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Visual Computing Group

CYENS CENTRE OF EXCELLENCE

University of Cyprus – MSc Artificial Intelligence

MAI644 – COMPUTER VISION

Lecture 8: Feature Descriptors and Image Transforms

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Last time

- Features
- Self-difference
- Harris corner detection

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Today’s Agenda

- Basic feature descriptor and matching
- Histogram of Oriented Gradients
- SIFT
- Image transformations
- Estimate transformations

[material based on Joseph Redmon’s course]

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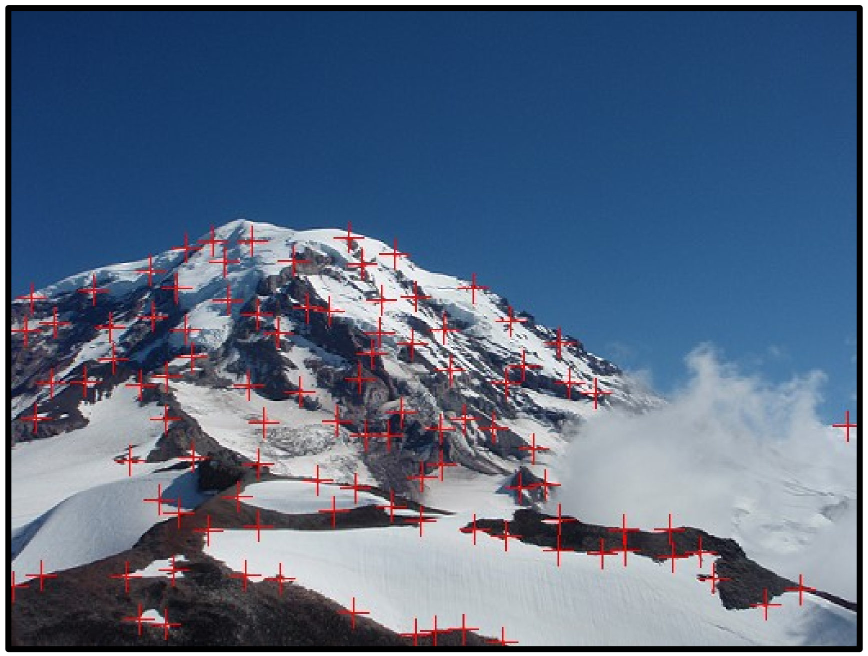
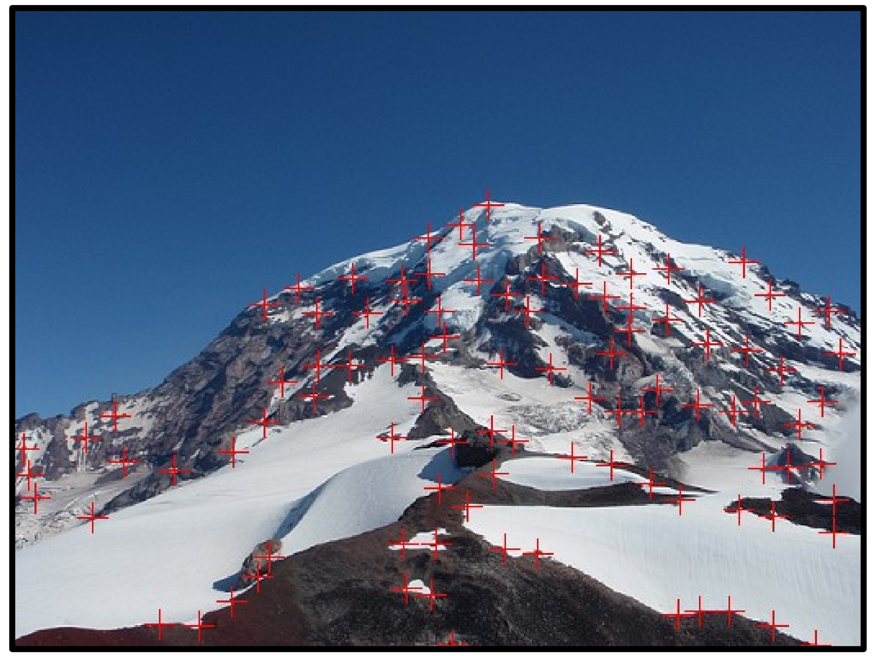
Ok, we found corners, now what?

- Need to match image patches to each other
- Need to figure out transform between images





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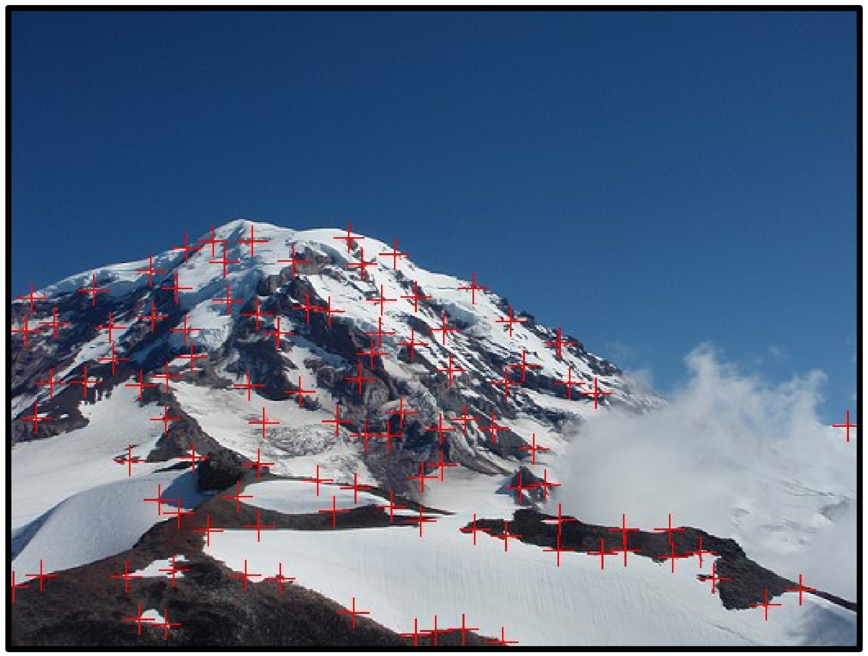
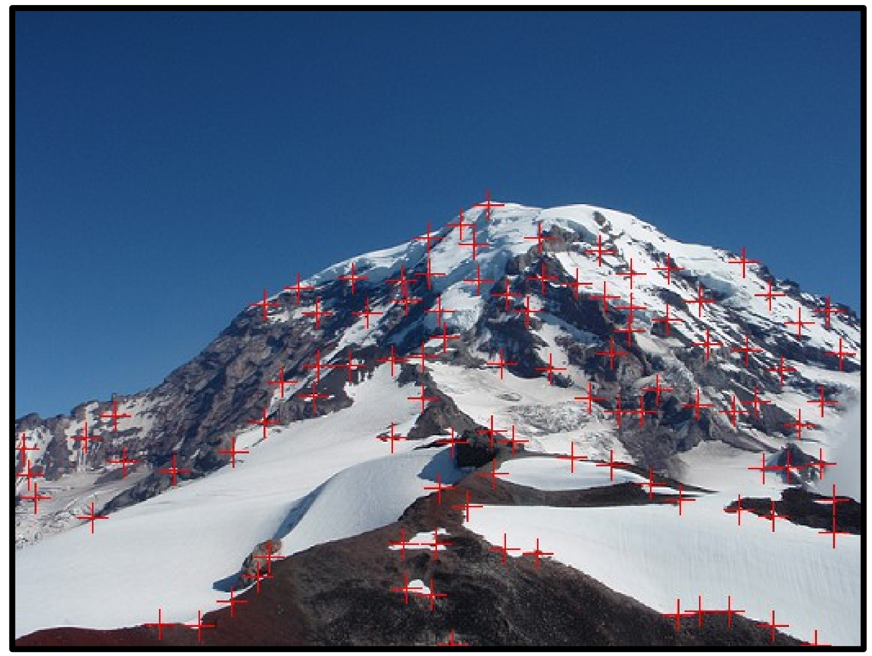
Ok, we found corners, now what?

- Need to match image patches to each other
  - What is a match? How do we look for matches? Pixels?
- Need to figure out transform between images





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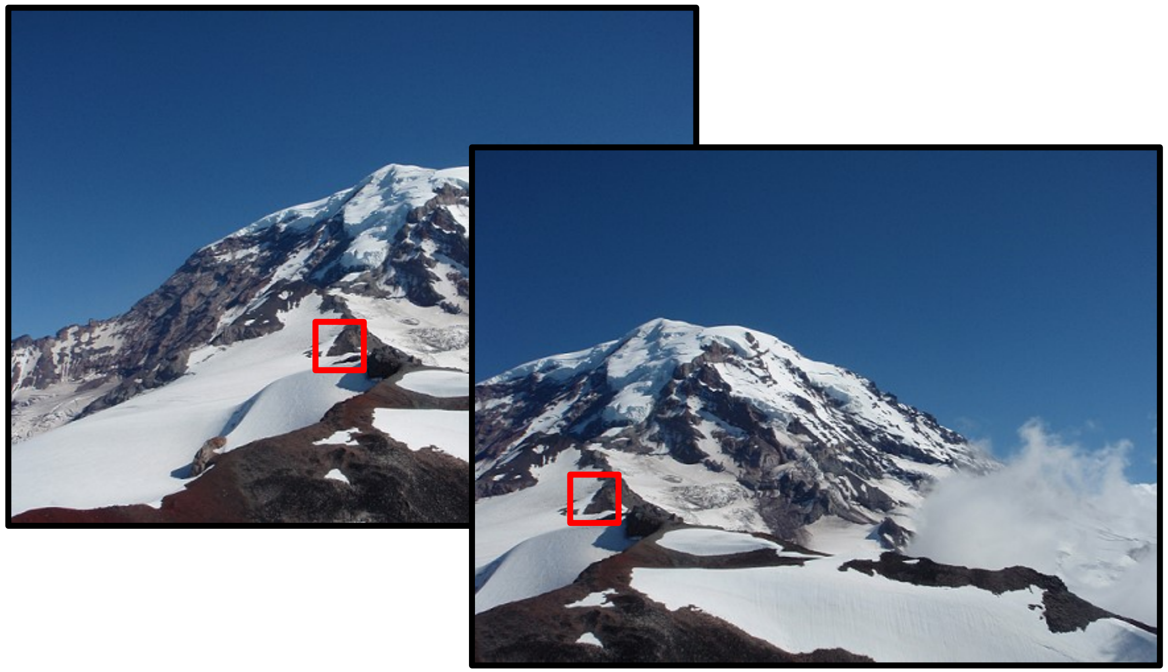
Matching patches: descriptors!

- We want a way to represent an image patch
- Can be very simple, just pixels!
- Finding matching patch is easy, distance metric:
  - $\Sigma_{x,y} (I(x,y) - J(x,y))^2$
  - What problems are there with just using pixels?



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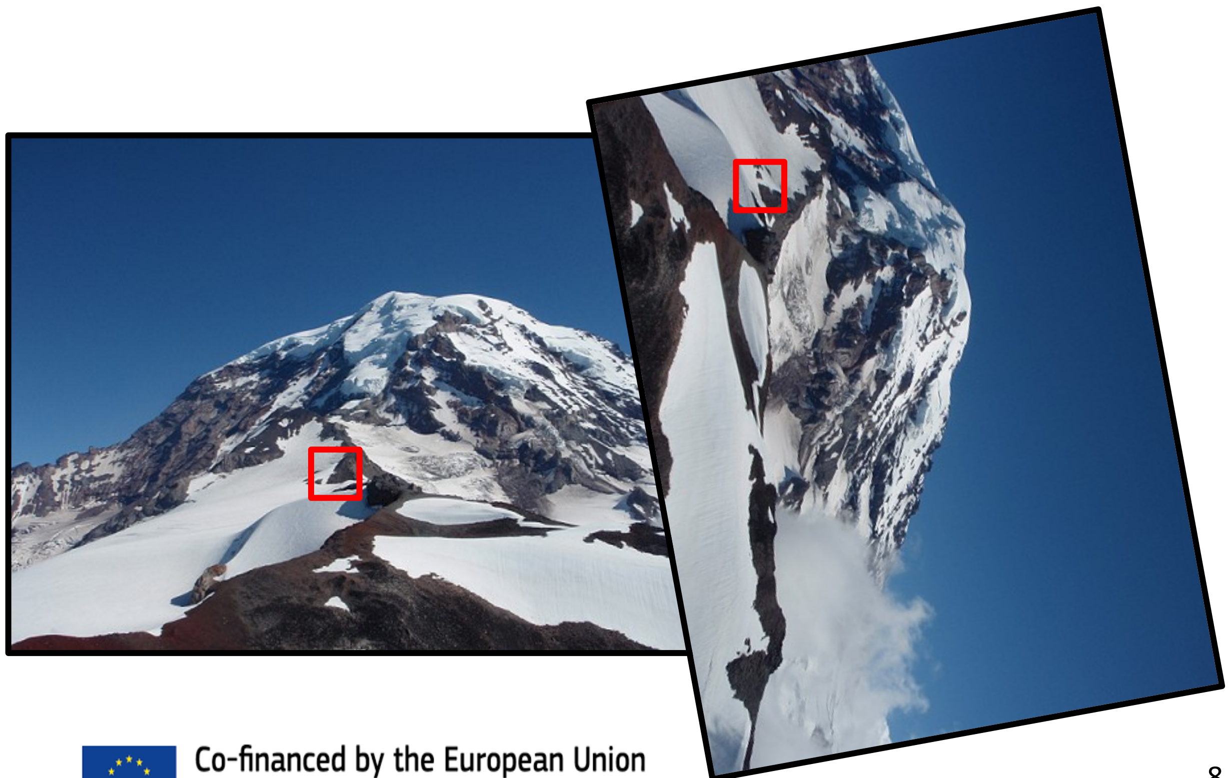
Matching patches: descriptors!

- We want a way to represent an image patch
- Can be very simple, just pixels!
- Finding matching patch is easy, distance metric:
  - $\Sigma_{x,y} (I(x,y) - J(x,y))^2$
  - Not invariant to some image transformations (e.g. rotation, scaling) !



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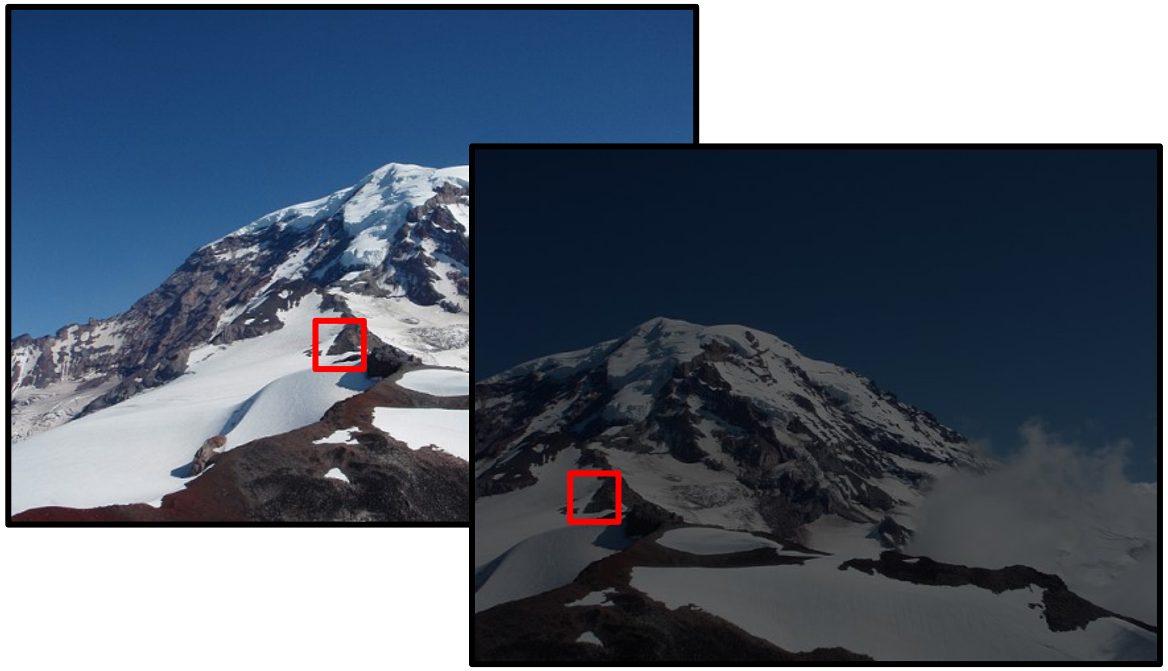
Matching patches: descriptors!

- We want a way to represent an image patch
- Can be very simple, just pixels!
- Finding matching patch is easy, distance metric:
  - $\Sigma_{x,y} (I(x,y) - J(x,y))^2$
  - Not invariant to lighting changes !



Matching patches: descriptors!

- We want feature descriptors invariant to lighting and image transforms !
- Descriptors can be more complex
  - Gradient information
  - How much context?

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Histogram of Oriented Gradients (HOG)

- By Dalal and Triggs 2005

- Better image descriptor

- Not reliant on magnitude, just direction
    - Invariant to some lighting changes

- They used it to train an SVM to recognize people

Binning from http://aishack.in/tutorials/sift-scale-invariant-feature-transform-features/

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Histogram of Oriented Gradients (HOG)

Steps to calculate HOG Feature Descriptor

1. Compute gradients

2. Bin gradient directions to create histogram

3. Normalize histograms of gradients

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Histogram of Oriented Gradients (HOG)

Steps to calculate HOG Feature Descriptor

1. Compute gradients

Gaussian smoothing (experimented with various σ), followed by a derivative filter

- σ =0 , i.e., no smoothing gave best results
- 1D filter [-1, 0, 1] gave best results

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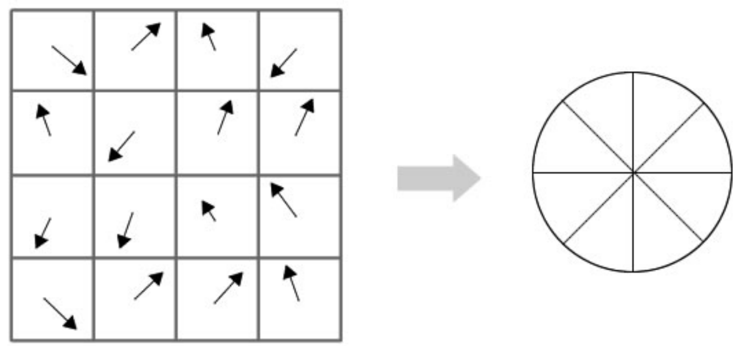
Histogram of Oriented Gradients (HOG)

Steps to calculate HOG Feature Descriptor

2. Bin gradient directions to create histogram

Split image into 8x8 'cells' and compute histogram for each cell

- Unsigned gradients, i.e., θ=0-180 degrees gave best results
- 9 bins gave best results



Histogram of Oriented Gradients (HOG)

Steps to calculate HOG Feature Descriptor

3. Normalize histograms of gradients

Gather overlapping ‘cells’ into ‘blocks’, concatenate histograms and normalize

- 16x16 blocks of 4 (2x2) cells gave best results
- L2 Normalization gave best results

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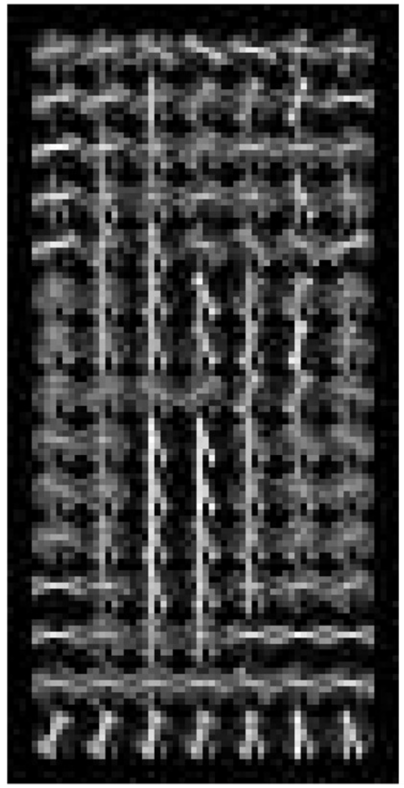
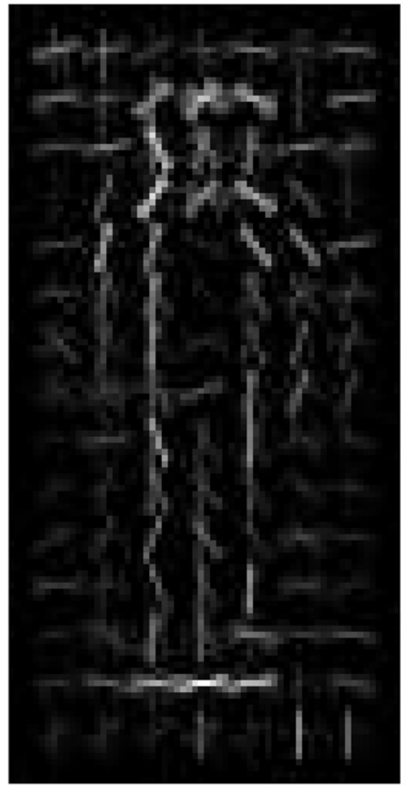
Histogram of Oriented Gradients (HOG)

For each training image of 64x128 there are 7x15 blocks, so the overall descriptor is 7x15x36 = 3780 dimensions

Training image



HOG descriptor of the image visualized for each 16x16 block



Descriptor weighted by the SVM weights

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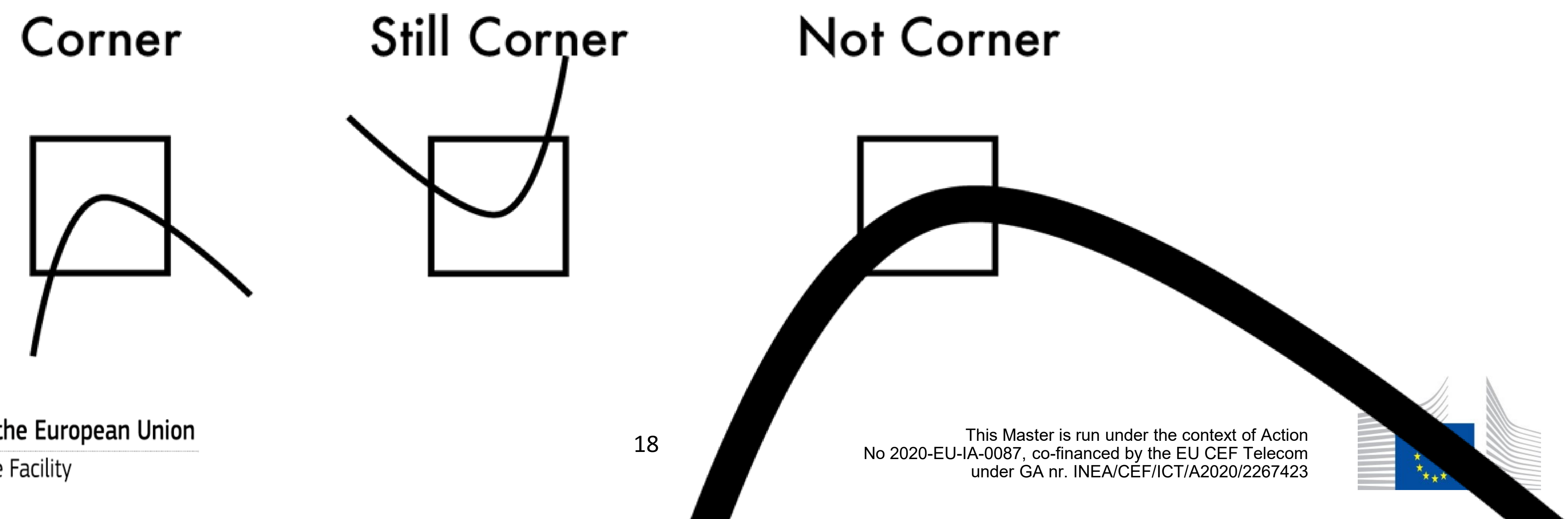
This is as good as it gets ?

- Not so fast…
- Harris has some issues:
  - Corner detection is rotation invariant
  - Harris not invariant to scale
- Descriptors are also hard
  - Just looking at pixels is not rotation invariant!
  - HOG also not rotation invariant



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Want features invariant to scaling, rotation, etc.

- Scale Invariant Feature Transform (SIFT)
  - Lowe et al. 2004, many images from that paper
- Get scale-invariant response map
- Find keypoints
- Extract rotation-invariant descriptors
  - Normalize based on orientation
  - Normalize based on lighting

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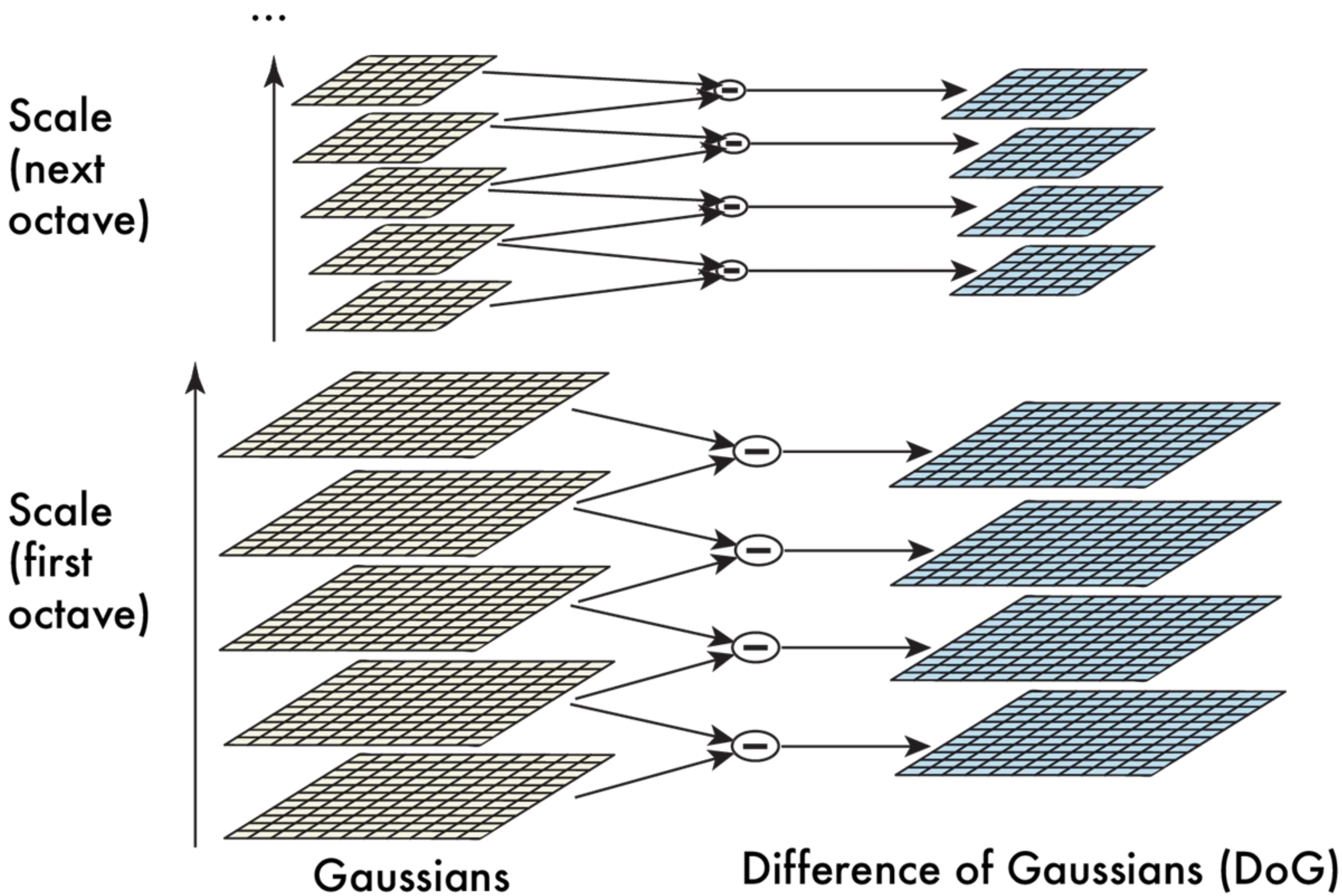
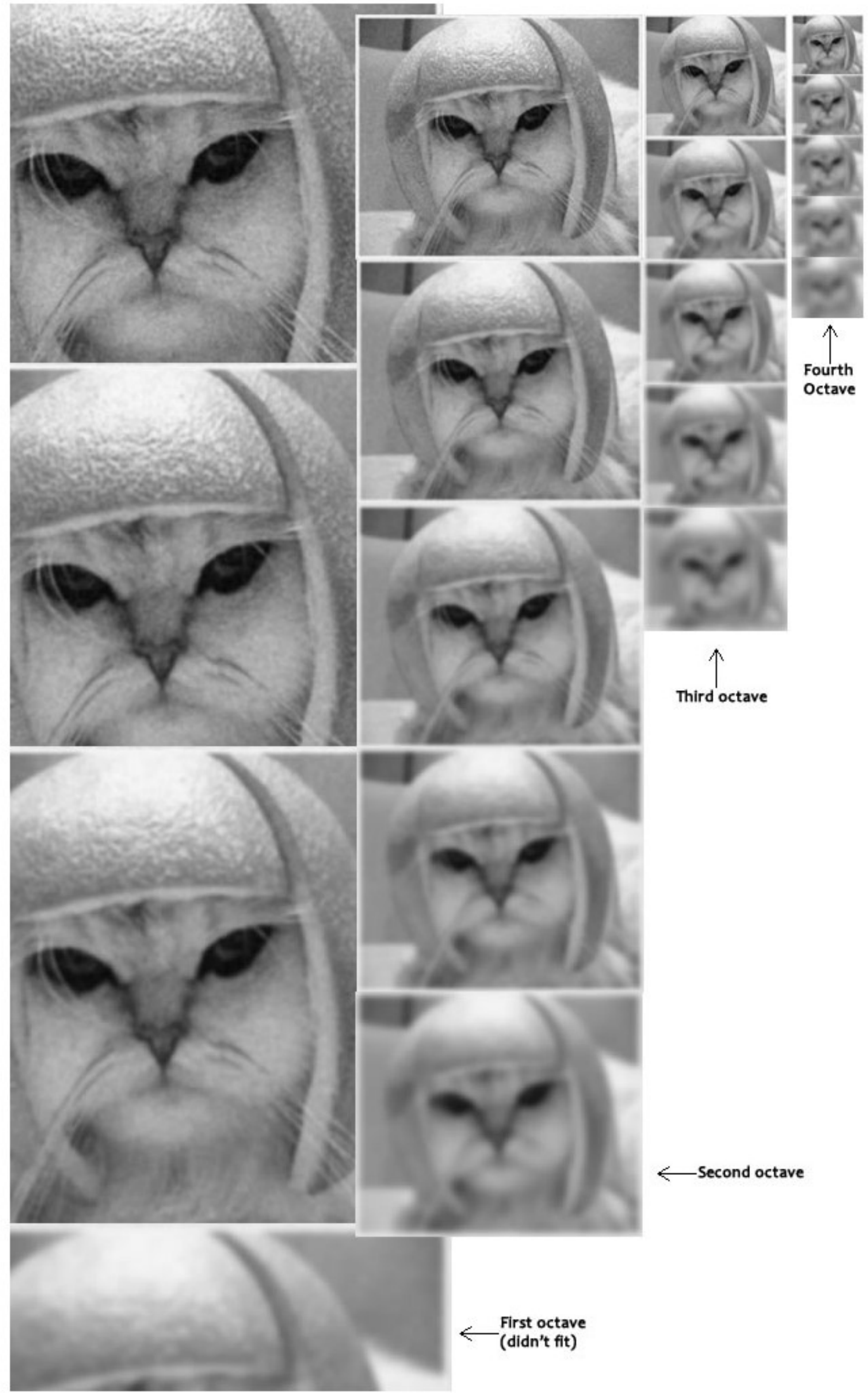
Extract DoG features at multiple scales



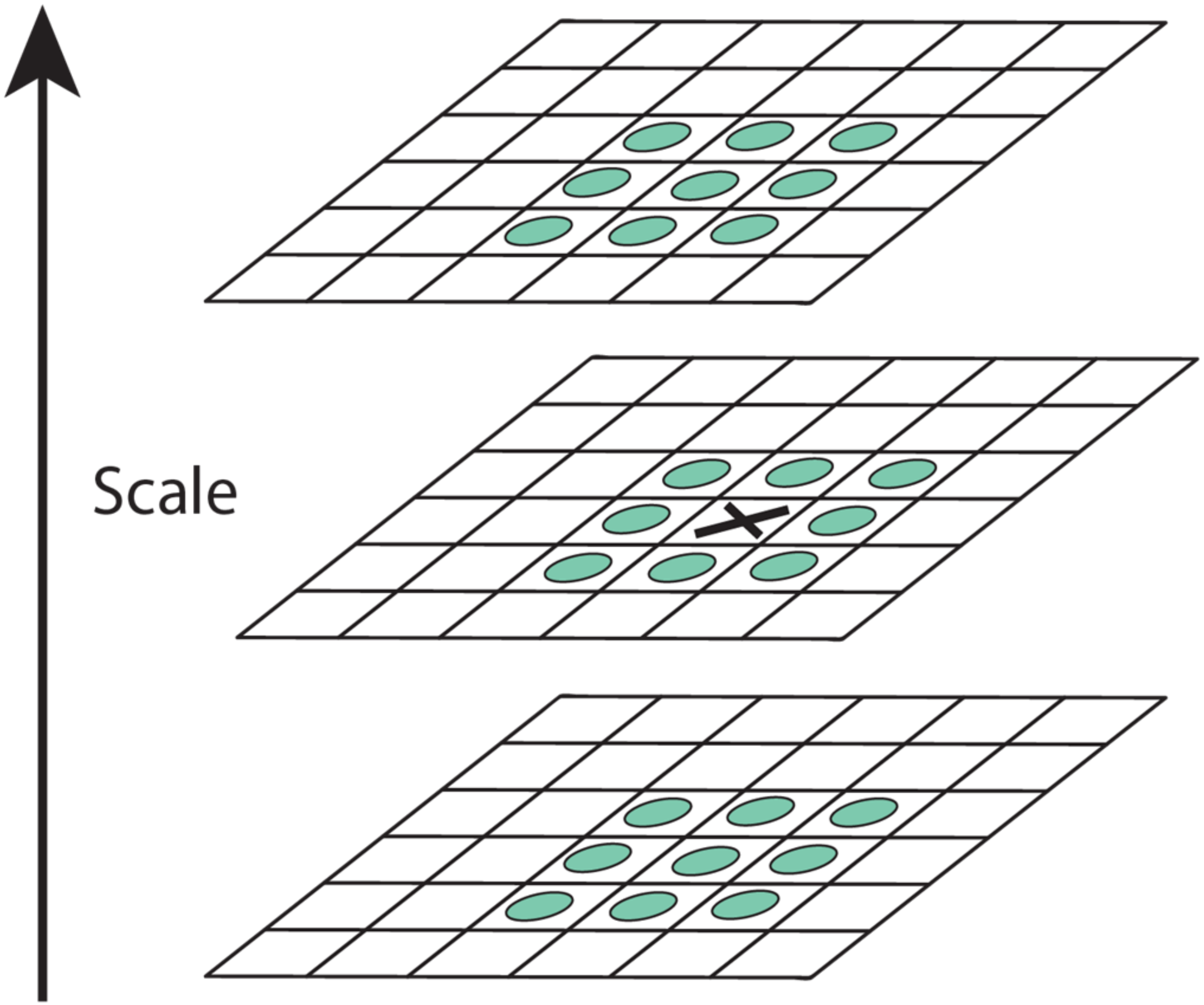
Scale space

scale →

octave				
0.707107	1.000000	1.414214	2.000000	2.828427
1.414214	2.000000	2.828427	4.000000	5.656854
2.828427	4.000000	5.656854	8.000000	11.313708
5.656854	8.000000	11.313708	16.000000	22.627417



Find local-maxima in location and scale



Throw out weak responses and edges

- Estimate gradients
  - Similar to before, look at nearby responses
  - Not whole image, only a few points! Faster!
  - Throw out weak responses
- Find cornery things
  - Same deal, structure matrix, use det and trace information
  - r : ratio of larger to smaller eigenvalue

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r},$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$$

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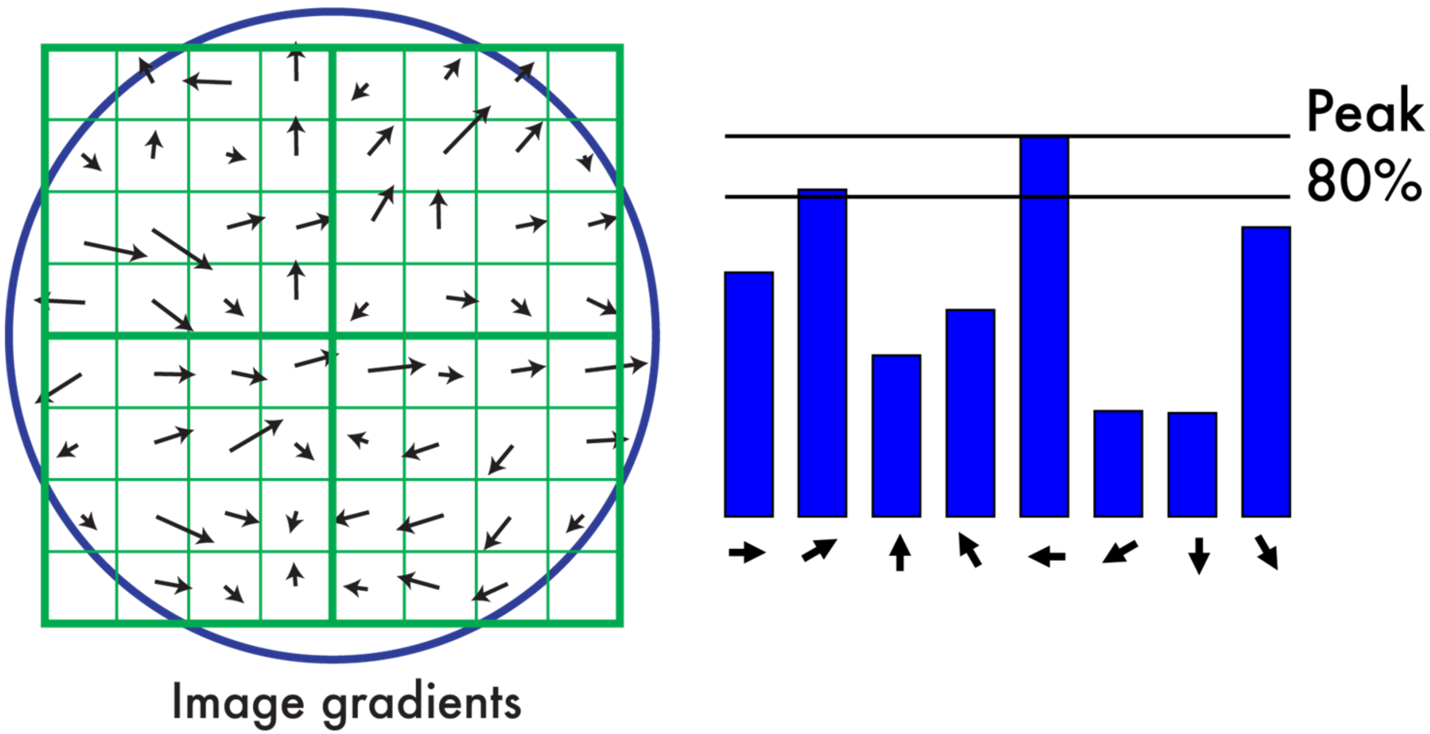
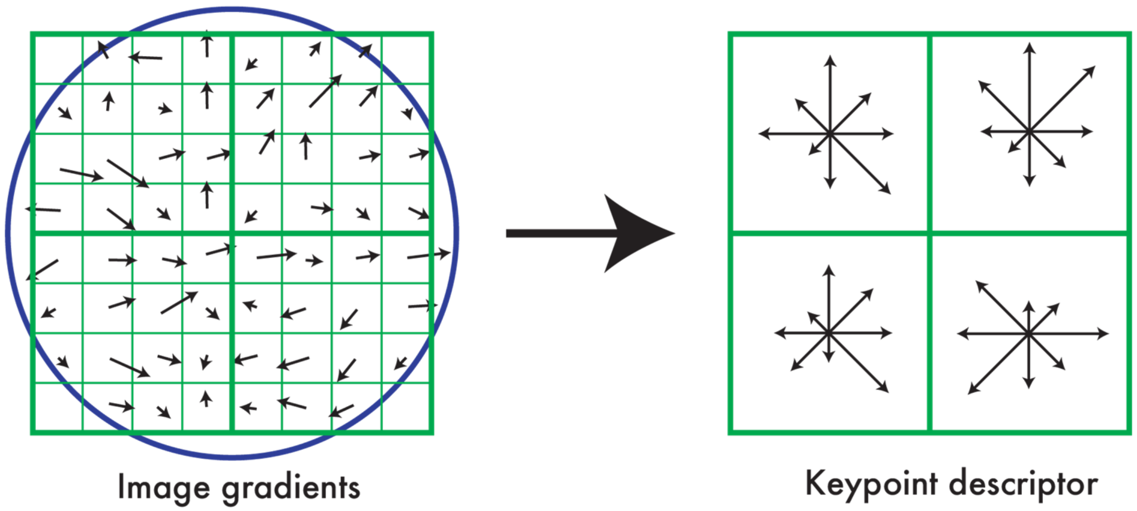
Find main orientation of patches

- Look at weighted histogram of nearby gradients
  - Any gradient within 80% of peak gets its own descriptor
    - Multiple keypoints per pixel
  - Descriptors are normalized based on main orientation



Keypoints are normalized gradient histograms

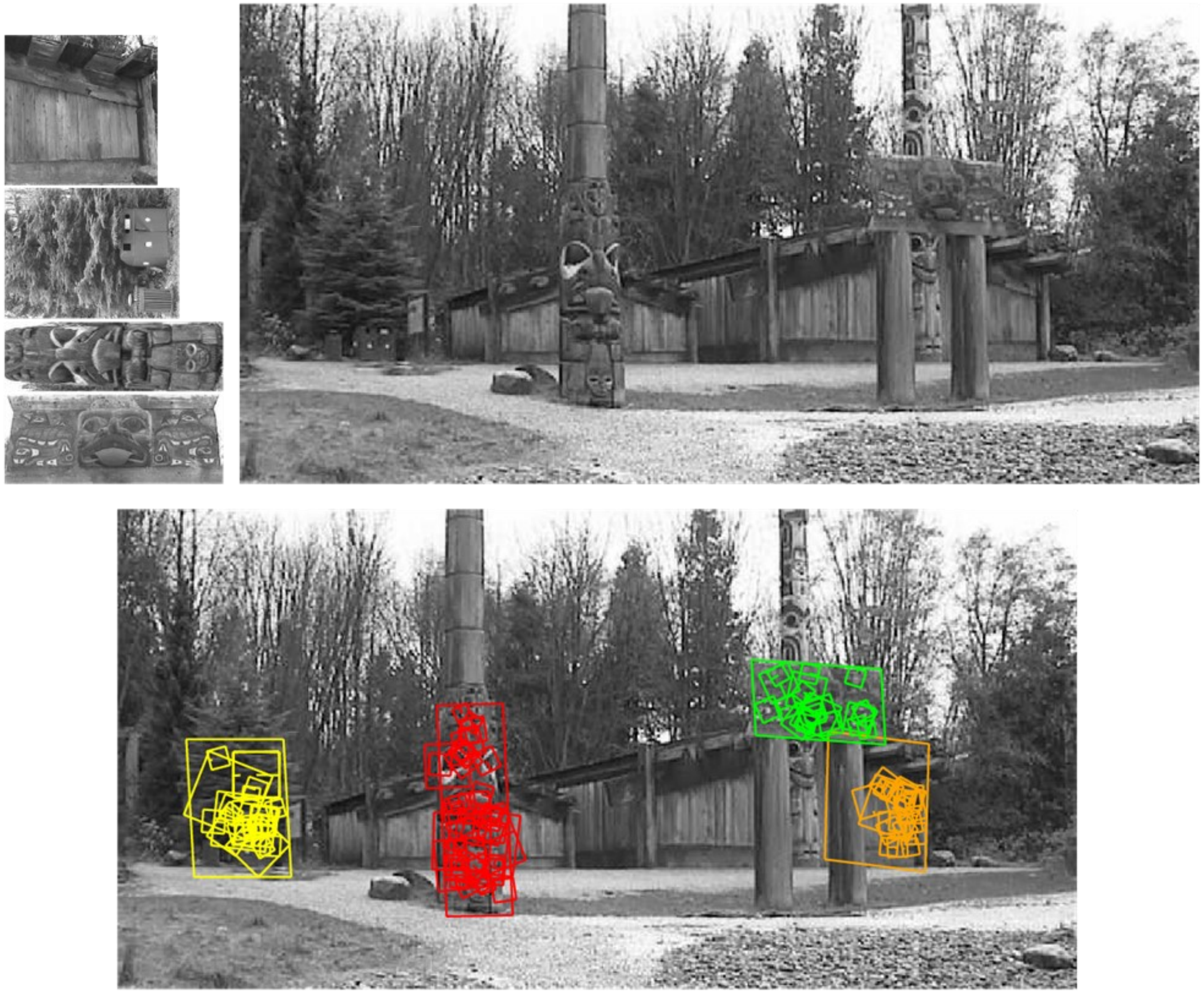
- Divide into subwindows (4x4)
- Bin gradients within subwindow, get histogram
    - Normalize to unit length
    - Clamp at maximum .2
    - Normalize again
    - Helps with lighting changes!



SIFT is great!

- Find good keypoints, describe them
- Finding objects, recognition, panoramas, etc.

SIFT is great!



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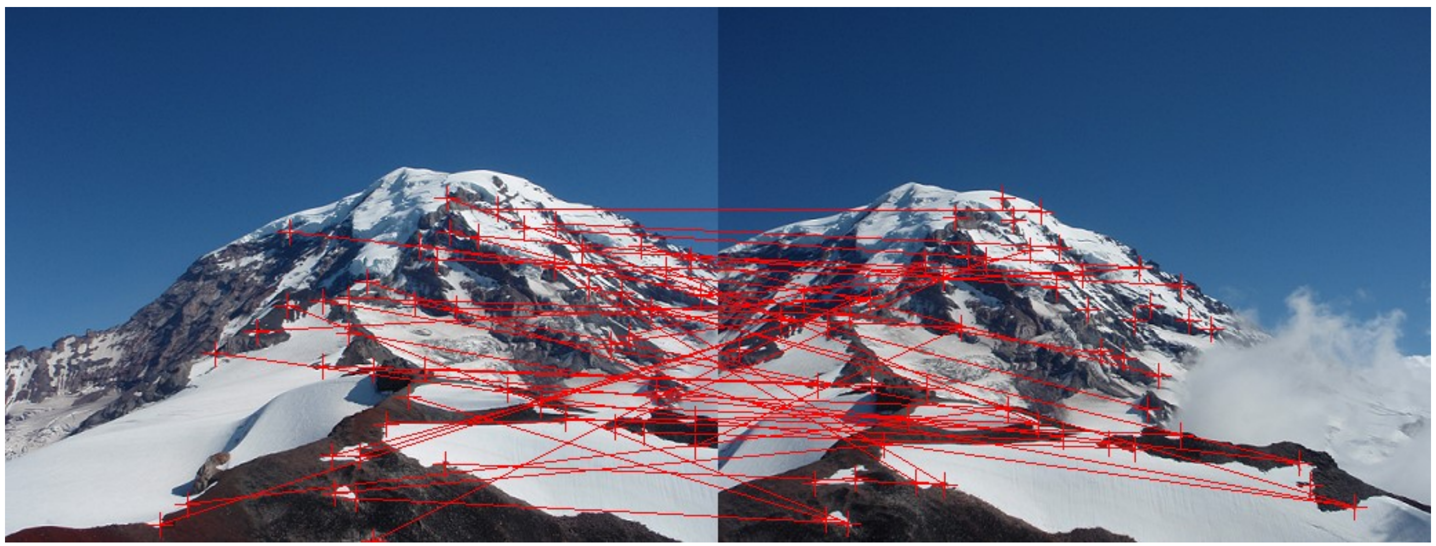
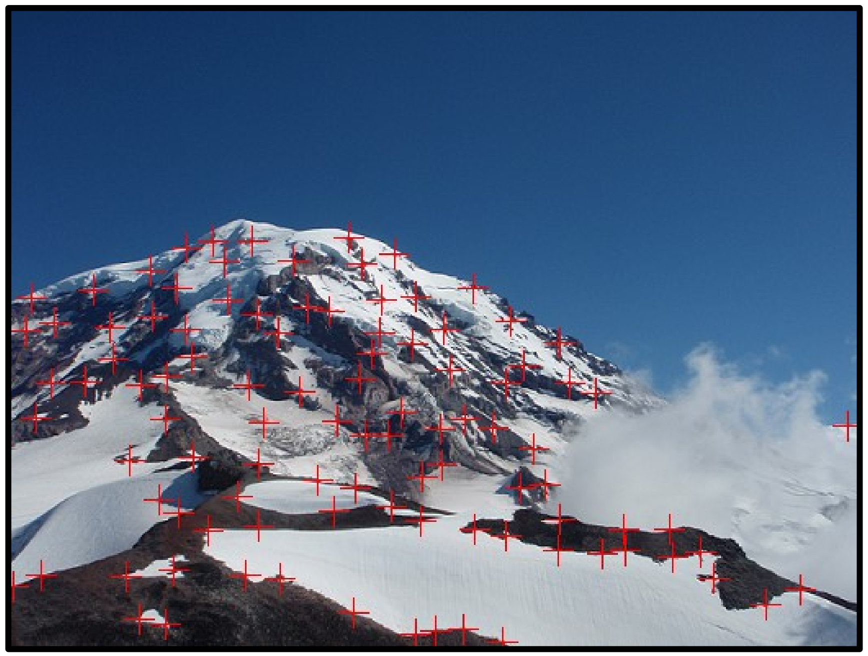
Matching patches: descriptors!

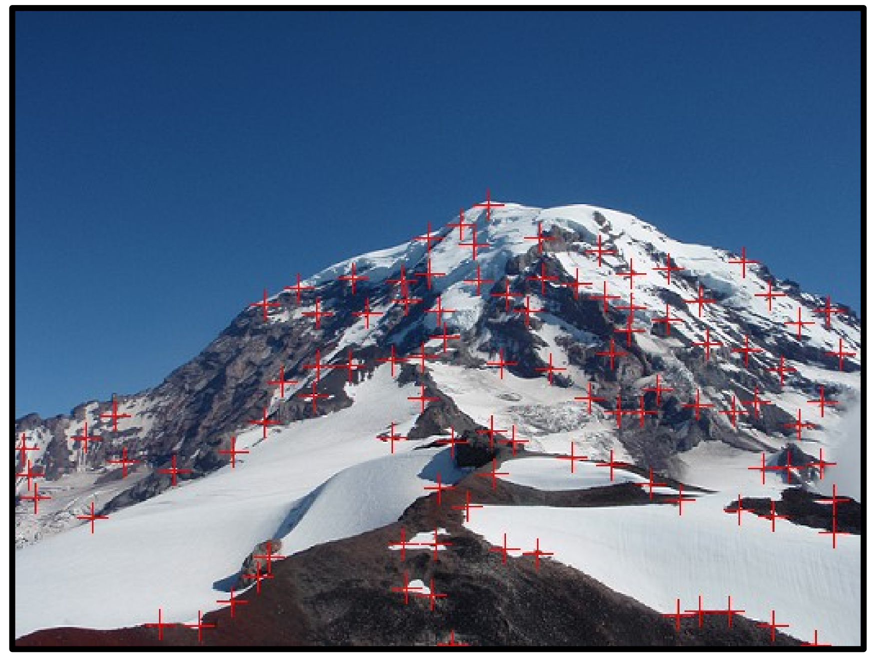
- Already have our patches that are likely “unique”-ish
- Loop over good patches in one image
  - Find best match in other image
- Do something with them?



Ok, we found corners, now what?

- Need to match image patches to each other
- Need to figure out transform between images





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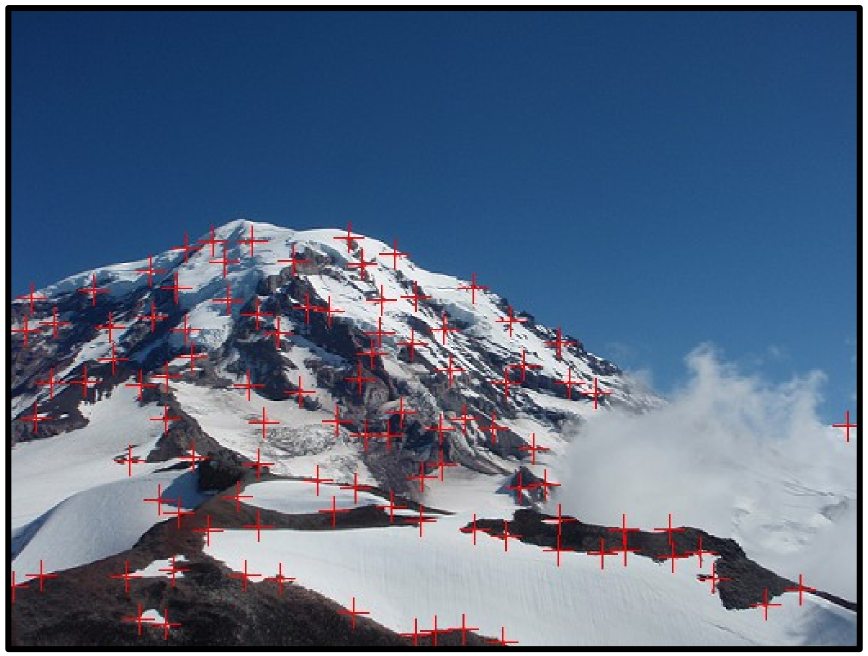
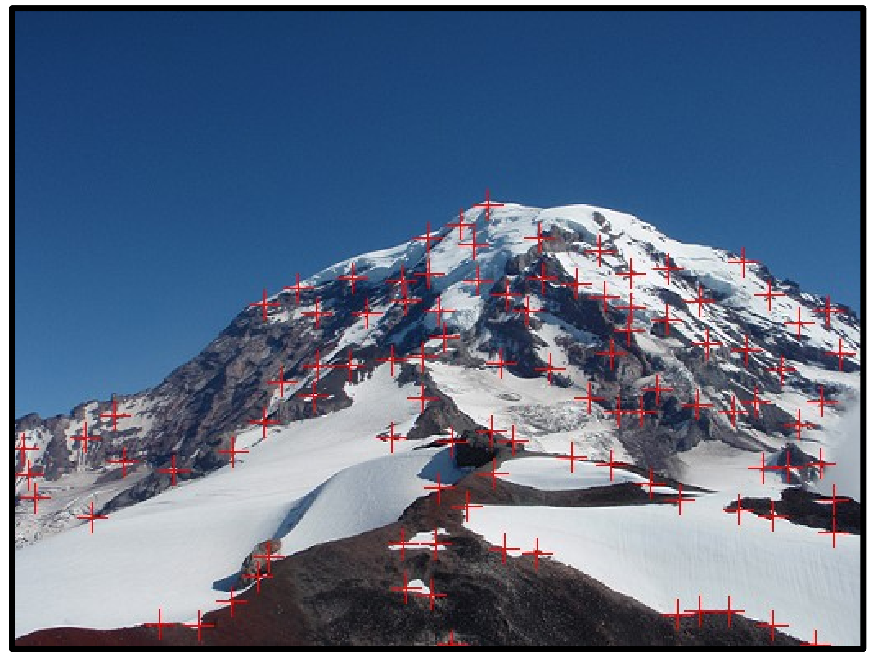
Ok, we found corners, now what?

- Need to match image patches to each other
- Need to figure out transform between images
  - How can we transform images?
  - How do we solve for this transformation given matches?





How can we transform images?

- Need to warp one image into the other
- Many different image transforms
  - Nested hierarchy of transformations

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How can we transform images?

- **x** is a point in our image where:
  - **x** = (x, y) or in matrix terms

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

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Say we want new coordinate system

- Map points from one image into another
- Often we can use matrix operations
- Given a point x, map to new point x’ using M

$$\mathbf{x}' = \mathbf{M}\,\mathbf{x}$$

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Scaling is just a matrix operation

- Map points from one image into another
- Often we can use matrix operations
- Given a point x, map to new point x' using M

$$\mathbf{x}' = \mathbf{S}\,\mathbf{x}$$

$$\mathbf{x}' = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \mathbf{x}$$

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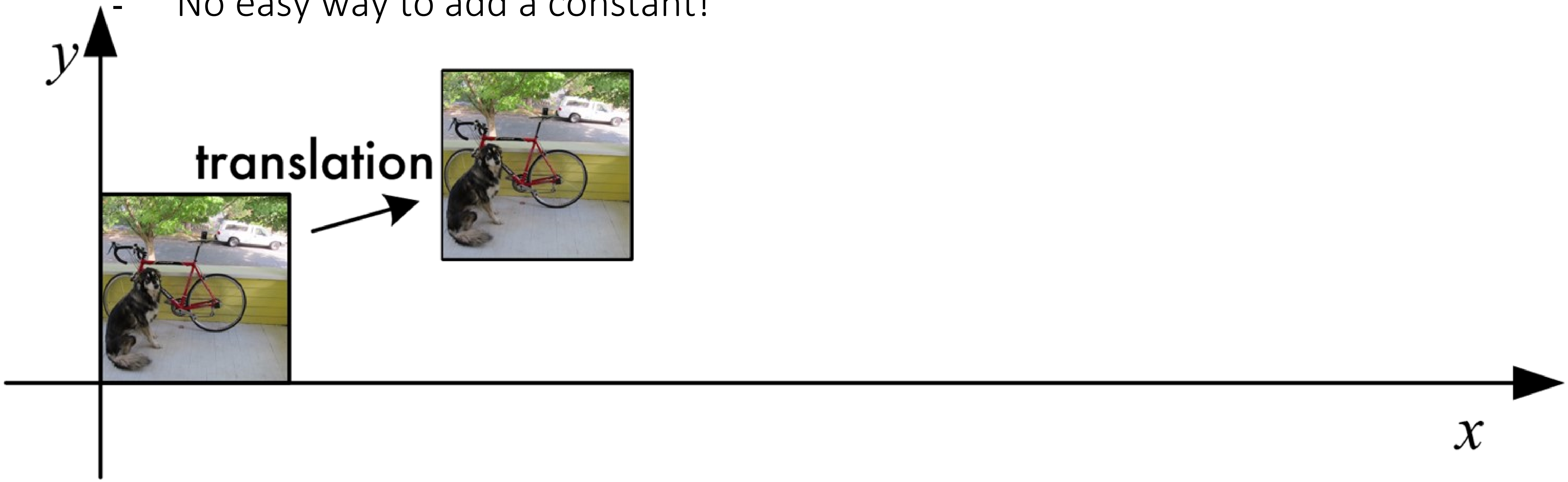
Translation is harder...

- x' = M x
  - Want to move x' by dx and y' by dy
  - How do we pick **M**?
  - Can only add up multiples of x or y
  - No easy way to add a constant!



Translation: add another row

- $\bar{\mathbf{x}}$ is $\mathbf{x}$ but with an added 1
- *Augmented vector*

$$\bar{\mathbf{x}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Translation: add another row

- $\bar{\mathbf{x}}$ is $\mathbf{x}$ but with an added 1
- *Augmented vector*
- Now translation is easy

$$\bar{\mathbf{x}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x}' = \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \bar{\mathbf{x}}$$

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Reminder, $\mathbf{I}$ = Identity

Common to just use $\mathbf{I}$ as a generic, whatever size identity fits here.

$$\mathbf{I}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ & & & \dots & \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$$\mathbf{I}_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Translation: add another row

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- *Augmented vector*
- Now translation is easy

$$\bar{\mathbf{x}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x}' = [\,\mathbf{I} \quad \mathbf{t}\,]\,\bar{\mathbf{x}}$$

Translation: add another row

- $\bar{\mathbf{x}}$ is **x** but with an added 1
- *Augmented vector*
- Now translation is easy
- x' = 1*x + 0*y + dx*1
- y' = 0*x + 1*y + dy*1

$$\bar{\mathbf{x}} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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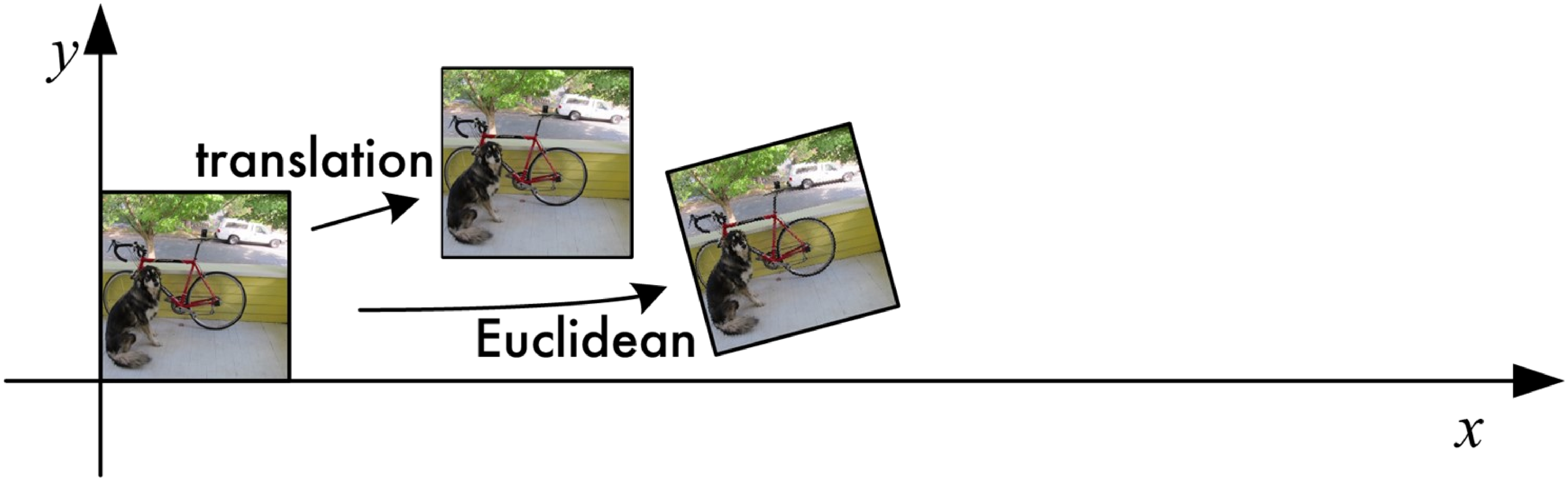
Euclidean: rotation + translation

- Want to translate and rotate at same time
- Still just matrix operation



Euclidean: rotation + translation

- Want to translate and rotate at same time
- Still just matrix operation

$$\mathbf{x}' = \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \bar{\mathbf{x}}$$

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Euclidean: rotation + translation

- Want to translate and rotate at same time
- Still just matrix operation
- **R** is rotation matrix, **t** is translation

$$\mathbf{x}' = [\,\mathbf{R} \quad \mathbf{t}\,]\,\bar{\mathbf{x}}$$

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Euclidean: rotation + translation

- Want to translate and rotate at same time
- Still just matrix operation
- **R** is rotation matrix, **t** is translation

$$\mathbf{x'} = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x'} = [\,\mathbf{R} \quad \mathbf{t}\,]\,\bar{\mathbf{x}}$$

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

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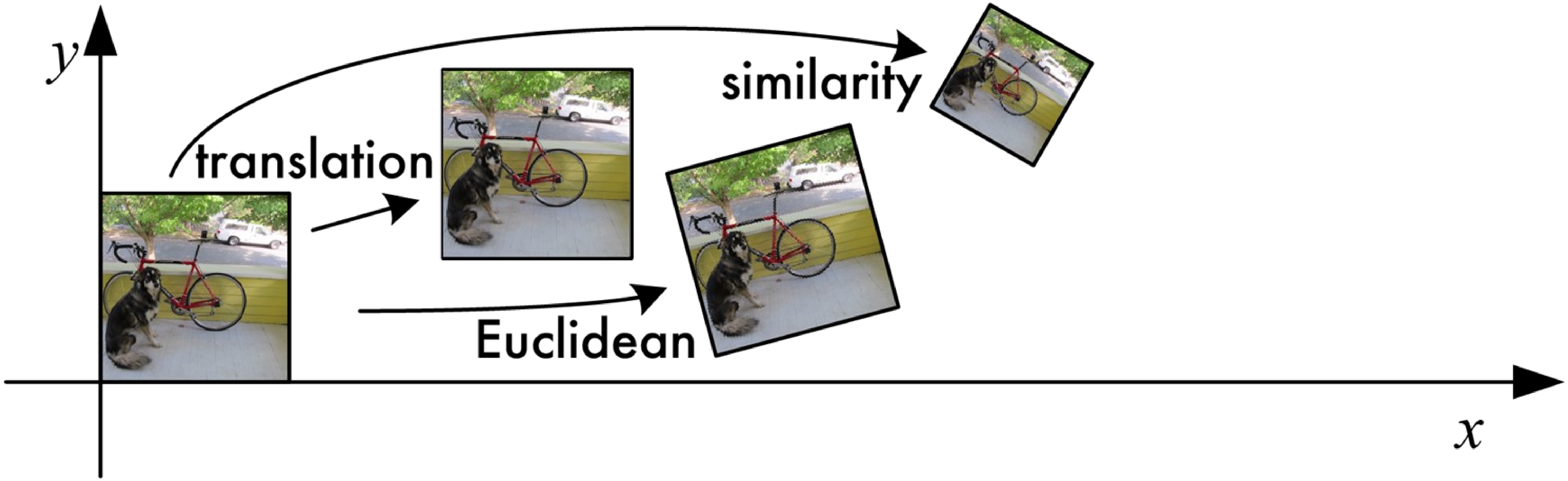
Similarity: scale, rotate, translate



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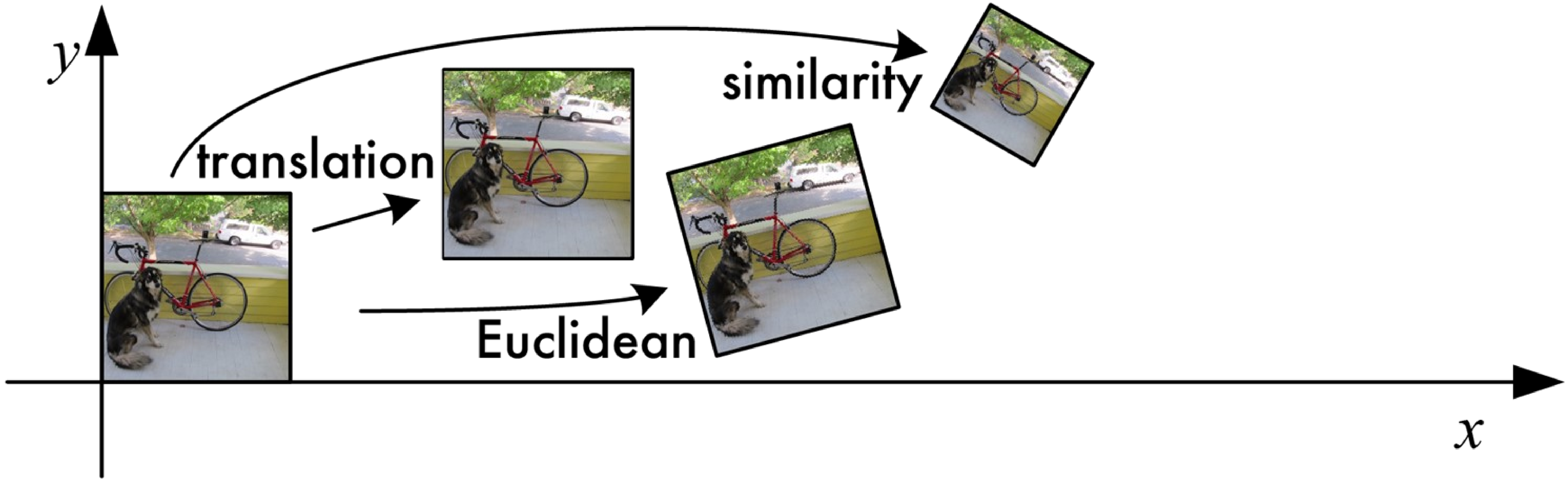
Similarity: scale, rotate, translate

$$\mathbf{x}' = [\, s\mathbf{R} \quad \mathbf{t} \,]\, \bar{\mathbf{x}}$$



Similarity: scale, rotate, translate

$$\mathbf{x}' = \begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix} \bar{\mathbf{x}}$$

$$\mathbf{x}' = \begin{bmatrix} a & -b & dx \\ b & a & dy \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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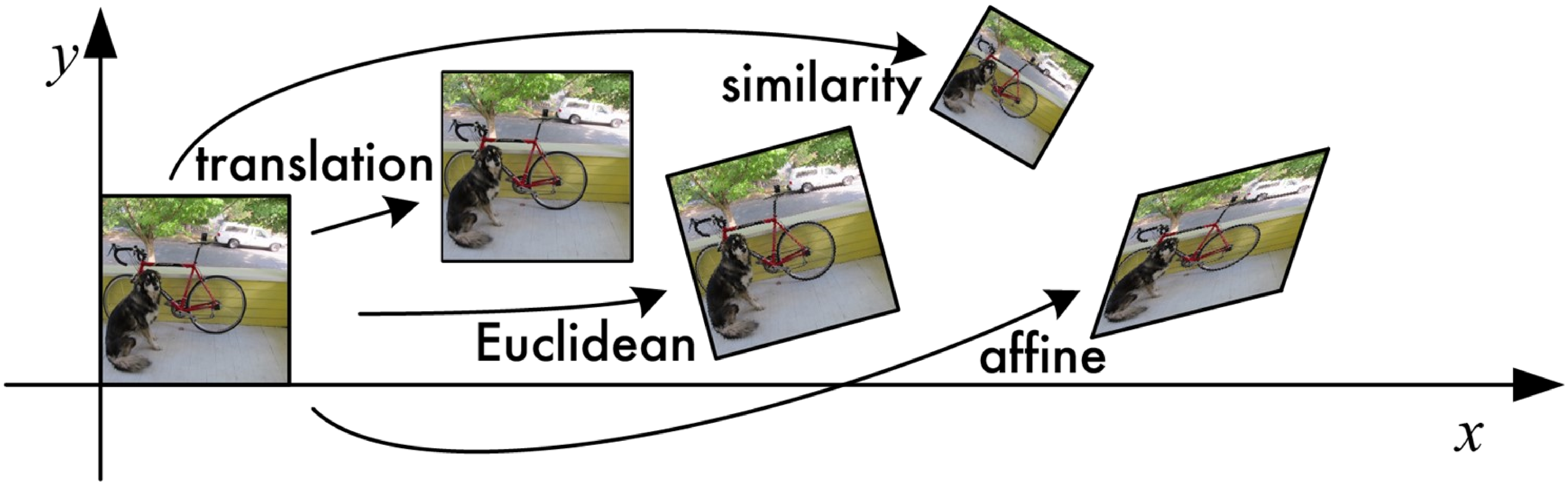
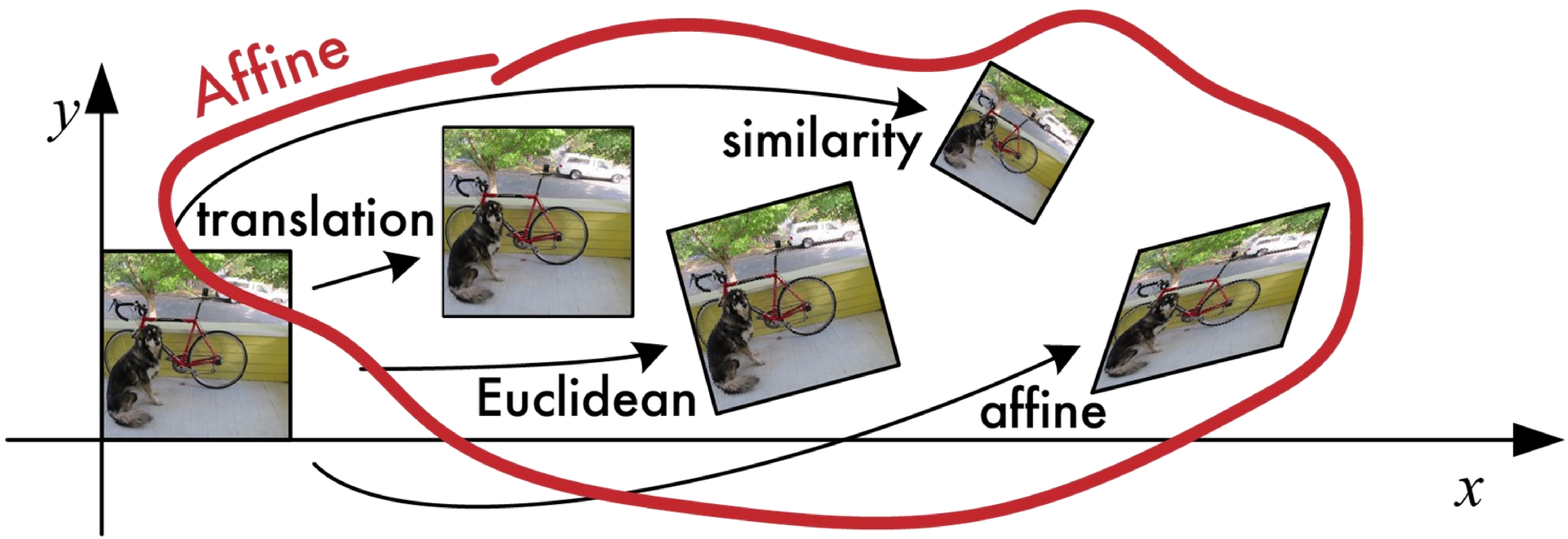
Affine: scale, rotate, translate, shear



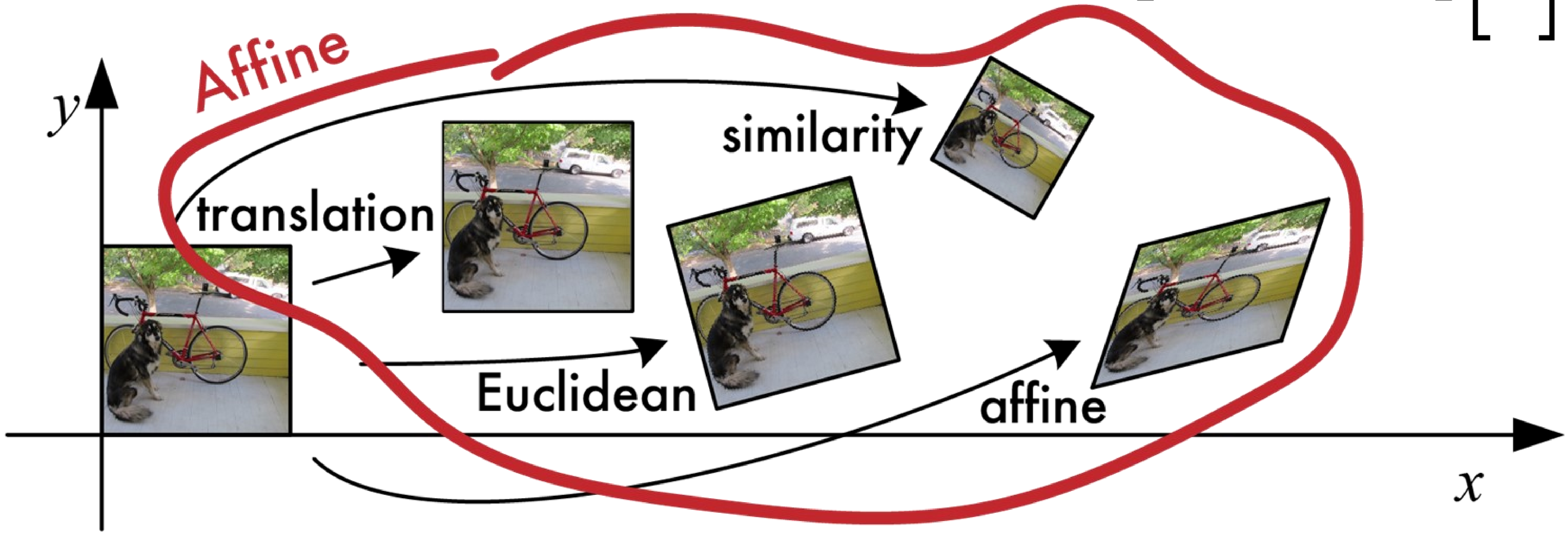
Affine: scale, rotate, translate, shear



Affine: scale, rotate, translate, shear

General case of 2x3 matrix

$$\mathbf{x}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Combinations are still affine

Say you want to translate, then rotate, then translate back, then scale.

$\mathbf{x}' = \mathbf{S}\,\mathbf{t}\,\mathbf{R}\,\mathbf{t}\,\bar{\mathbf{x}} = \mathbf{M}\,\bar{\mathbf{x}}$,

If $\mathbf{M} = (\mathbf{S}\,\mathbf{t}\,\mathbf{R}\,\mathbf{t})$

M is still affine transformation

Wait, but these are all 2x3, how to we multiply them together?

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Added row to transforms

$$\bar{\mathbf{x}}' = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a & -b & dx \\ b & a & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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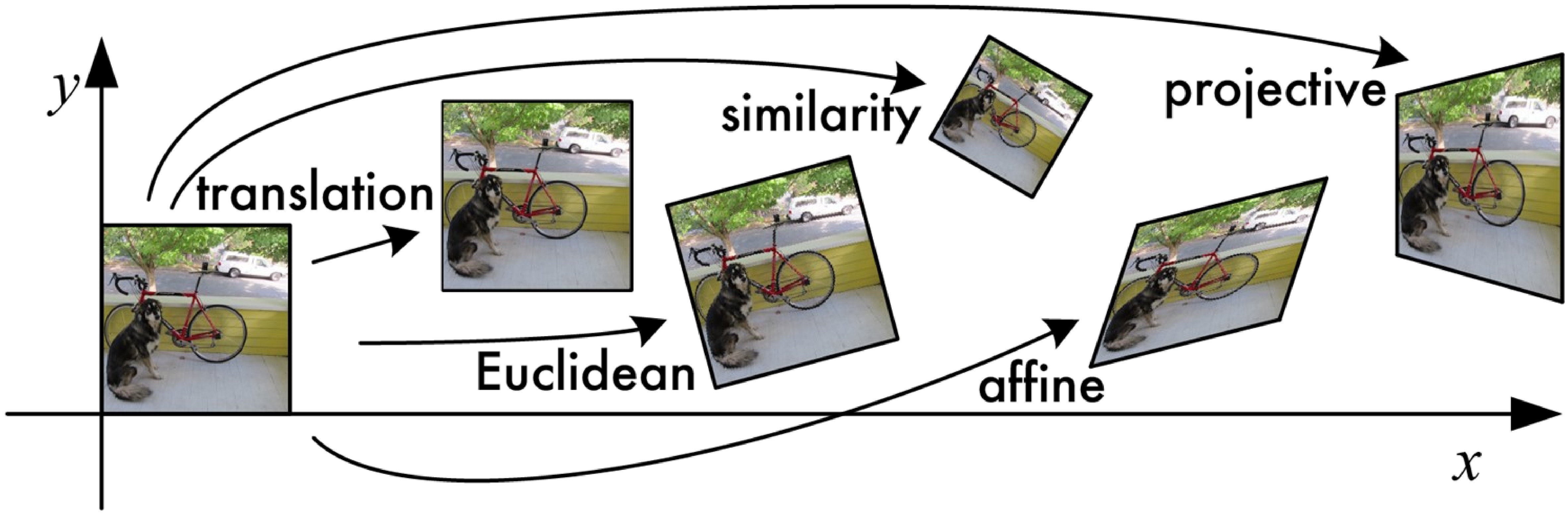
Projective transform

- Also known as homography
- Wait but affine was any 2x3 matrix...



Need some new coordinates!

- Homogeneous coordinate system
- Each point in 2d is actually a vector in 3d
- Equivalent up to scaling factor
- Have to normalize to get back to 2d

$$\tilde{\mathbf{x}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix} \qquad \bar{\mathbf{x}} = \tilde{\mathbf{x}} \, / \, \tilde{w}$$

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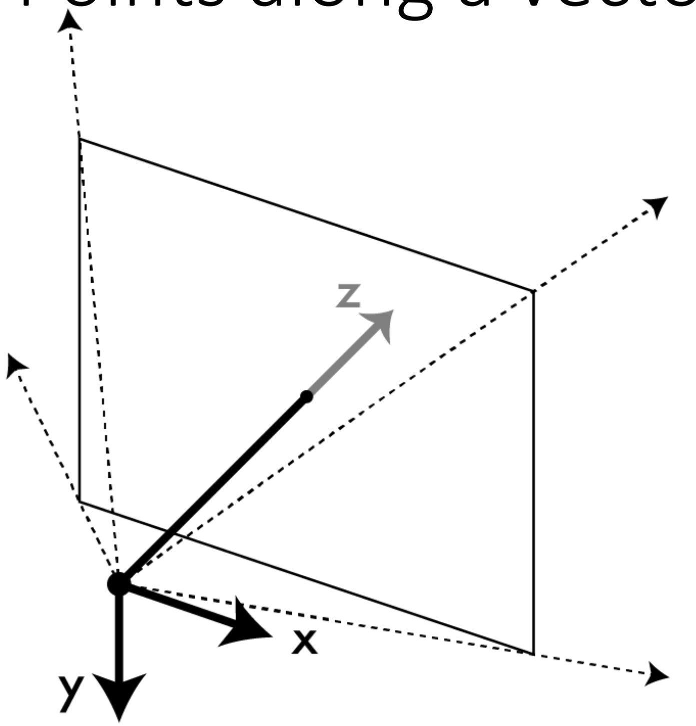
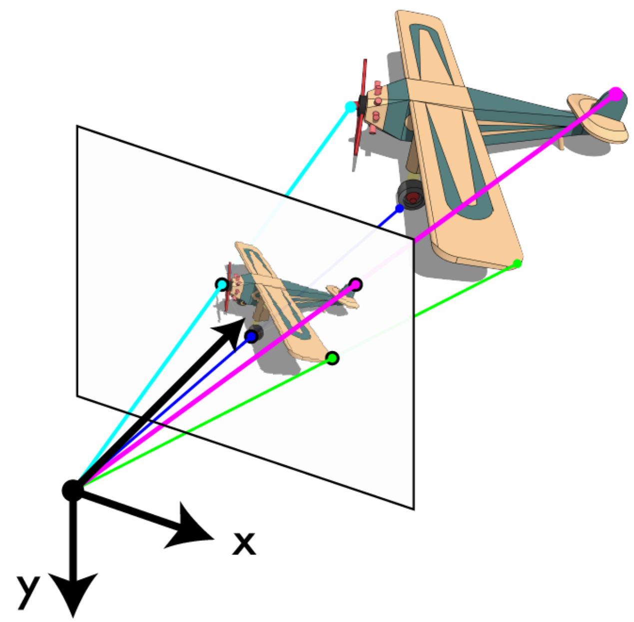
Why does this make sense?

- Remember our pinhole camera model
- Every point in 3d projects onto our viewing plane through our aperture
- Points along a vector are indistinguishable





Projective transform

- Also known as homography
- Wait but affine was any 2x3 matrix…
- Homography is general 3x3 matrix
- Multiplication by scalar is equivalent

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}}\,\tilde{\mathbf{x}}$$

Projective transform

- Also known as homography
- Wait but affine was any 2x3 matrix…
- Homography is general 3x3 matrix
- Multiplication by scalar: equivalent projection
  - 3*H ~ H

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}}\,\tilde{\mathbf{x}}$$

$$\tilde{\mathbf{x}}' = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

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Using homography to project point

- Multiply **x~** by **H~** to get **x'~**
- Convert to **x'** by dividing by **w'~**

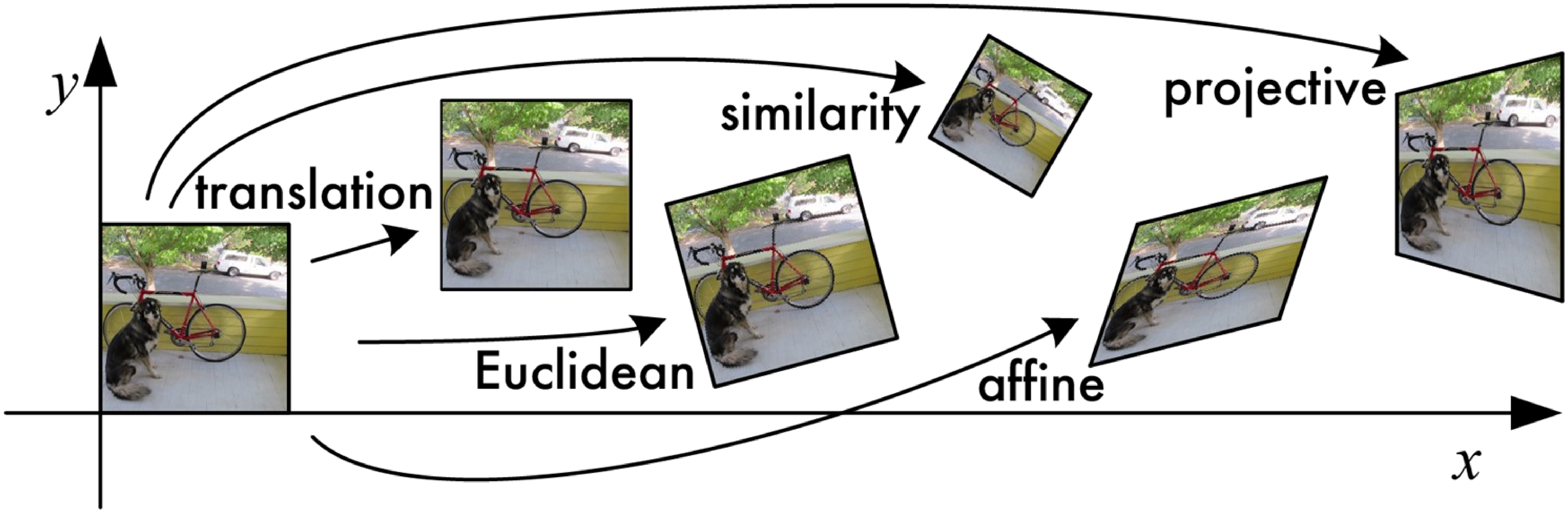
$$\tilde{\mathbf{x}}' = \tilde{\mathbf{H}}\, \tilde{\mathbf{x}}$$

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{w}' \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

$$\bar{\mathbf{x}} = \tilde{\mathbf{x}} / \tilde{w}$$

Lots to choose from

- What do each of them do?
- Which is right for panorama stitching?



Today's Agenda

- Basic descriptor and matching
- Image transformations
- Estimate transformations

How hard are they to recover?

$$\bar{\mathbf{x}}' = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a & -b & dx \\ b & a & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\bar{\mathbf{x}}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{x}}' = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

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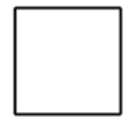
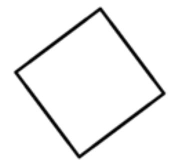
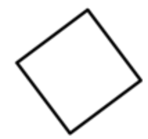
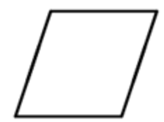
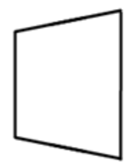
Lots to choose from

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mid & \mathbf{t} \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mid & \mathbf{t} \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\mathbf{R} & \mid & \mathbf{t} \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

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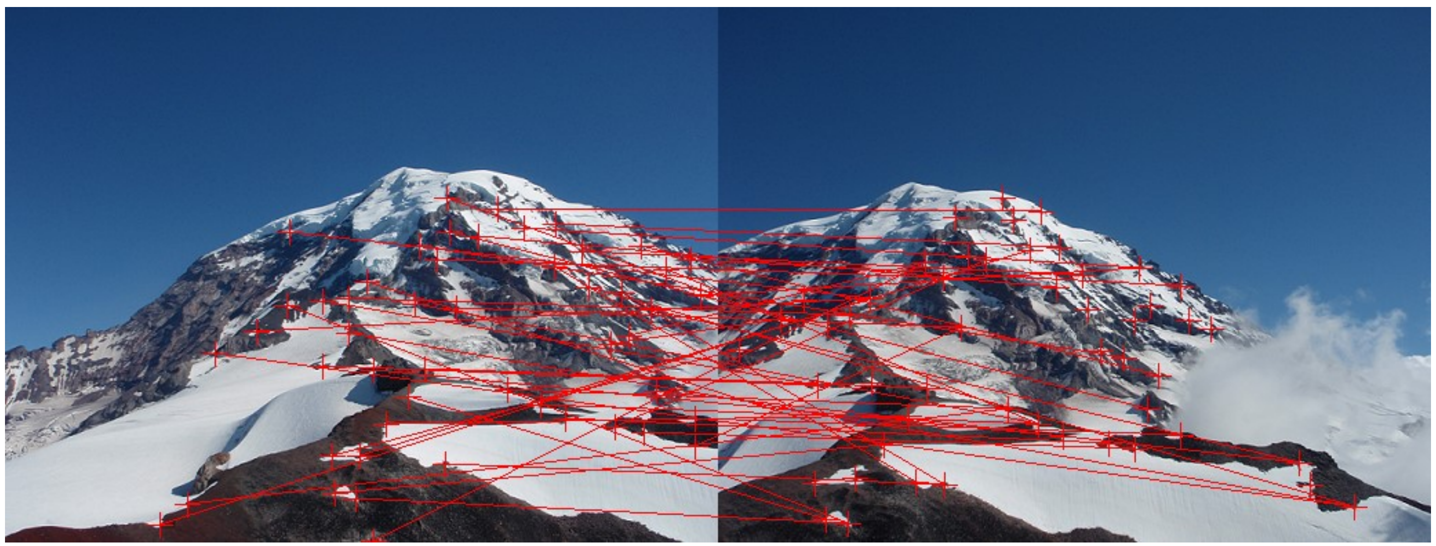
Say we want affine transformation

- Have our matched points
- Want to estimate **A** that maps from **x** to **x'**
- **Ax** = **x'**



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Say we want affine transformation

- Have our matched points
- Want to estimate **A** that maps from **x** to **x'**
- **Ax = x'**
- How many degrees of freedom?

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Say we want affine transformation

- Have our matched points
- Want to estimate **A** that maps from **x** to **x'**
- **A**x = x'
- How many degrees of freedom?
  - 6
- How many knowns do we get with one match?

$$\mathbf{x'} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Say we want affine transformation

- Have our matched points
- Want to estimate **A** that maps from **x** to **x'**
- **Ax** = **x'**
- How many degrees of freedom?
  - 6
- How many knowns do we get with one match?
  - 2
  - $n_x = a_{00}*m_x + a_{01}*m_y + a_{02}*1$
  - $n_y = a_{10}*m_x + a_{11}*m_y + a_{12}*1$

$$\mathbf{x'} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Say we want affine transformation

- How many knowns do we get with one match?
  - $n_x = a_{00}*m_x + a_{01}*m_y + a_{02}*1$
  - $n_y = a_{10}*m_x + a_{11}*m_y + a_{12}*1$
  - Solve linear system of equations **M a = b**
    - $M^{-1} M a = M^{-1} b \Rightarrow a = M^{-1} b$
    - But M^{-1} does not exist in general - Why?
  - Still works if overdetermined
    - Why???
    - Pseudoinverse – least squares solution
    - $M^T M a = M^T b$
    - $(M^T M)^{-1} (M^T M) a = (M^T M)^{-1} M^T b$
    - $\Rightarrow a = (M^T M)^{-1} M^T b$

$$\underbrace{\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 \\ & & \cdots & & & \\ & & \cdots & & & \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ \cdots \\ \cdots \end{bmatrix}}_{\mathbf{b}}$$

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Thank you.

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