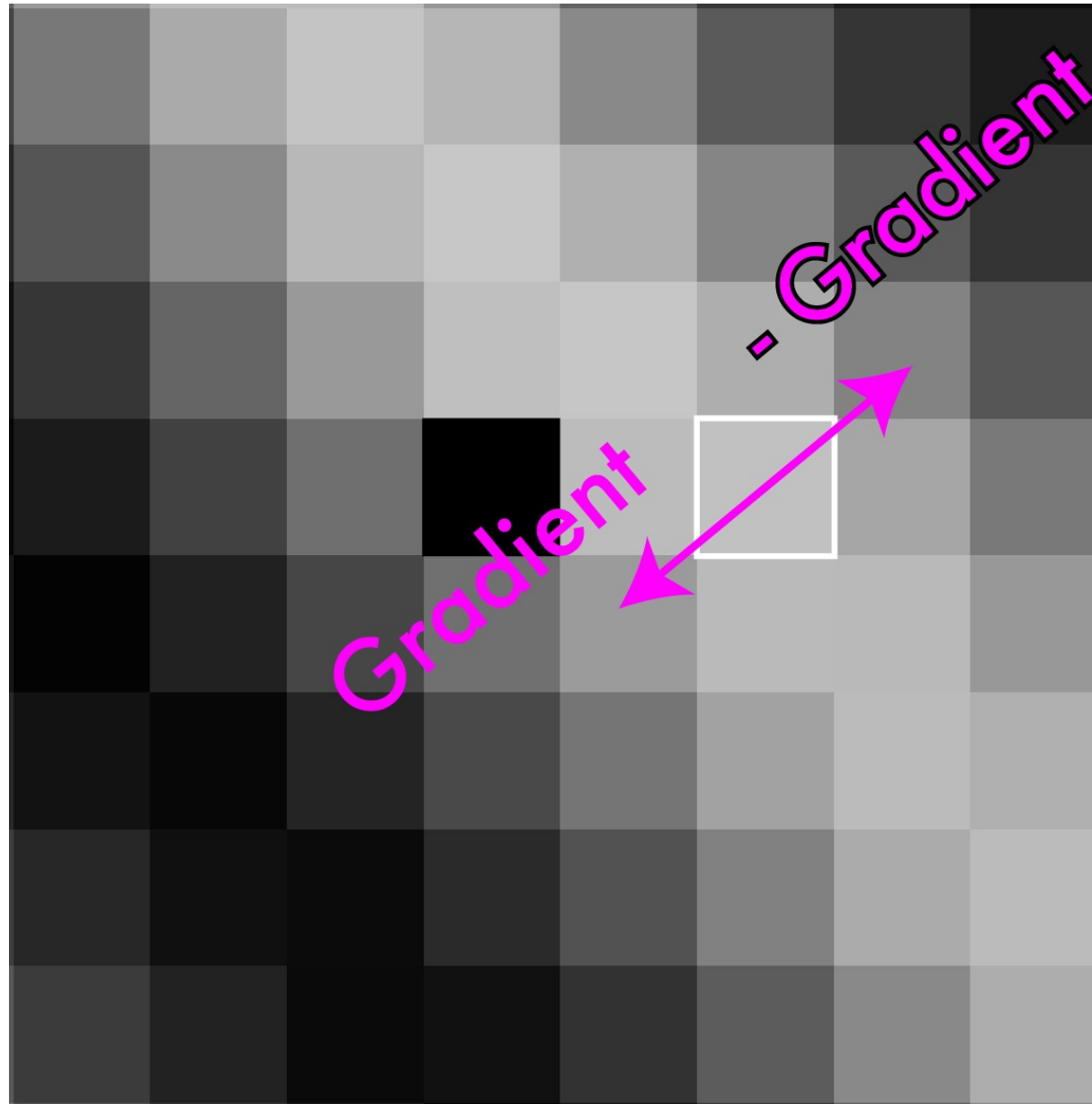
## Non-maximum suppression



## Non-maximum suppression

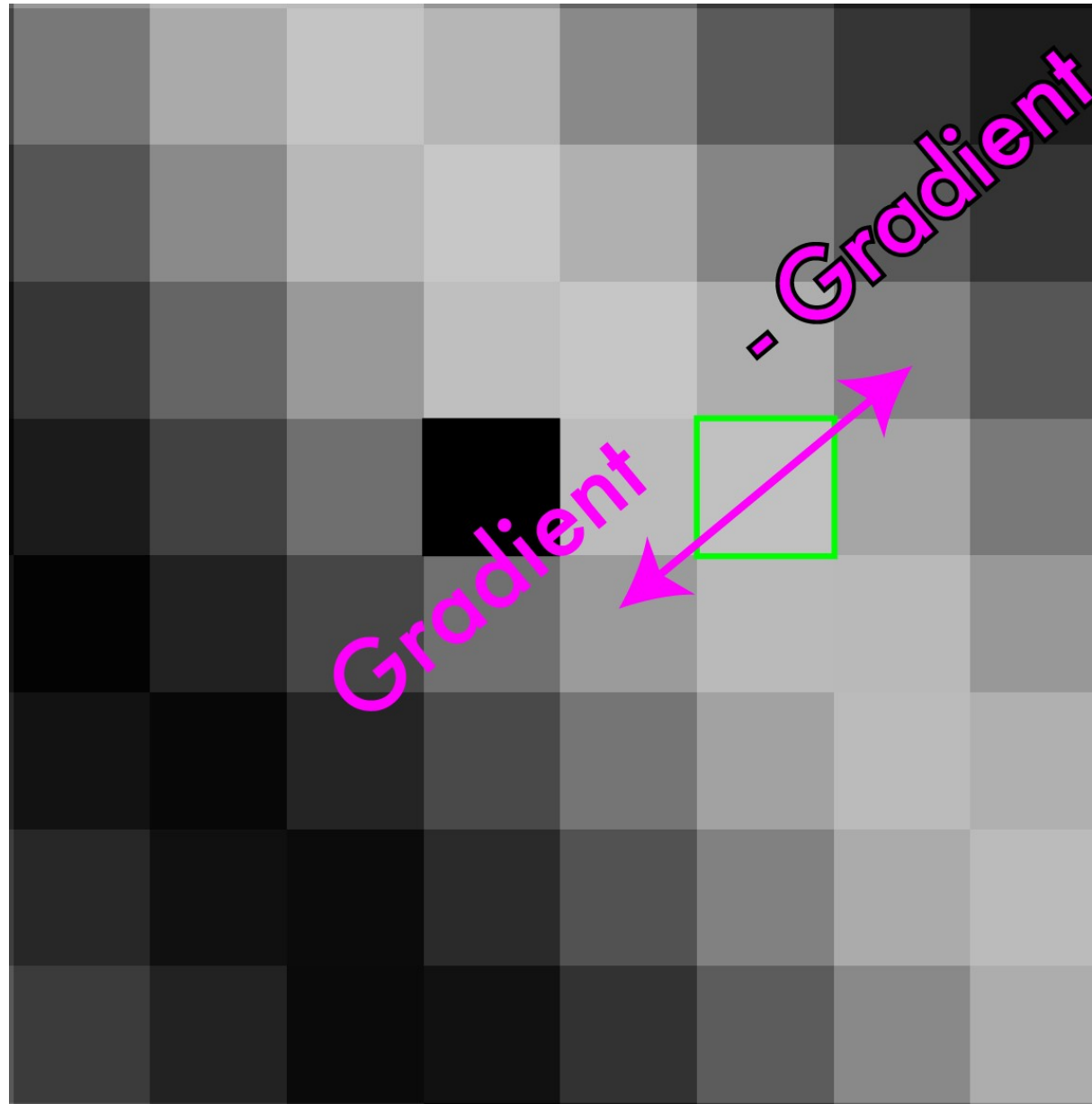


## Non-maximum suppression

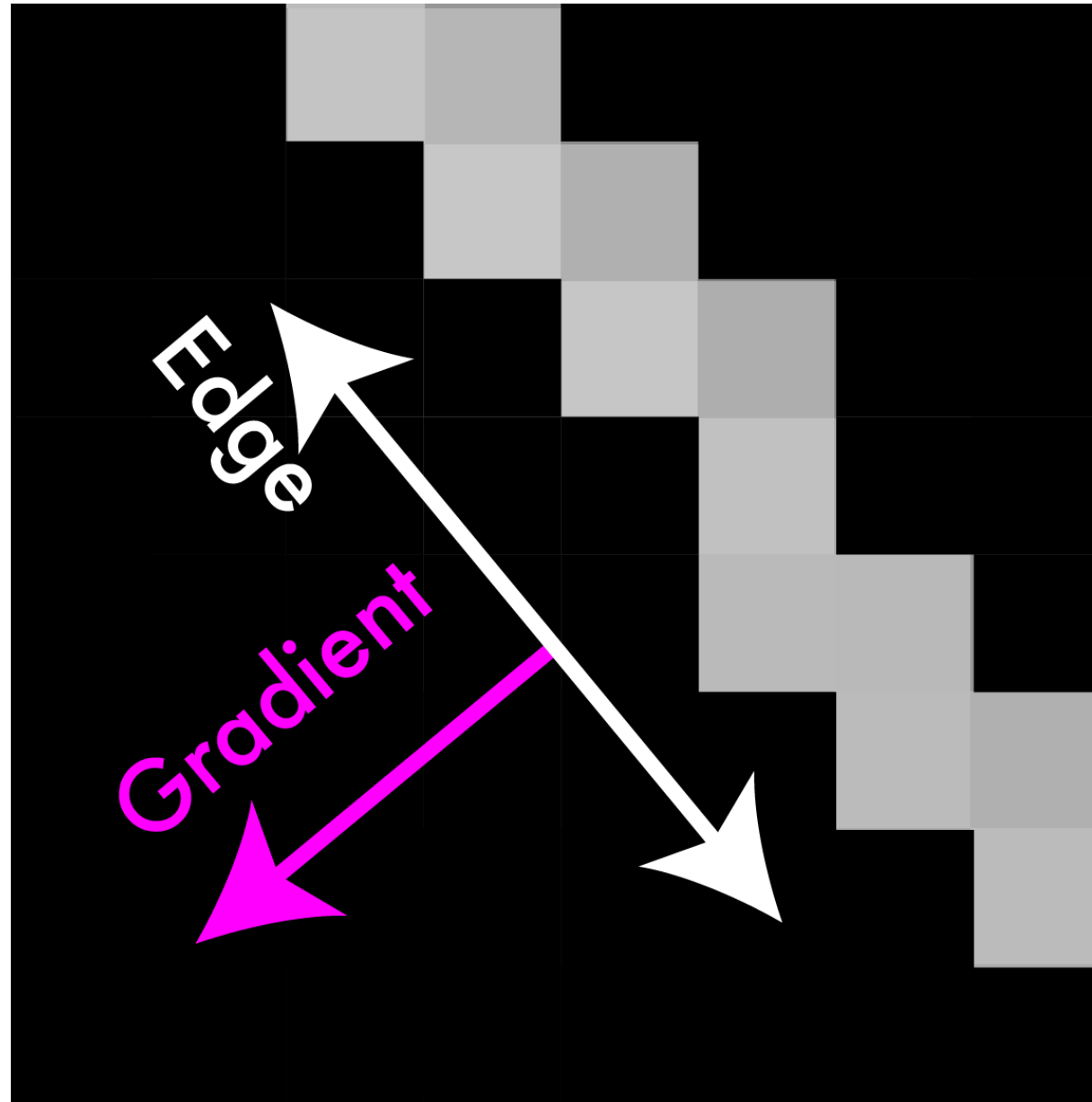




# Non-maximum suppression



## Non-maximum suppression



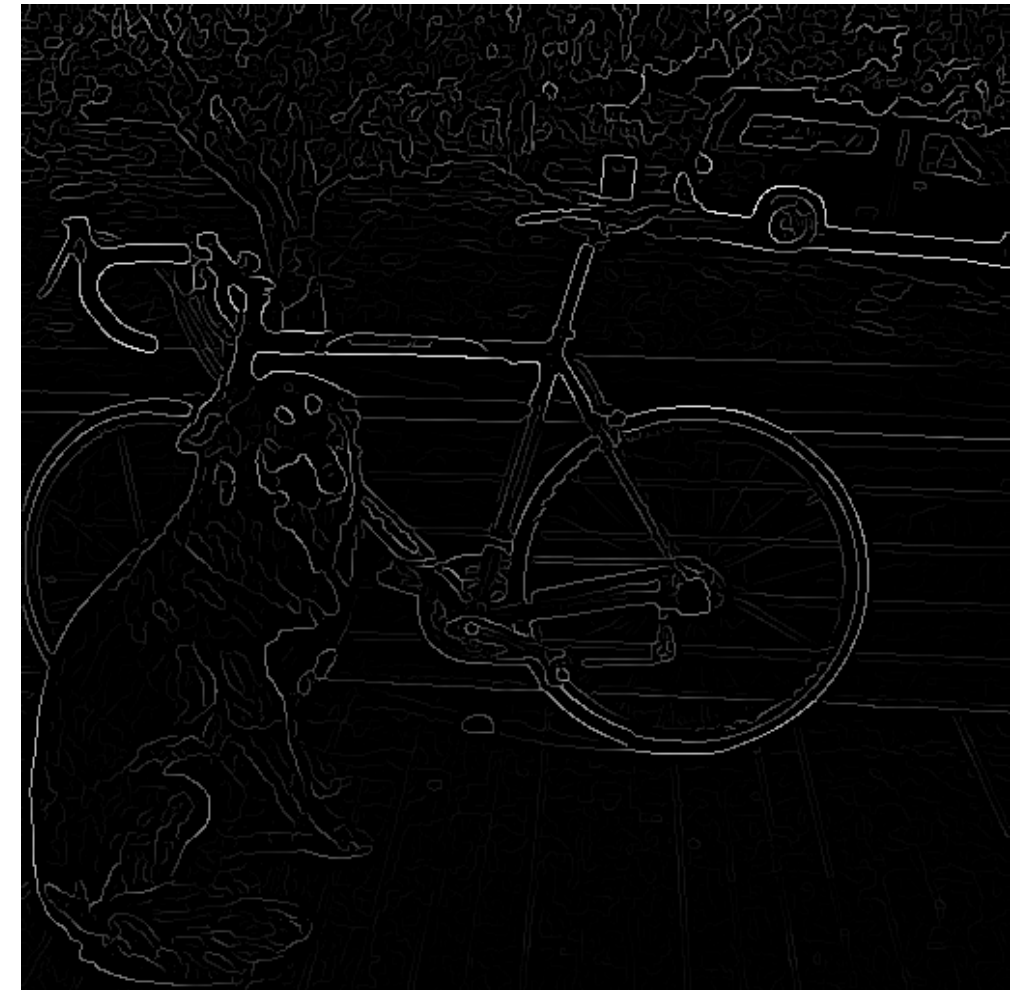
## Non-maximum suppression



<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

# Threshold edges

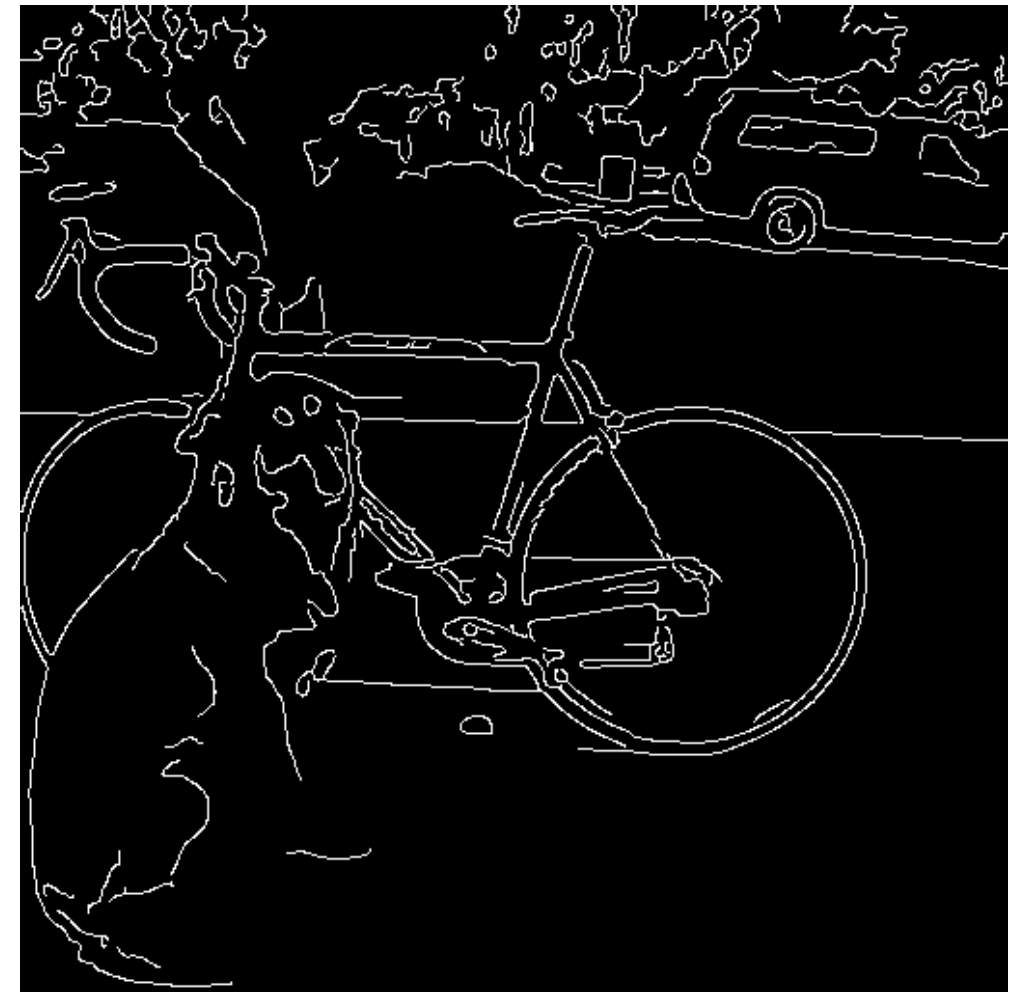
- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
  - $R > T$ : strong edge
  - $R < T$  but  $R > t$ : weak edge
  - $R < t$ : no edge
- Why two thresholds?



<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

# Connect them up!

- Strong edges are edges!
- Weak edges are edges iff they connect to strong
- Look in some neighborhood (usually 8 closest)



<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

# Canny Edge Detection

## Algorithm:

- Smooth image (only want “real” edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Connect together components
- Tunable: Sigma, thresholds

<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>



## Canny Edge Detection



<http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/>

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# Thank you.

