MAI4CAREU Master programmes in Artificial Intelligence 4 Careers in Europe

Visual Computing Group

CYENS CENTRE OF EXCELLENCE

University of Cyprus – MSc Artificial Intelligence

MAI644 – COMPUTER VISION

Lecture 9: RANSAC, Panorama Stitching

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Last time

- Basic feature descriptor and matching
- Histogram of Oriented Gradients
- SIFT
- Image transformations
- Estimate transformations

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Today's Agenda

- Linear least-squares
- RANSAC
- Panorama Stitching

[material based on Joseph Redmon's course]

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Linear least squares

Want to solve overdetermined linear system:

- $M a = b$

Want to minimize squared error:

$|| b - M a ||^2 =$

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Want to solve overdetermined linear system:

- $M a = b$

Want to minimize squared error:

$|| b - M a ||^2 =$

$(b - M a)^T(b - M a)$

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Linear least squares

Want to solve overdetermined linear system:

- $M a = b$

Want to minimize squared error:

$|| b - M a ||^2 =$

$(b - M a)^T(b - M a) =$

$b^Tb - a^TM^Tb - b^TMa + a^TM^TMa$

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Linear least squares

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$b^Tb - a^TM^Tb - b^TMa + a^TM^TMa =$

$b^Tb - 2a^TM^Tb + a^TM^TMa$

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Linear least squares

Want to minimize squared error: $|| b - M a ||^2 =$

$b^Tb - 2a^TM^Tb + a^TM^TMa$

This is convex and minimized when gradient = 0. So we take the derivative wrt a and set = 0.

$-M^Tb + (M^TM)a = 0$

$(M^TM)a = M^Tb$

$a = (M^TM)^{-1}M^Tb$

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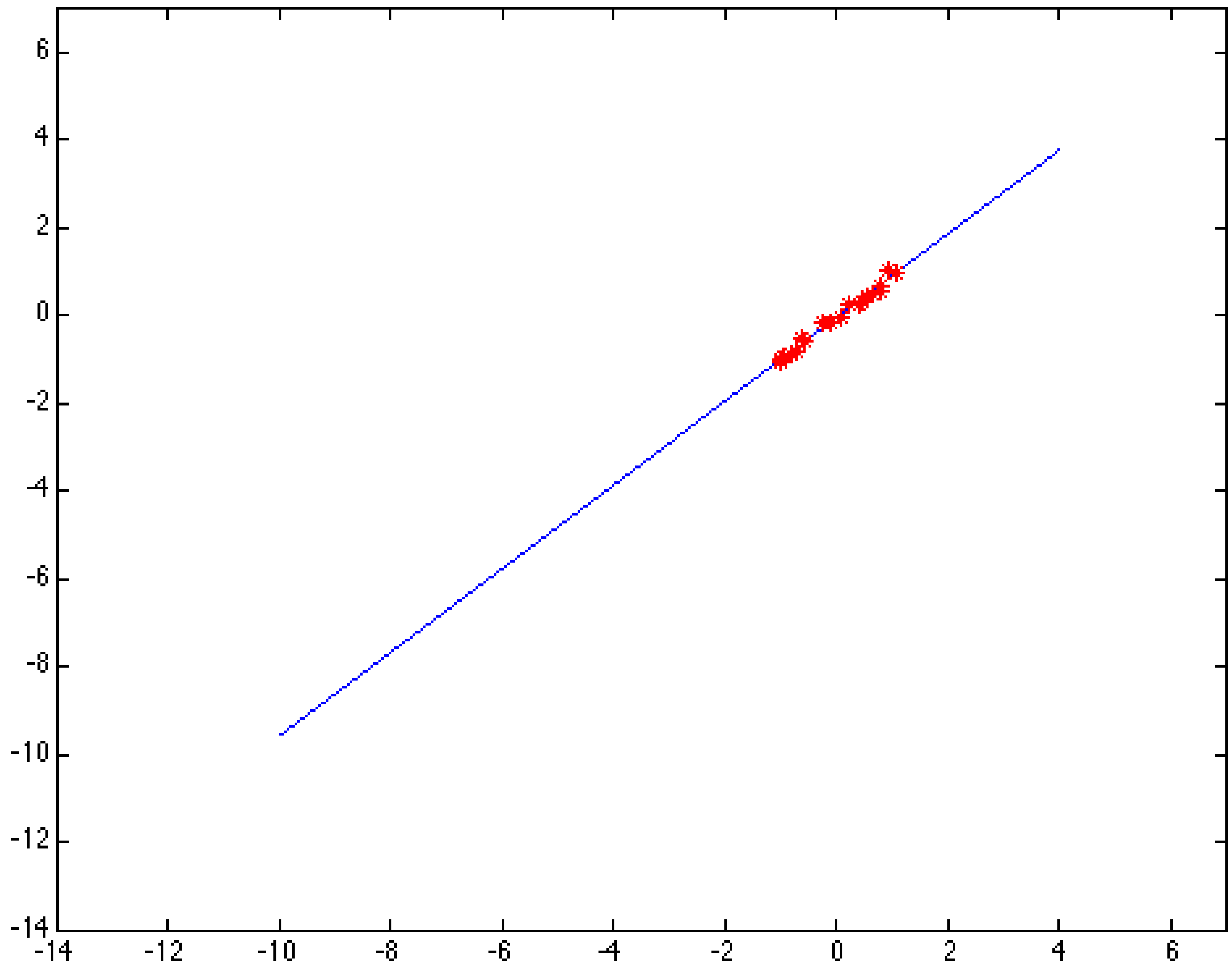
So what does linear least squares do?

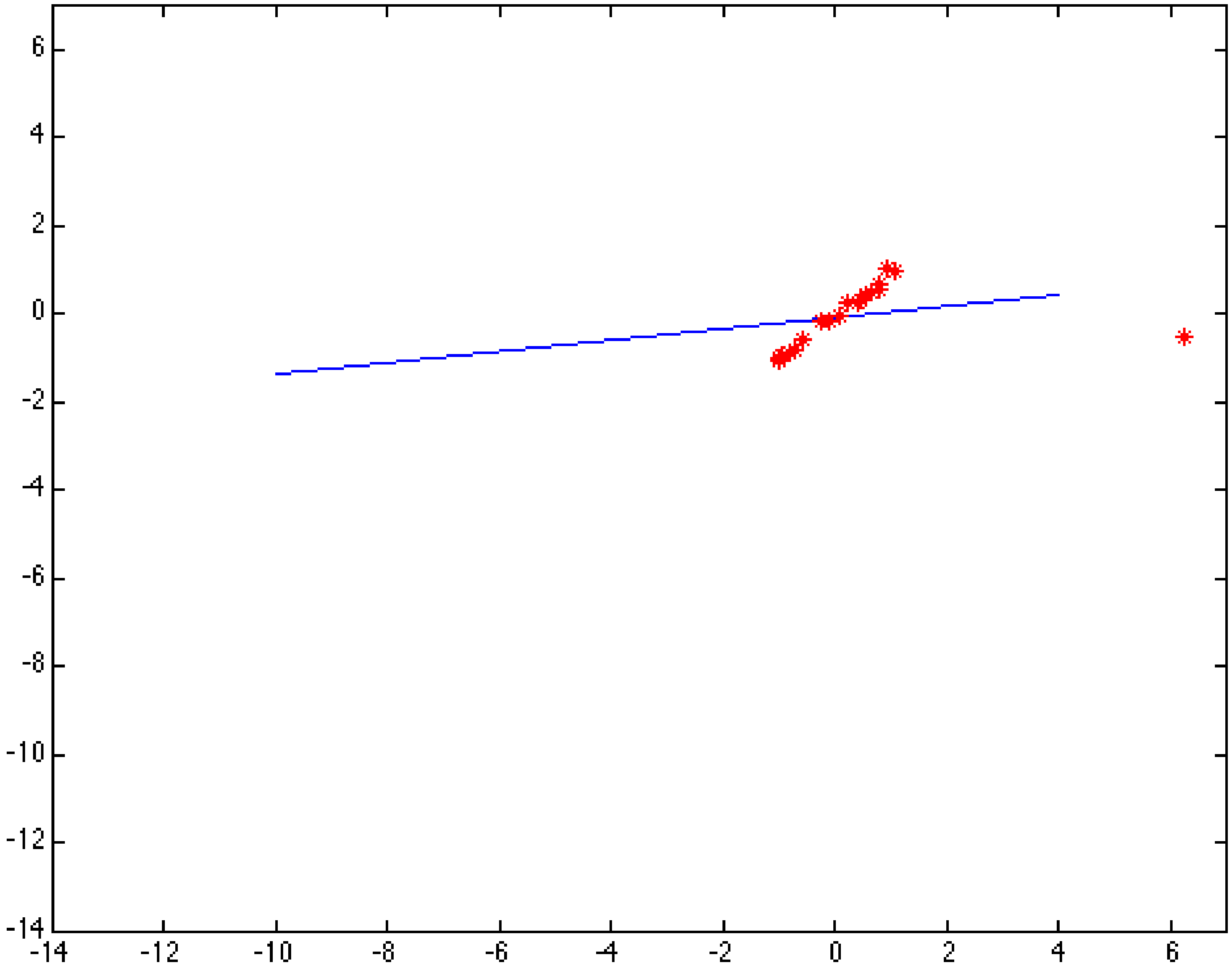


So what does linear least squares do?

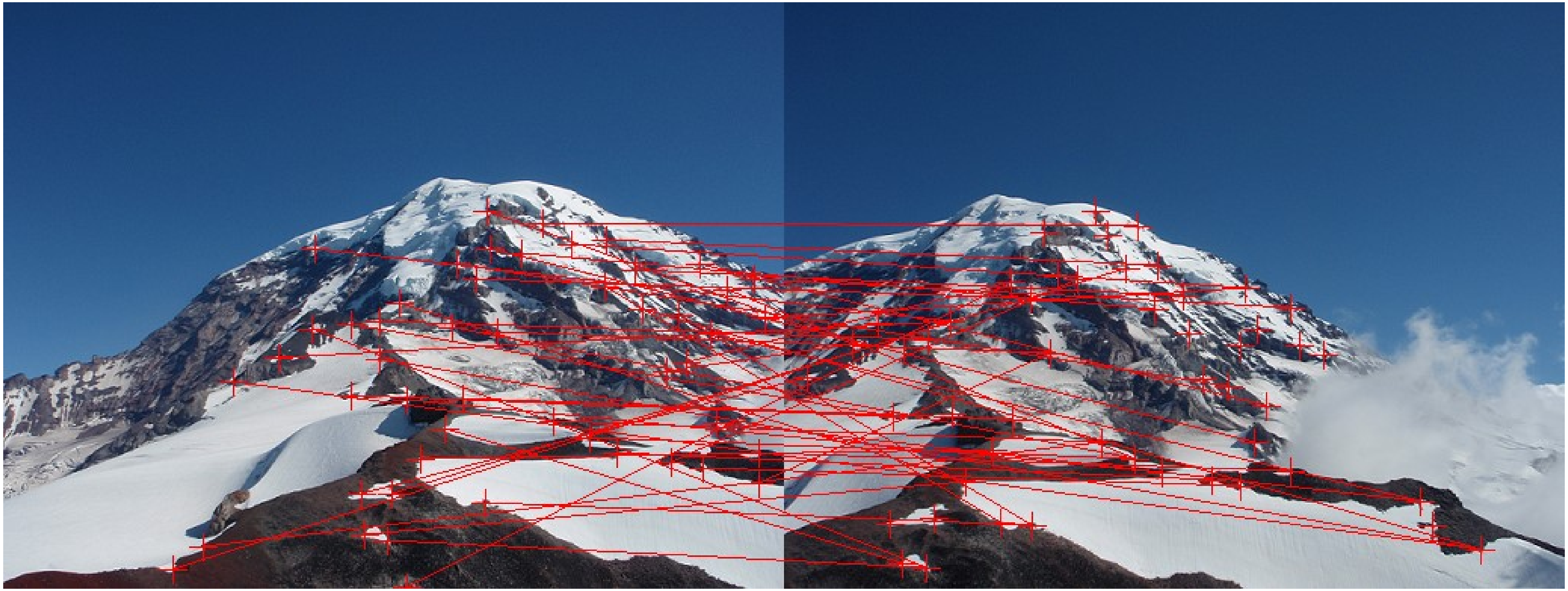
Error based on **squared** residual

Very scared of being wrong, even for just one point

Very bad at handling outliers in data



Not a problem for us, our data is perfect…



Not really …

Today's Agenda

- Linear least-squares
- RANSAC
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RANSAC: RANdom SAmple Consensus

- How can we fit model to inliers but ignore outliers?
- Try a bunch of models, see which ones are best!
- Inliers will all agree on a model
- Outliers are basically random, will not agree

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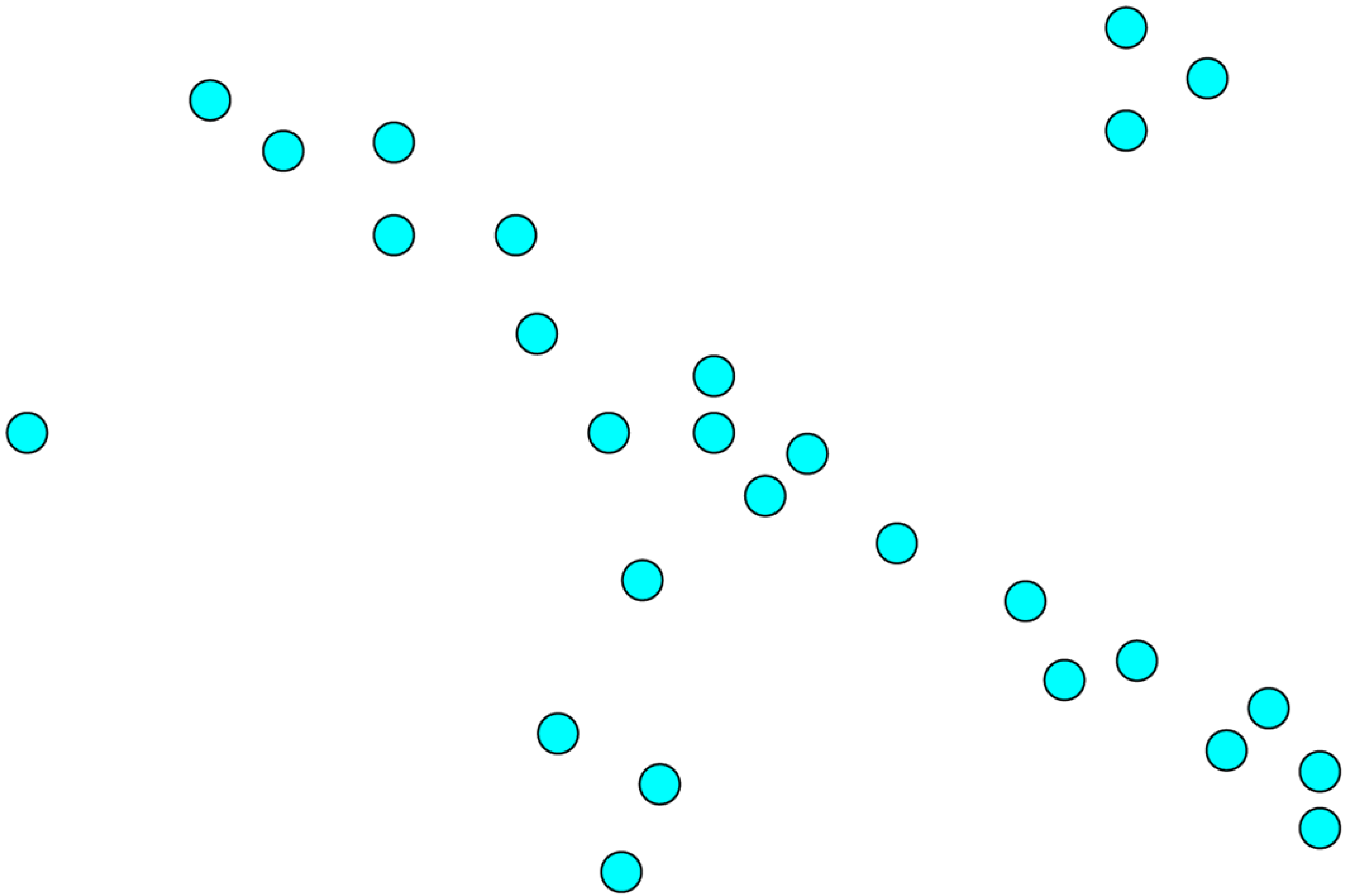
RANSAC: RANdom SAmple Consensus



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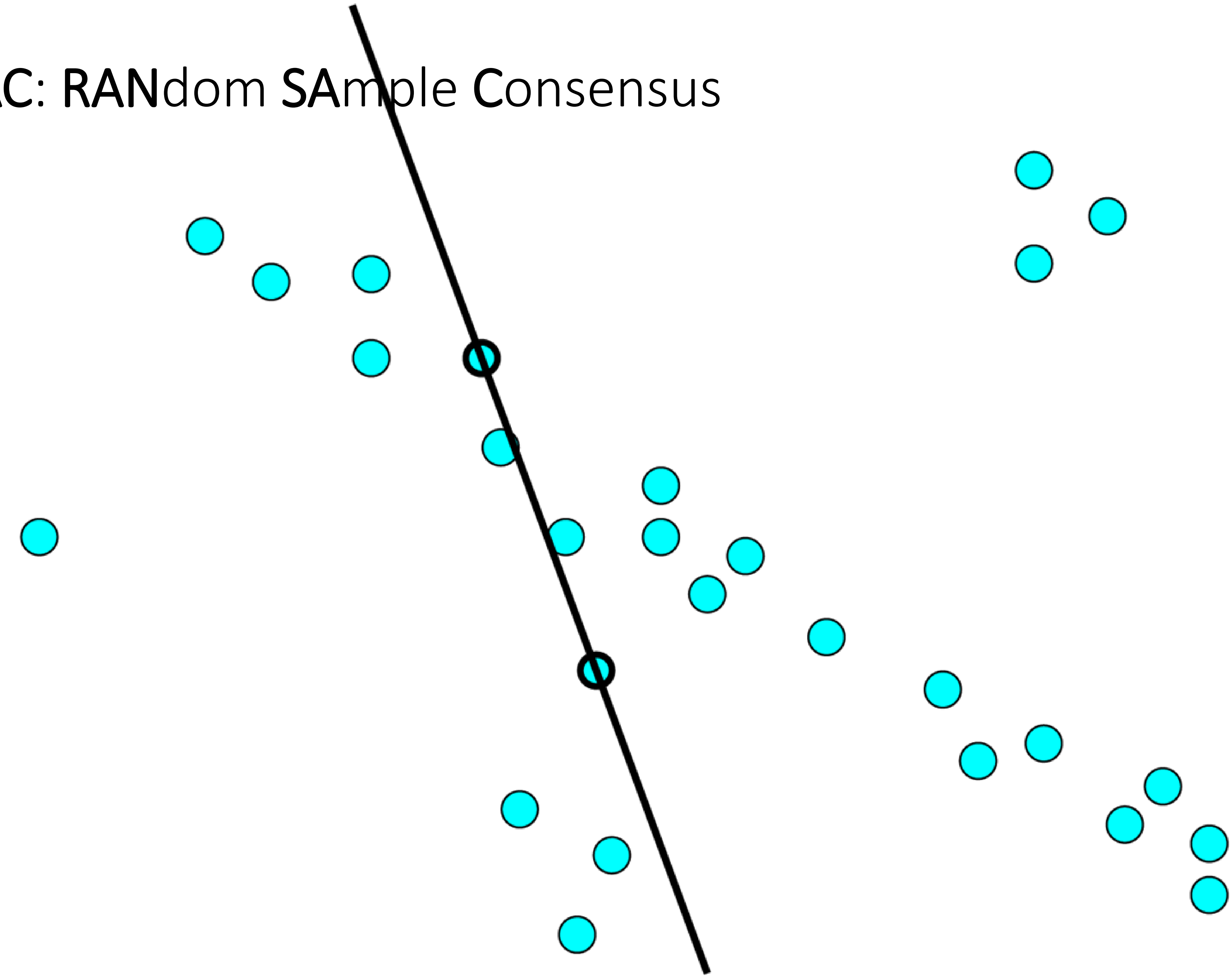

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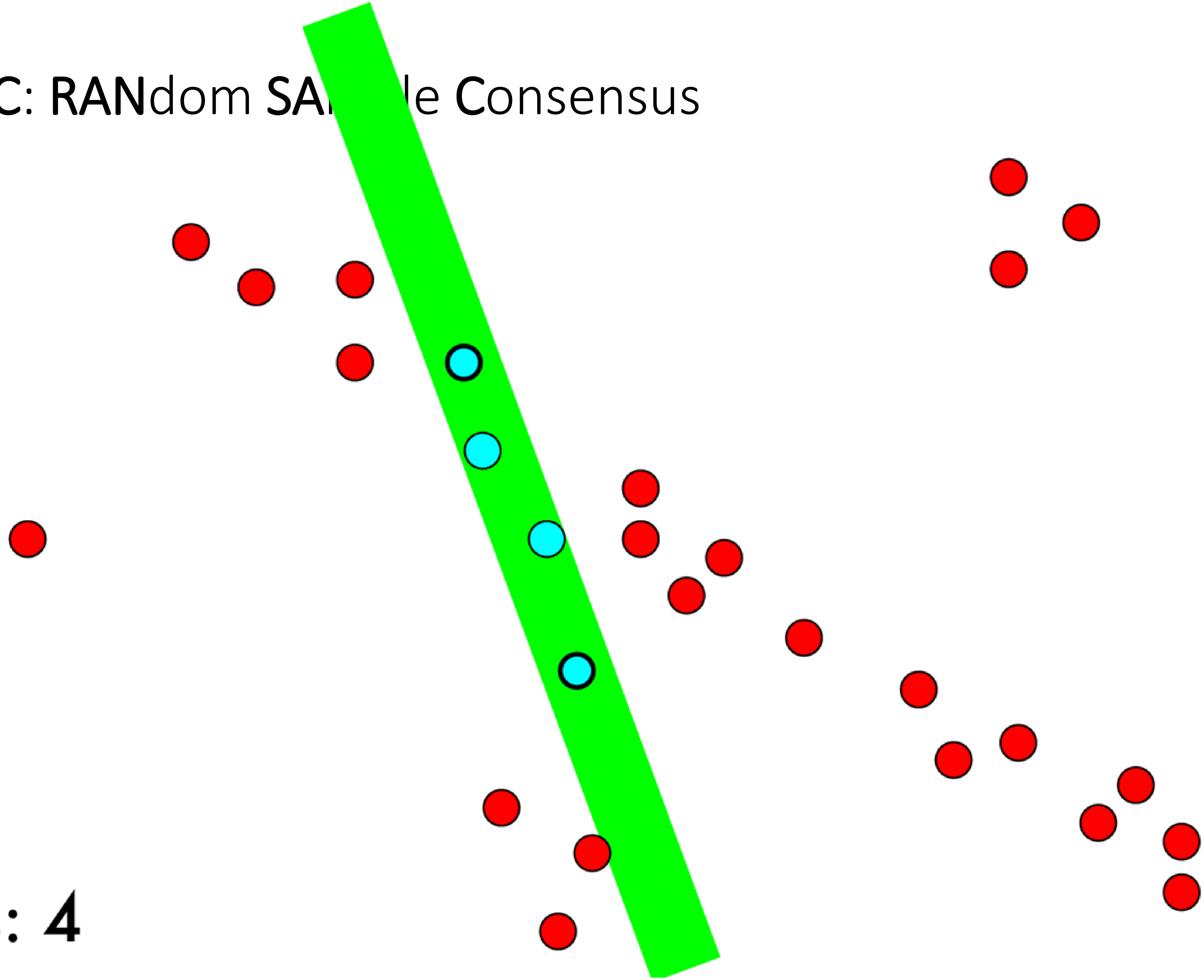
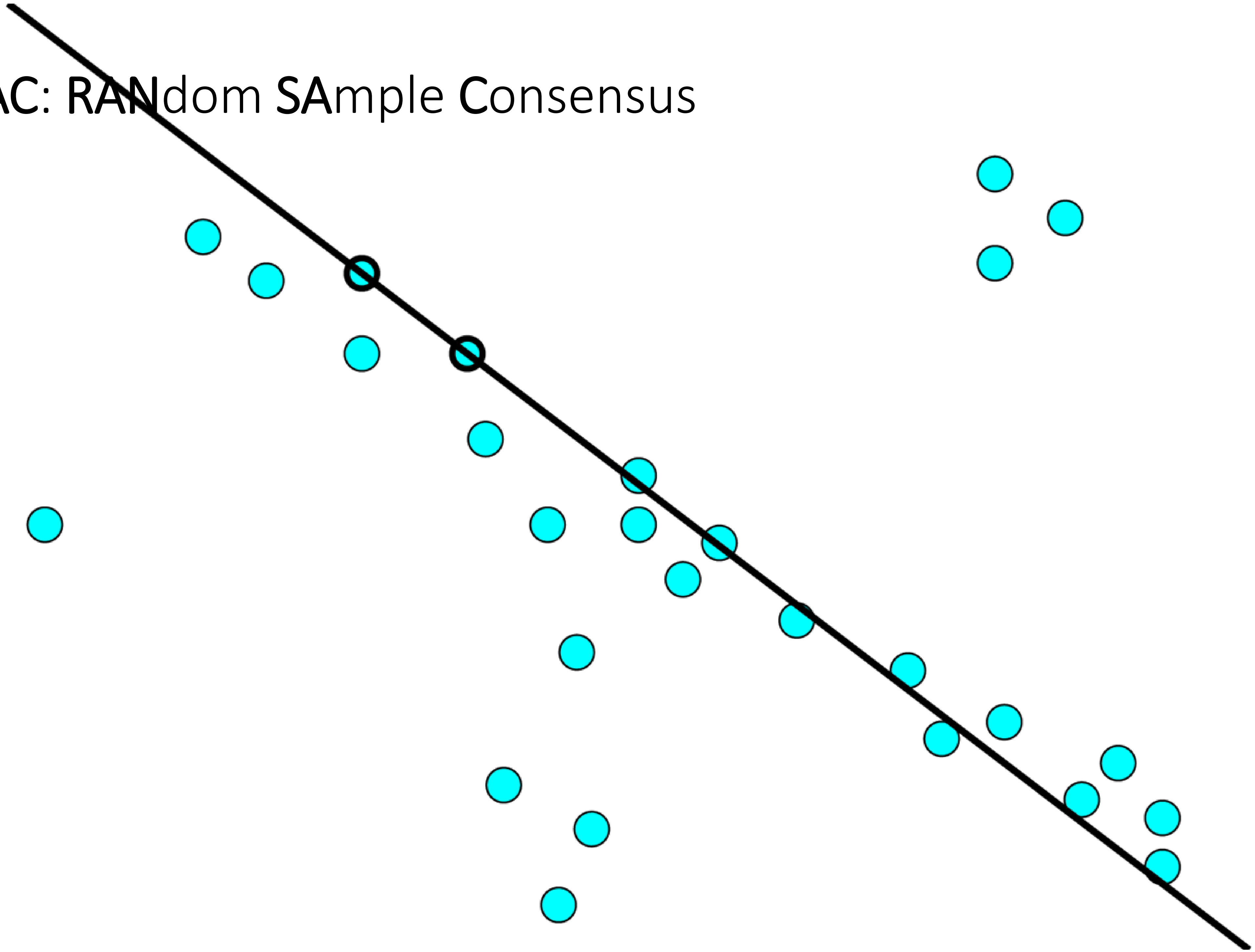
RANSAC: RANdom SA le Consensus



Inliers: 4

RANSAC: RANdom SAmple Consensus



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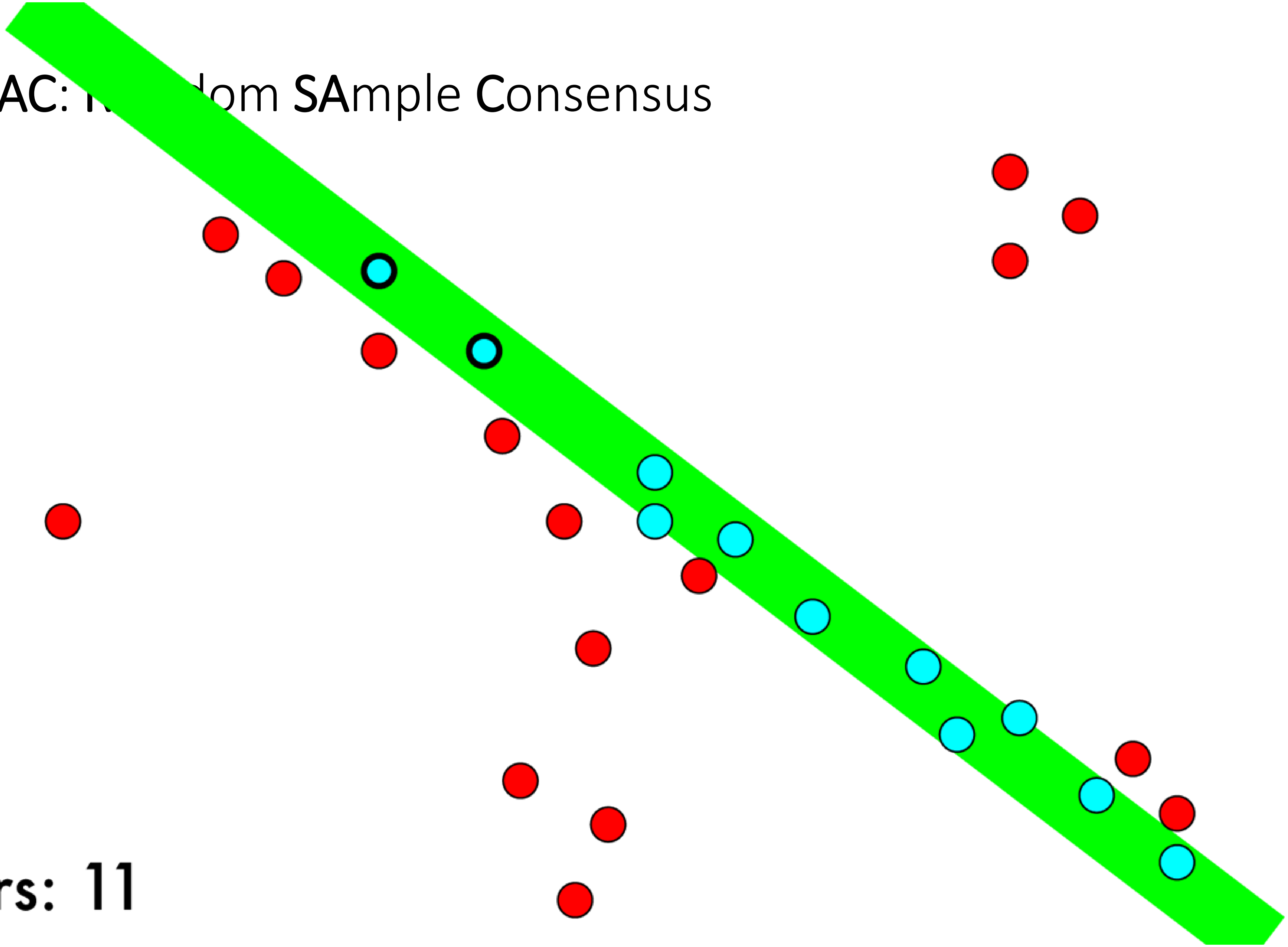
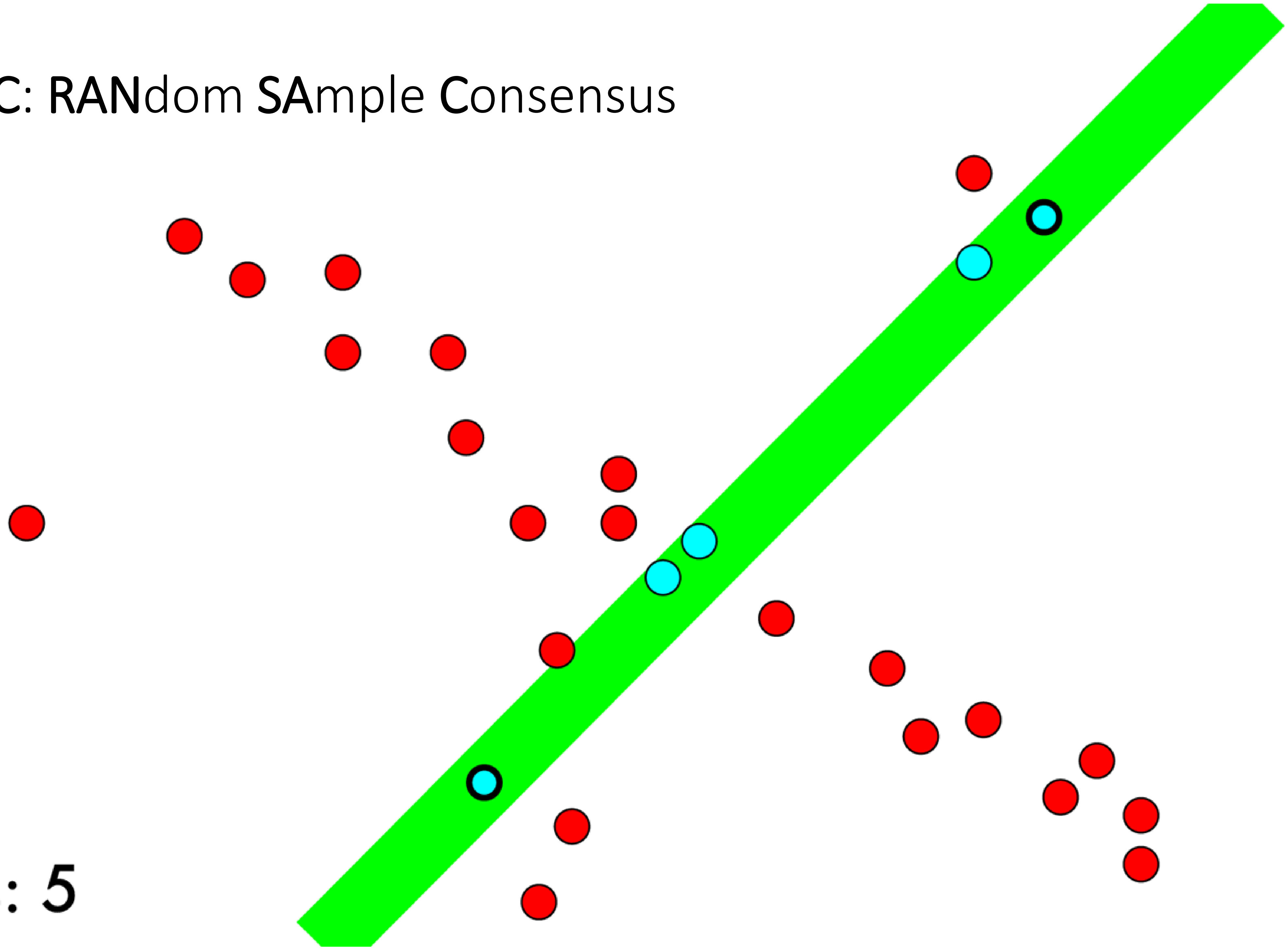
RANSAC: Random SAmple Consensus



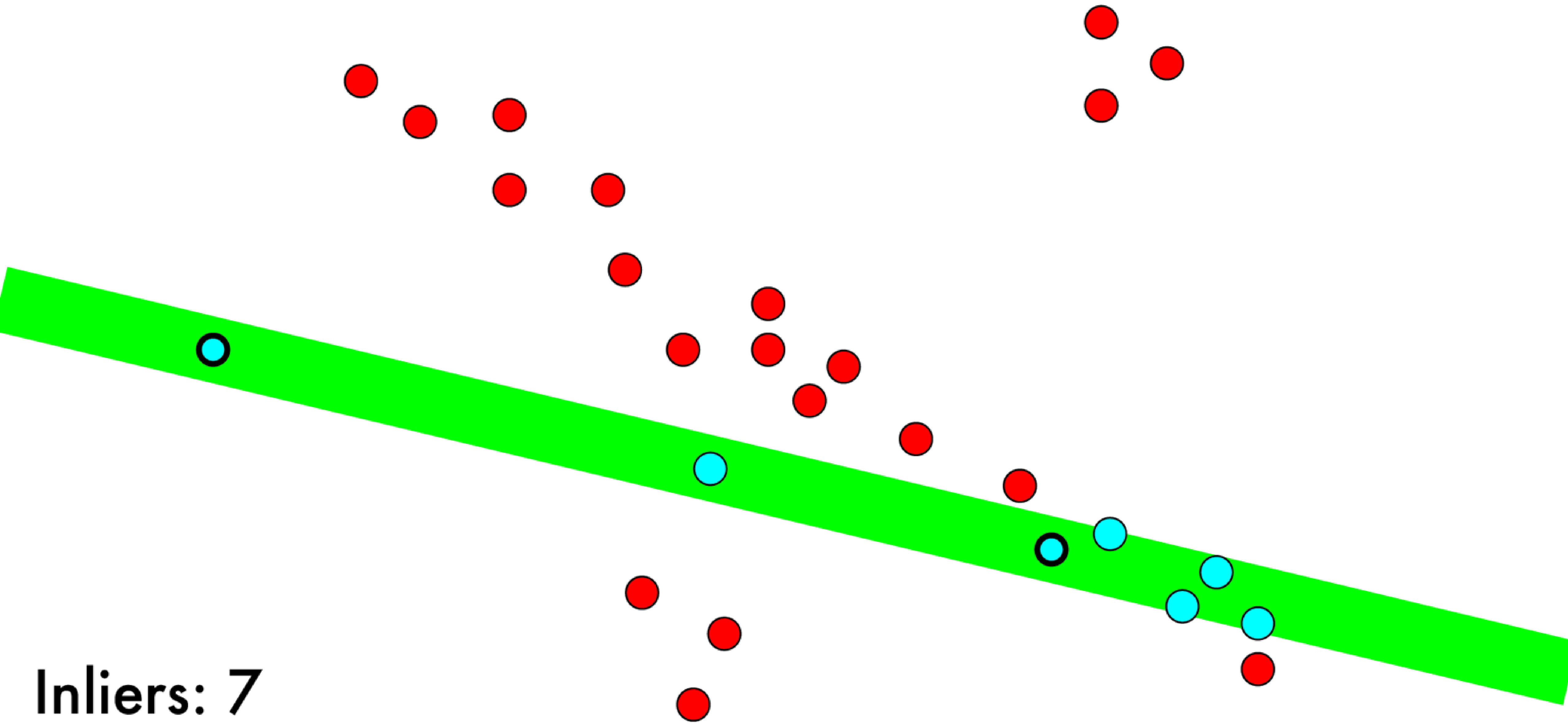
Inliers: 11

RANSAC: RANdom SAmple Consensus



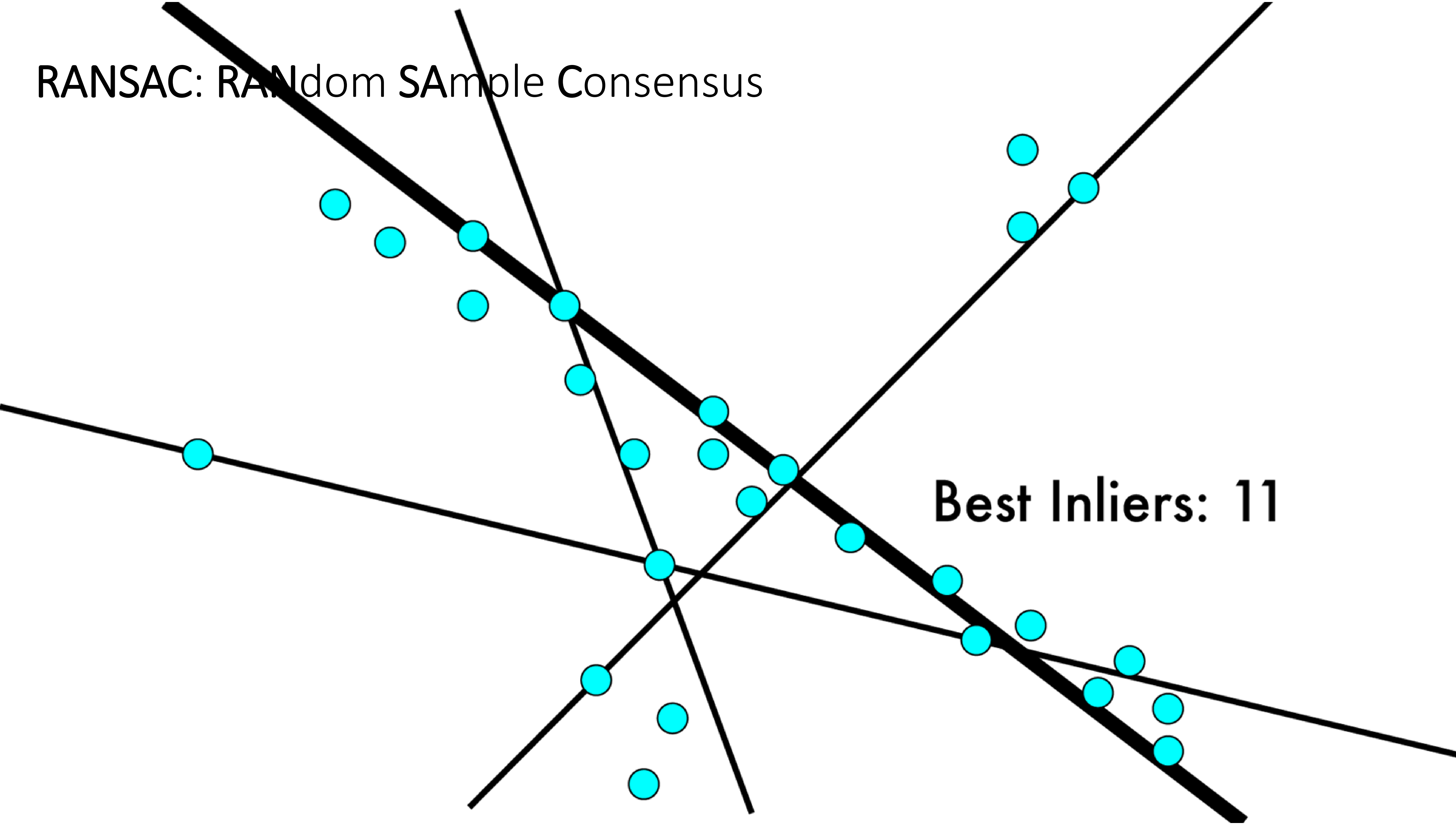
Inliers: 5

RANSAC: RANdom SAmple Consensus



Inliers: 7

RANSAC: RANdom SAmple Consensus



RANSAC: RANdom SAmple Consensus

- Parameters: data, model, n points to fit model, k iterations, t threshold, d “good” fit cutoff

```
bestmodel = None
bestfit = -INF
While i < k:
        sample = draw n random points from data
        Fit model to sample
        inliers = data within t of model
        if inliers > bestfit:
                Fit model to all inliers
                bestfit = fit
                bestmodel = model
                if inliers > d:
                        return model
return bestmodel
```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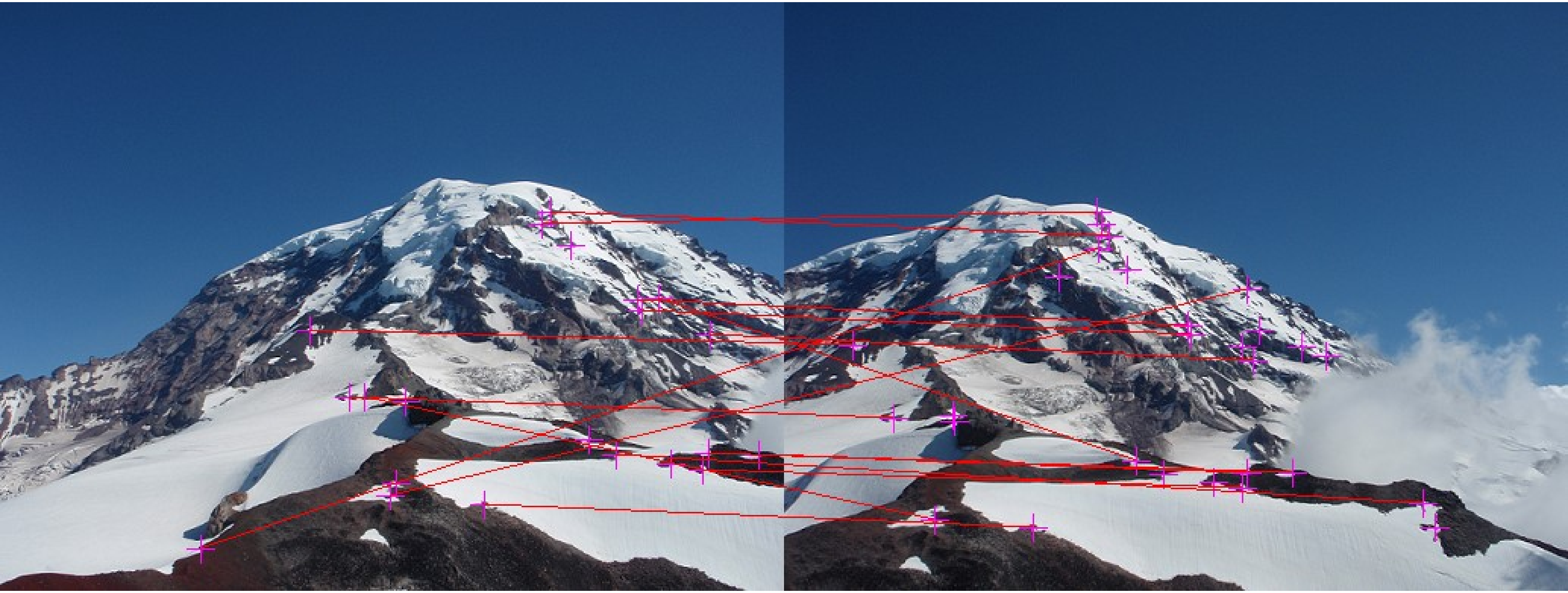
RANSAC: RANdom SAmple Consensus

- Works well even with extreme noise.



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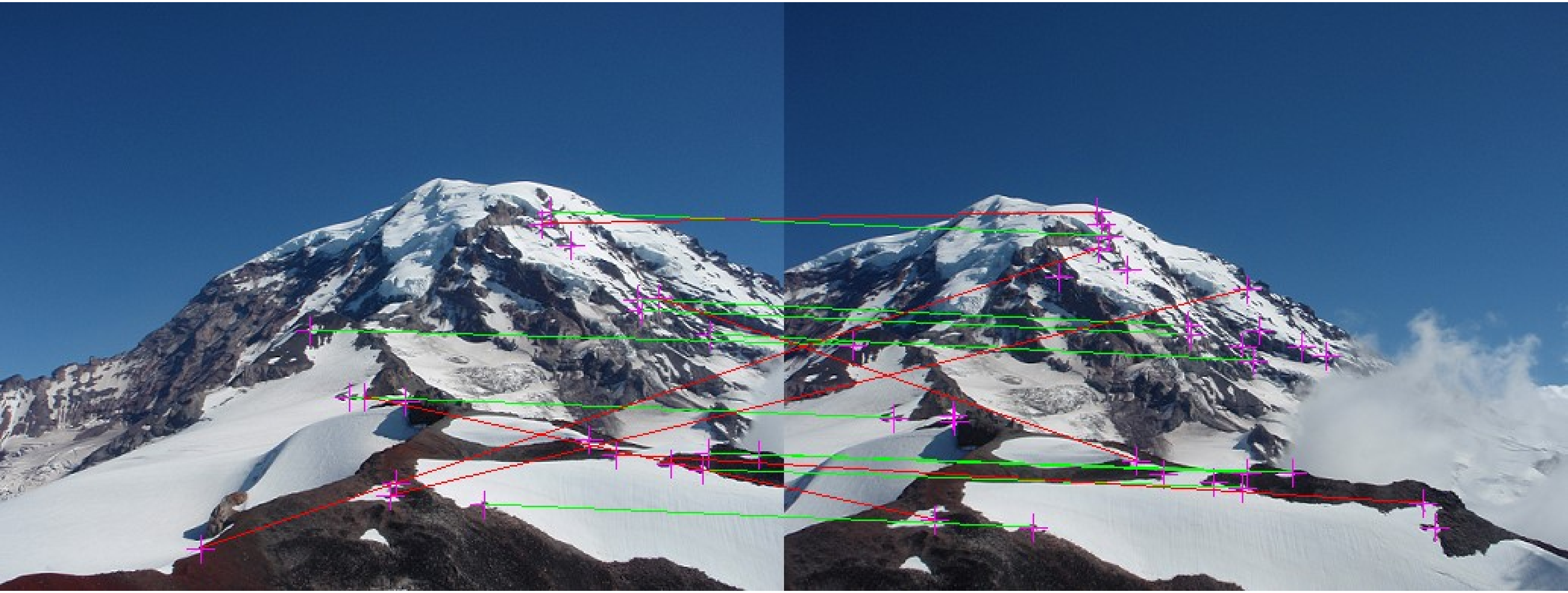

RANSAC: RANdom SAmple Consensus

- Parameters: data, model, n points to fit model, k iterations, t threshold, d “good” fit cutoff
- Lots of tunable parameters
- Want high probability of recovering “right” model
- t: often quite small, assume “good” inliers
- n: should be just enough to fit model, no extra
- k: can be very high
- d: should be >> n

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We can estimate affine..

- How many knowns do we get with one match?
  - $n_x = a_{00} * m_x + a_{01} * m_y + a_{02} * 1$
  - $n_y = a_{10} * m_x + a_{11} * m_y + a_{12} * 1$
  - Solve linear system of equations **M a** = **b**
    - $M^{-1} M a = M^{-1} b \Rightarrow a = M^{-1} b$
    - But M^{-1} does not exist in general - Why?
  - Still works if overdetermined
    - Why???
    - Pseudoinverse – least squares solution
    - $M^T M a = M^T b$
    - $(M^T M)^{-1} (M^T M) a = (M^T M)^{-1} M^T b$
    - $\Rightarrow a = (M^T M)^{-1} M^T b$

$$\underbrace{\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 \\ & & \cdots & & & \\ & & \cdots & & & \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ \cdots \\ \cdots \end{bmatrix}}_{\mathbf{b}}$$

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We want projective (homography)

- What are our equations now?
  - $n_x = (h_{00}*m_x + h_{01}*m_y + h_{02}*m_w) / (h_{20}*m_x + h_{21}*m_y + h_{22}*m_w)$
  - $n_y = (h_{10}*m_x + h_{11}*m_y + h_{12}*m_w) / (h_{20}*m_x + h_{21}*m_y + h_{22}*m_w)$

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{w}' \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

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- Assume h_{22} and m_w are 1, now 8 DOF
  - $n_x = (h_{00}*m_x + h_{01}*m_y + h_{02}) / (h_{20}*m_x + h_{21}*m_y + 1)$
  - $n_y = (h_{10}*m_x + h_{11}*m_y + h_{12}) / (h_{20}*m_x + h_{21}*m_y + 1)$

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{w}' \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{bmatrix}$$

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  - $n_x = (h_{00}*m_x + h_{01}*m_y + h_{02}) / (h_{20}*m_x + h_{21}*m_y + 1)$
  - $n_y = (h_{10}*m_x + h_{11}*m_y + h_{12}) / (h_{20}*m_x + h_{21}*m_y + 1)$
- More algebra on n_x
  - $n_x * (h_{20}*m_x + h_{21}*m_y + 1) = (h_{00}*m_x + h_{01}*m_y + h_{02})$
  - $n_x*h_{20}*m_x + n_x*h_{21}*m_y + n_x = h_{00}*m_x + h_{01}*m_y + h_{02}$
  - $n_x = h_{00}*m_x + h_{01}*m_y + h_{02} - n_x*h_{20}*m_x - n_x*h_{21}*m_y$
- Similar for n_y

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    - $n_y = h_{10}*m_x + h_{11}*m_y + h_{12} - n_x*h_{20}*m_x - n_x*h_{21}*m_y$
- New matrix equations:

$$\underbrace{\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{bmatrix}}_{\mathbf{b}}$$

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We want projective (homography)

- New matrix equations:

$$\underbrace{\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{bmatrix}}_{\mathbf{b}}$$

- Same procedure, Solve **M a = b**
  - Exact if #rows of M = 8
  - Least squares if #rows of M > 8

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Are there any problems with this??

- New matrix equations:

$$\underbrace{\begin{bmatrix} m_{x1} & m_{y1} & 1 & 0 & 0 & 0 & -m_{x1}n_{x1} & -m_{y1}n_{x1} \\ 0 & 0 & 0 & m_{x1} & m_{y1} & 1 & -m_{x1}n_{y1} & -m_{y1}n_{y1} \\ m_{x2} & m_{y2} & 1 & 0 & 0 & 0 & -m_{x2}n_{x2} & -m_{y2}n_{x2} \\ 0 & 0 & 0 & m_{x2} & m_{y2} & 1 & -m_{x2}n_{y2} & -m_{y2}n_{y2} \\ m_{x3} & m_{y3} & 1 & 0 & 0 & 0 & -m_{x3}n_{x3} & -m_{y3}n_{x3} \\ 0 & 0 & 0 & m_{x3} & m_{y3} & 1 & -m_{x3}n_{y3} & -m_{y3}n_{y3} \\ m_{x4} & m_{y4} & 1 & 0 & 0 & 0 & -m_{x4}n_{x4} & -m_{y4}n_{x4} \\ 0 & 0 & 0 & m_{x4} & m_{y4} & 1 & -m_{x4}n_{y4} & -m_{y4}n_{y4} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} n_{x1} \\ n_{y1} \\ n_{x2} \\ n_{y2} \\ n_{x3} \\ n_{y3} \\ n_{x4} \\ n_{y4} \end{bmatrix}}_{\mathbf{b}}$$

- Same procedure, Solve **M a = b**
  - Exact if #rows of **M** = 8
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Today's Agenda

- Linear least-squares
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- Panorama Stitching

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Panorama algorithm

Find corners in both images

Calculate descriptors

Match descriptors

RANSAC to find homography

Stitch together images with homography

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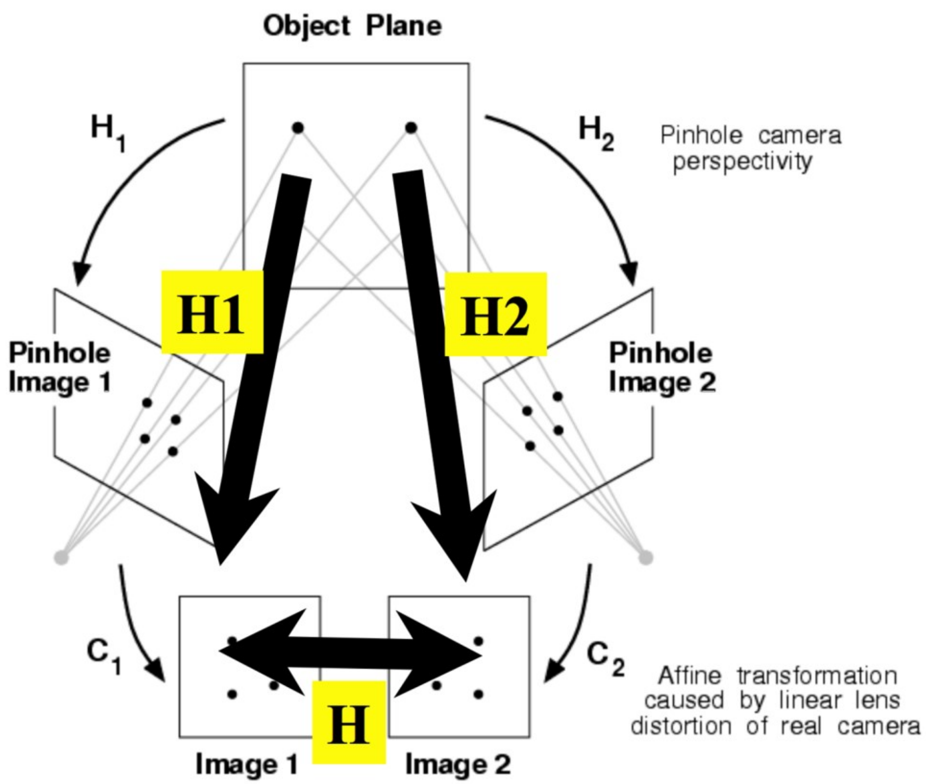
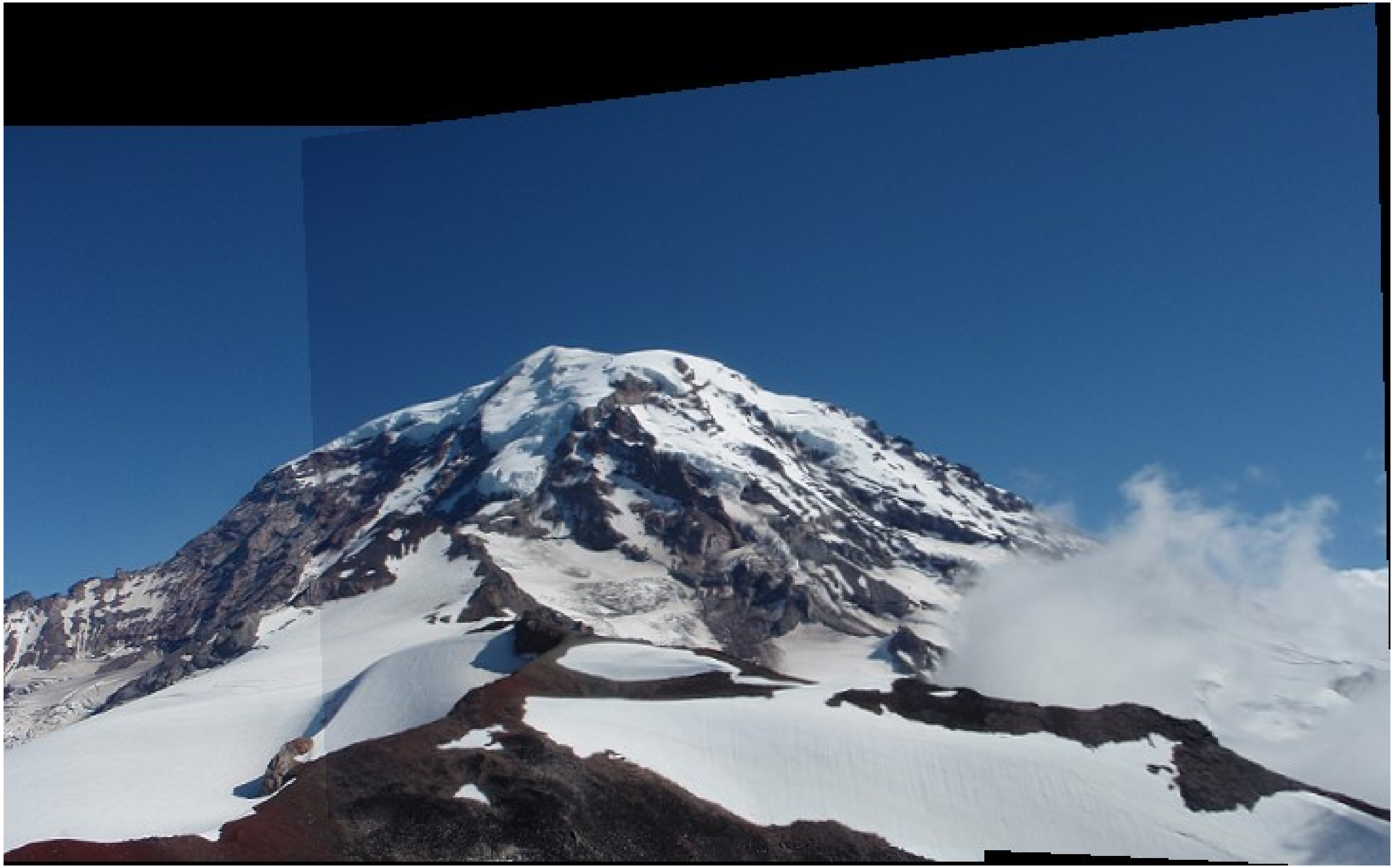
Stitching panoramas

- We know homography is right choice under certain assumption:
  - Assume we are taking multiple images of planar object

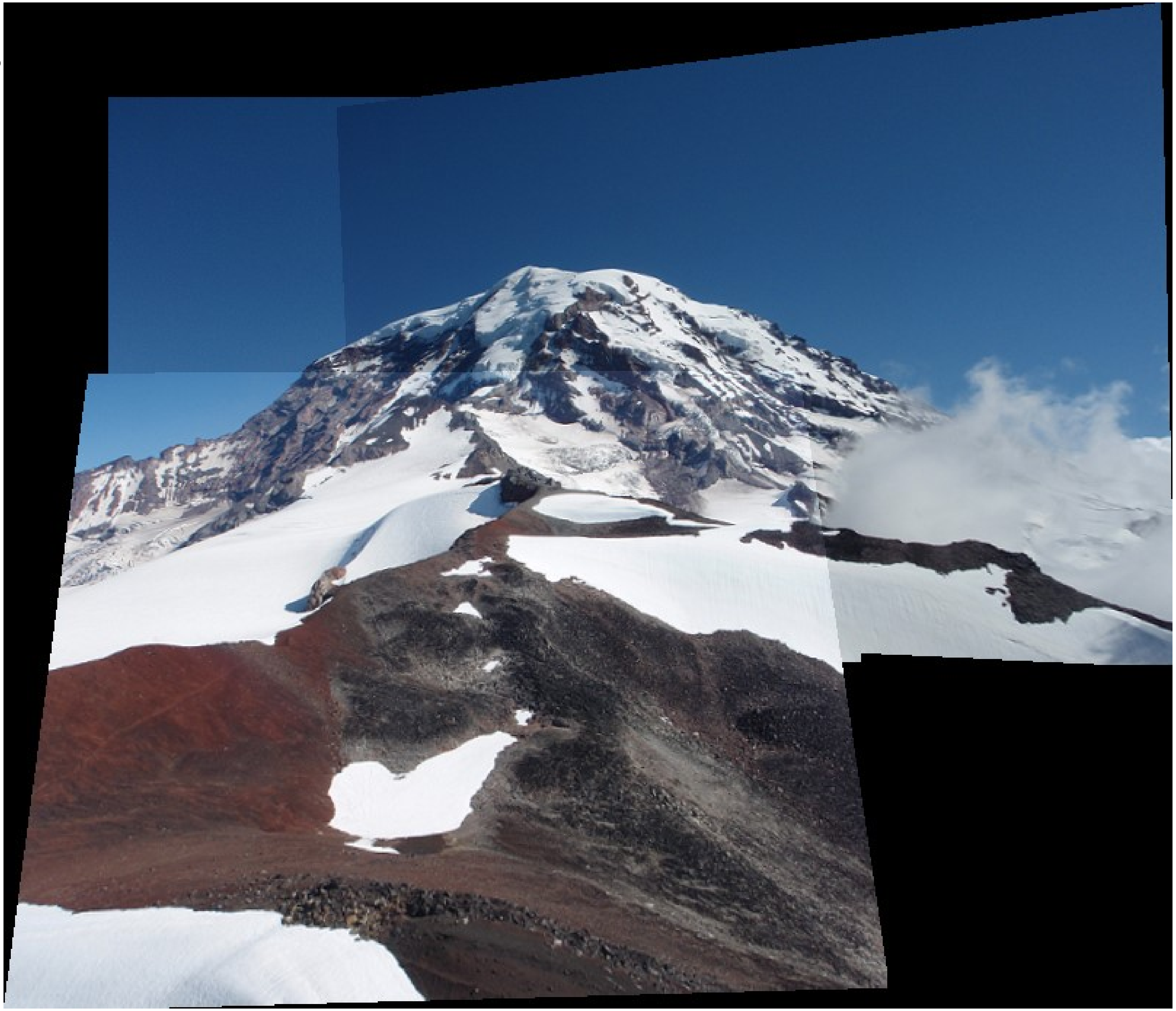


In practice

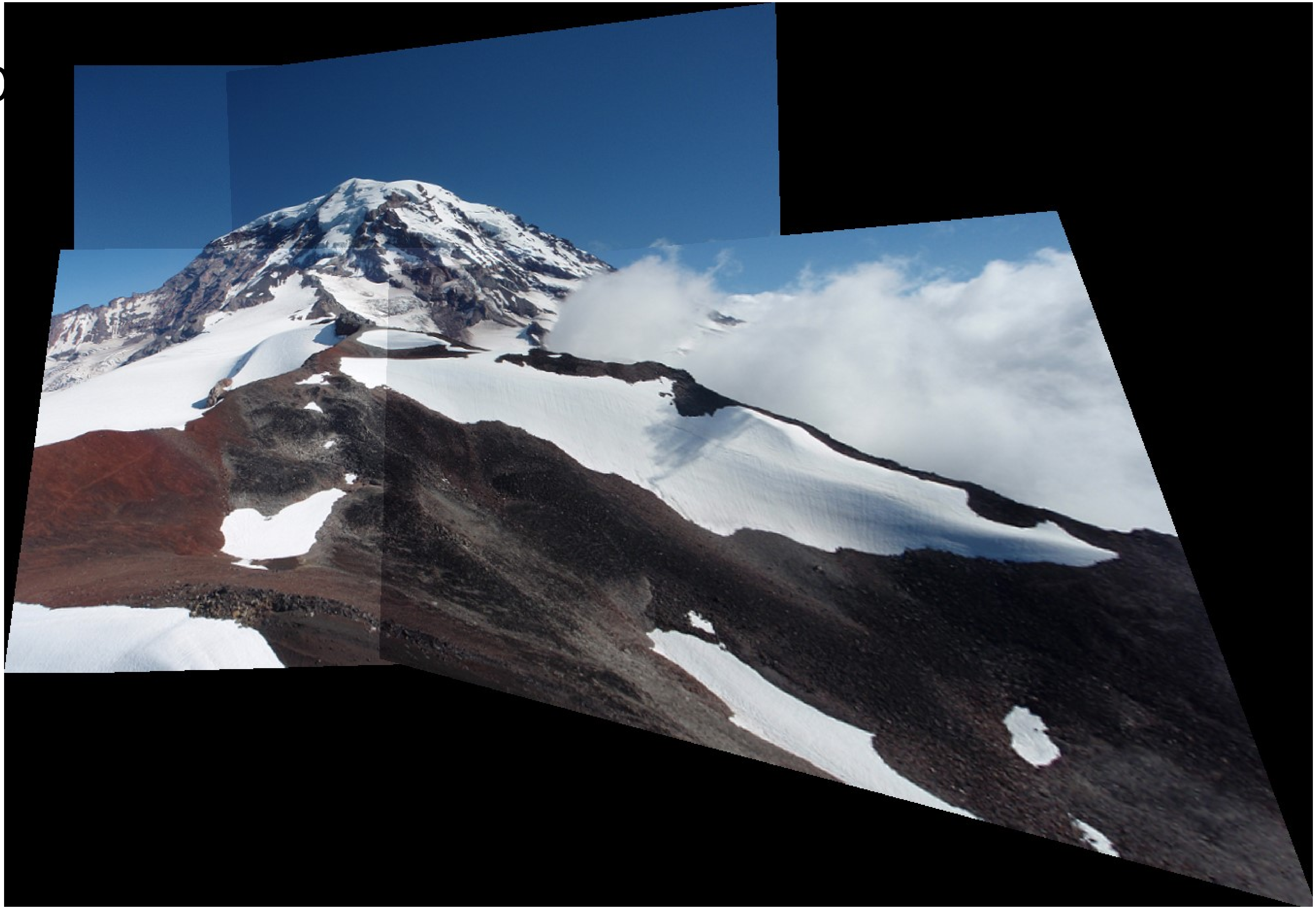
In practice



In practi



In p



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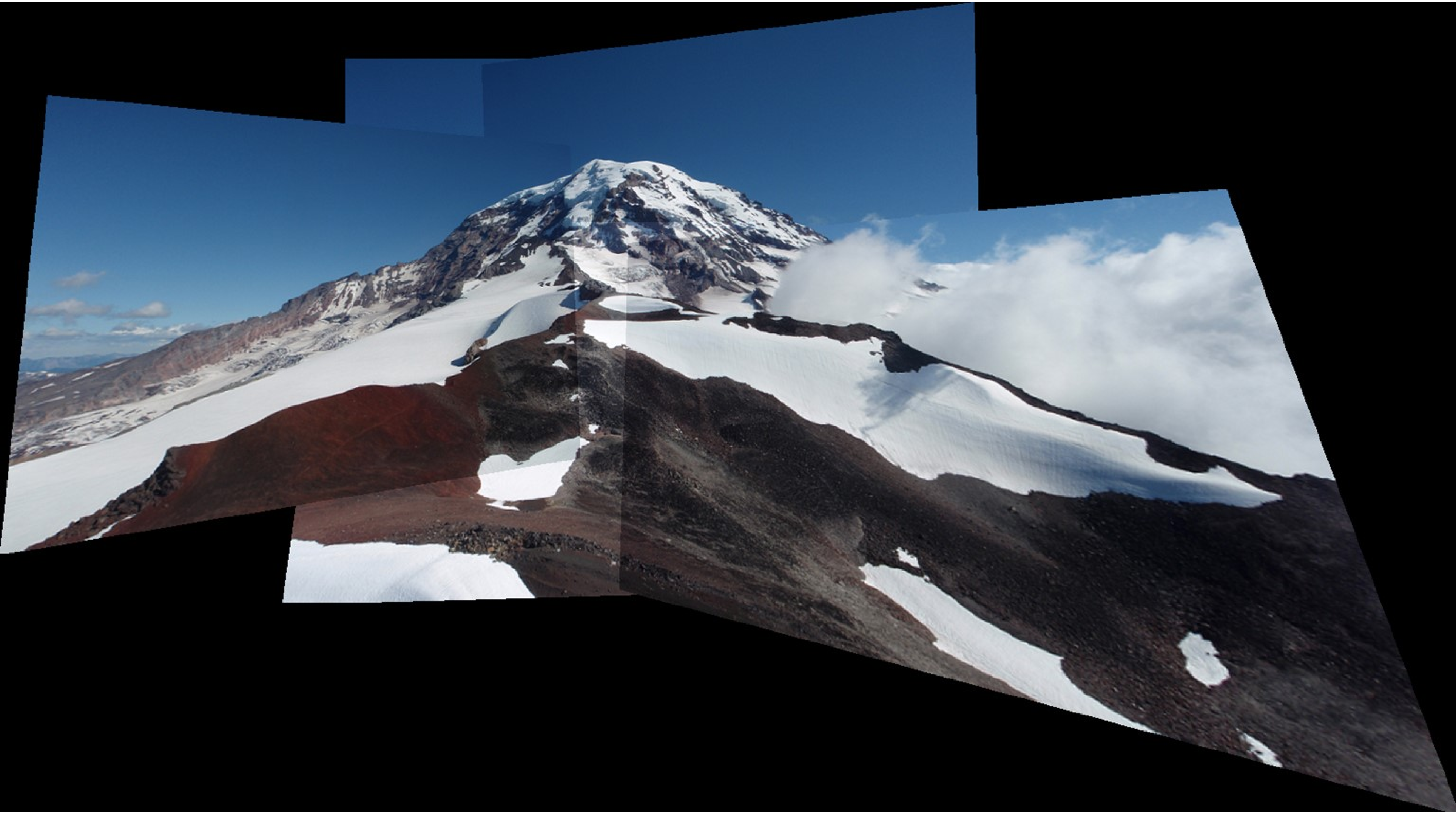

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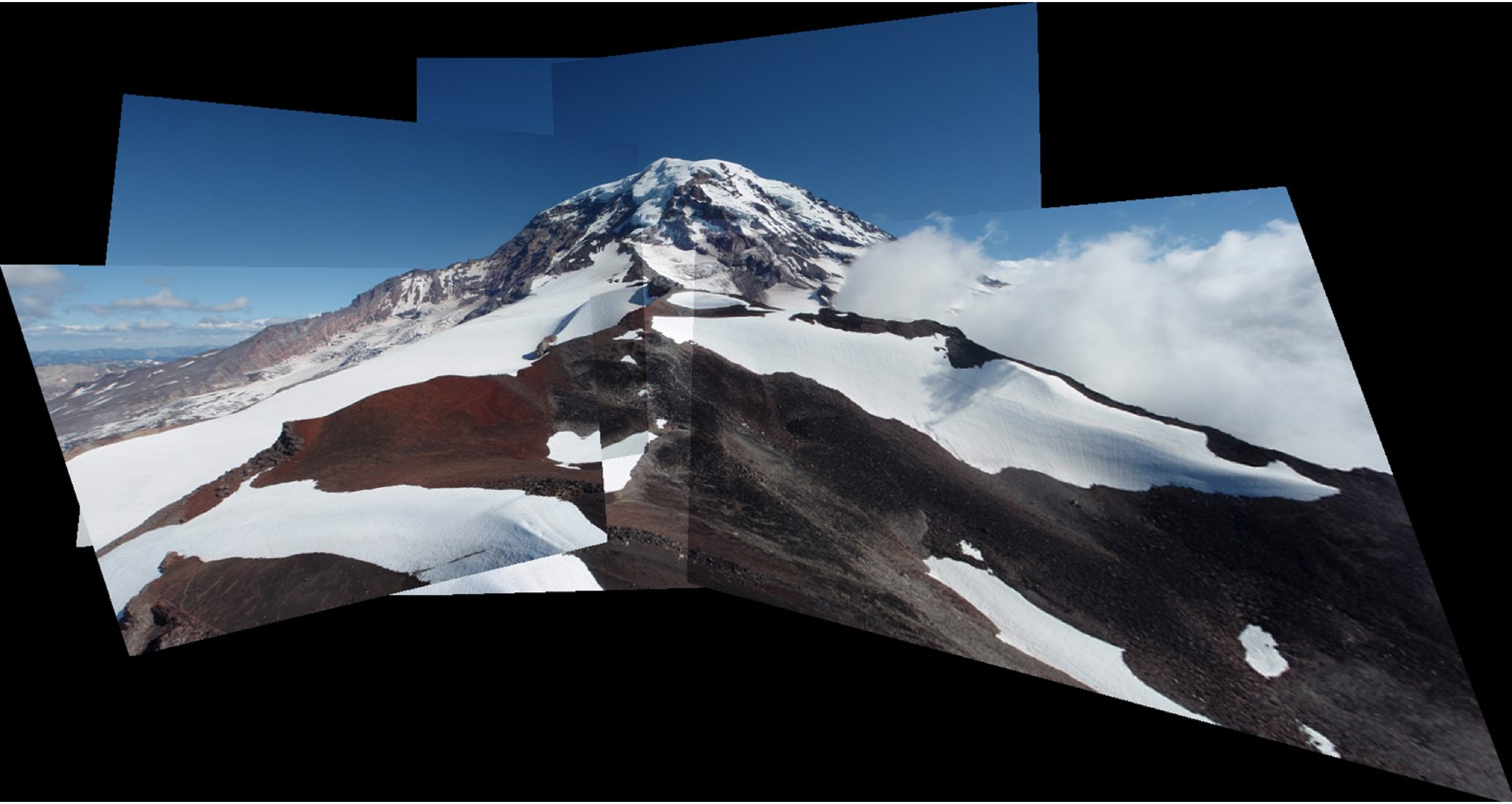
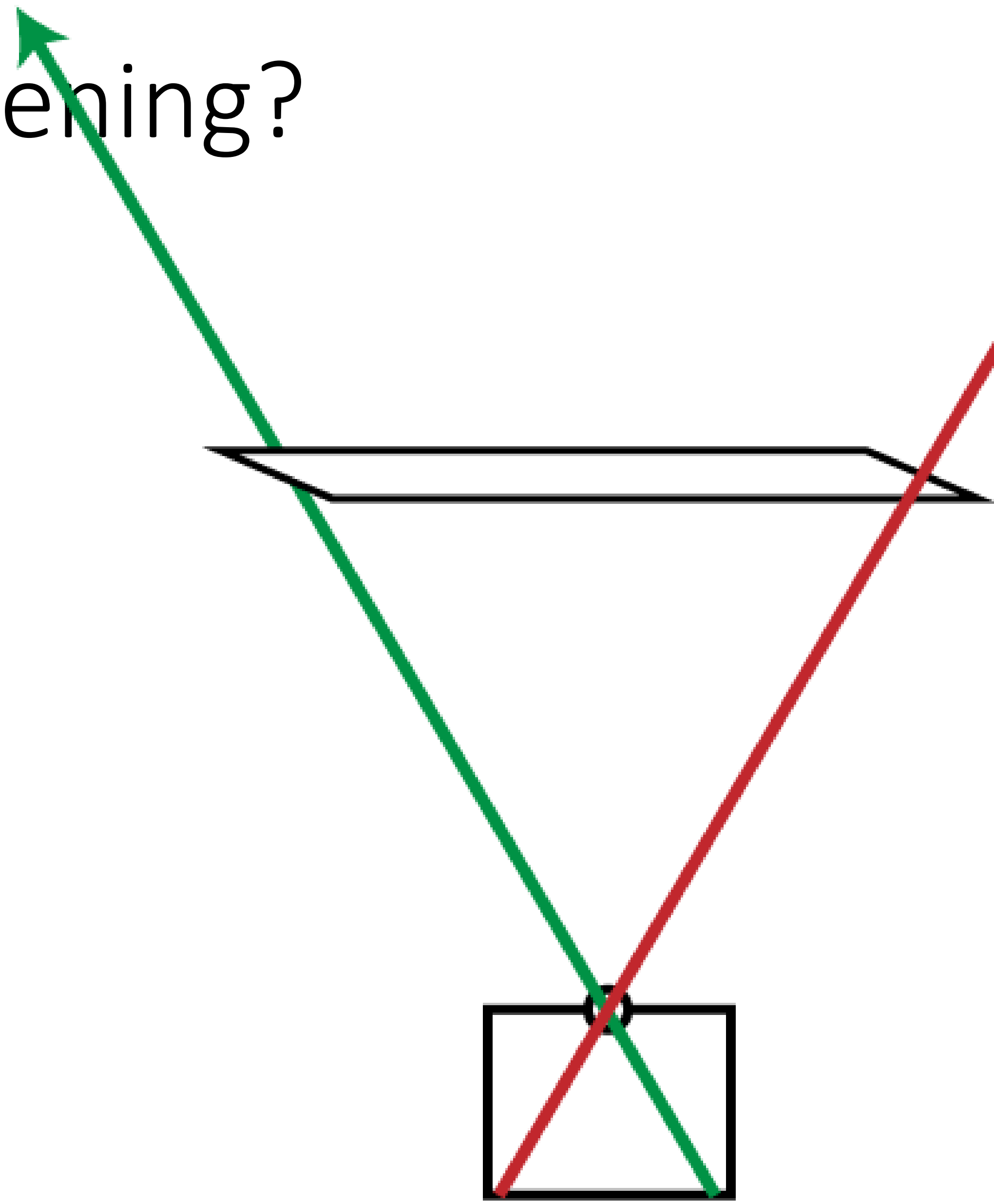


What’s happening?



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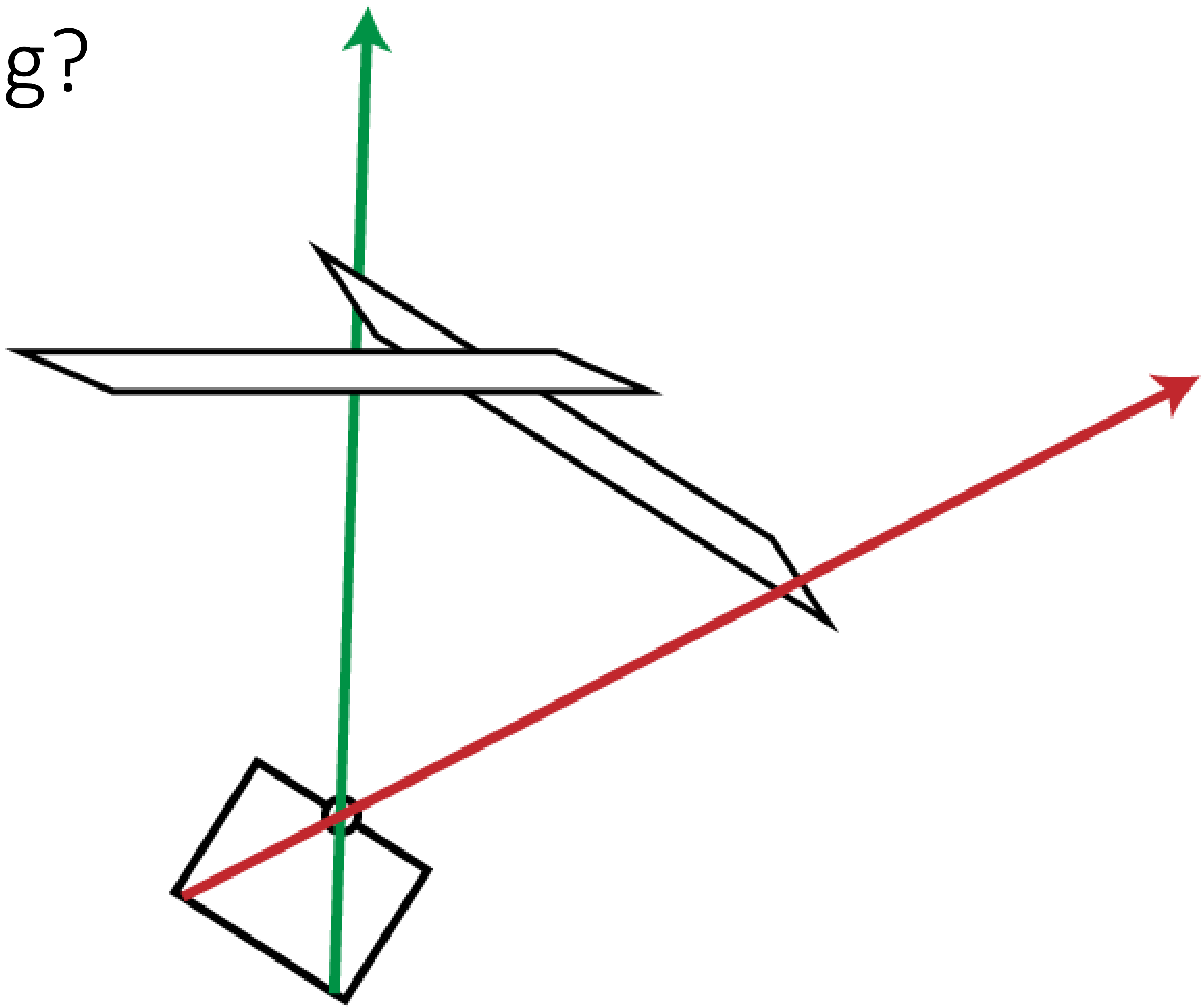

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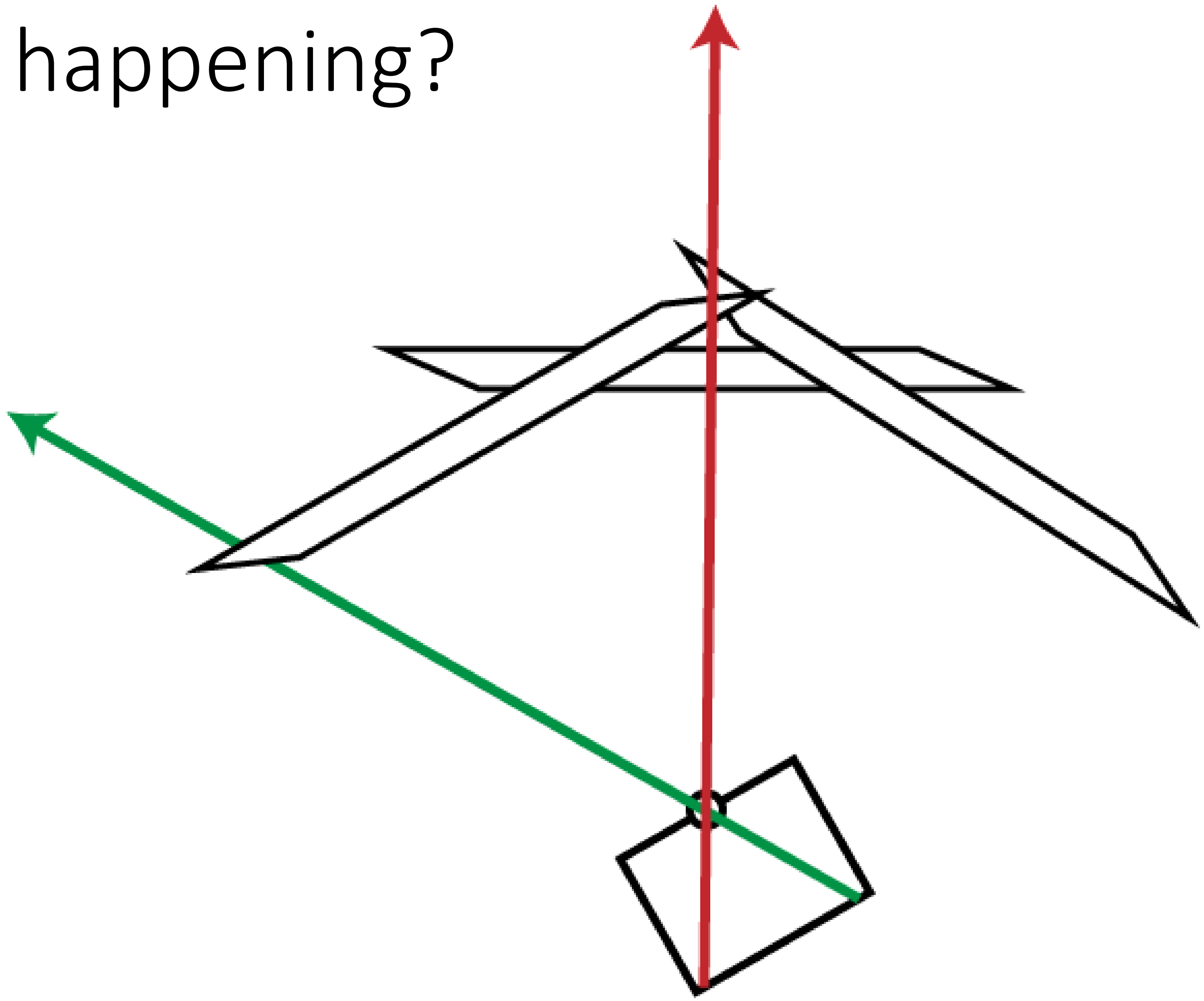
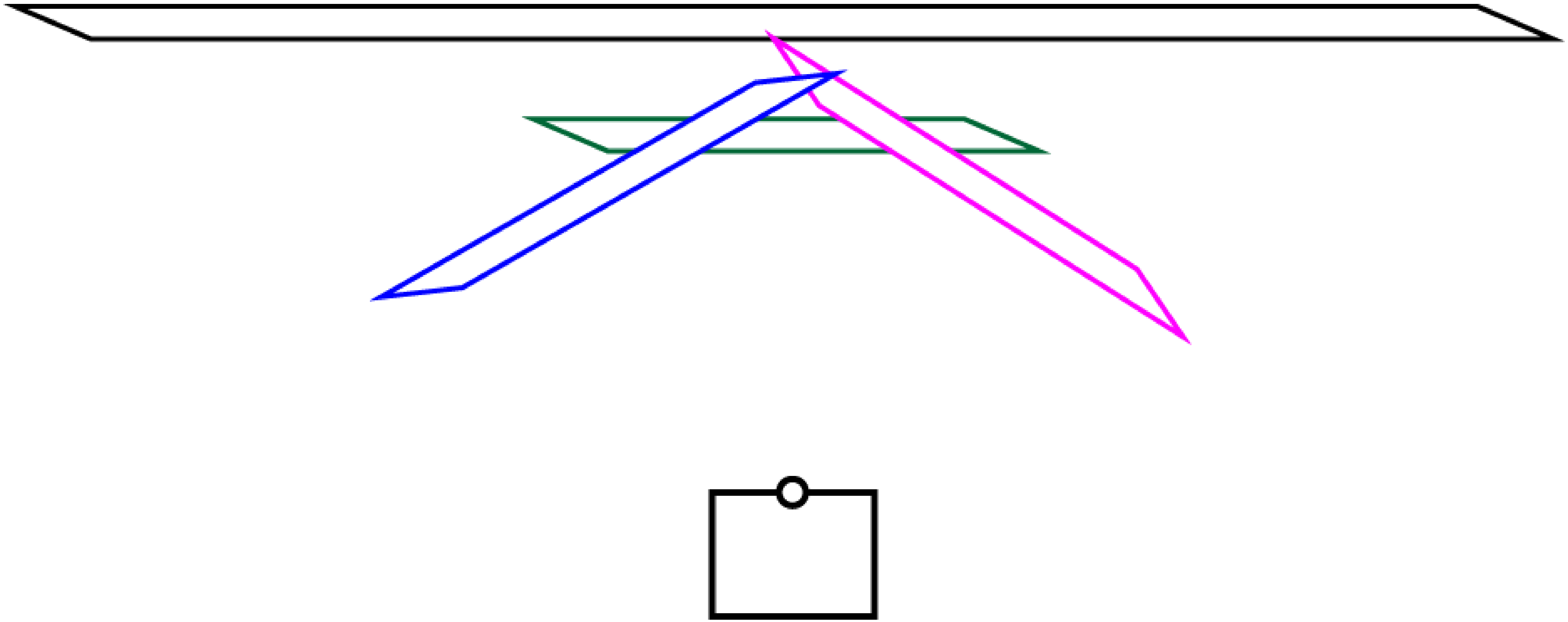


What's happening?



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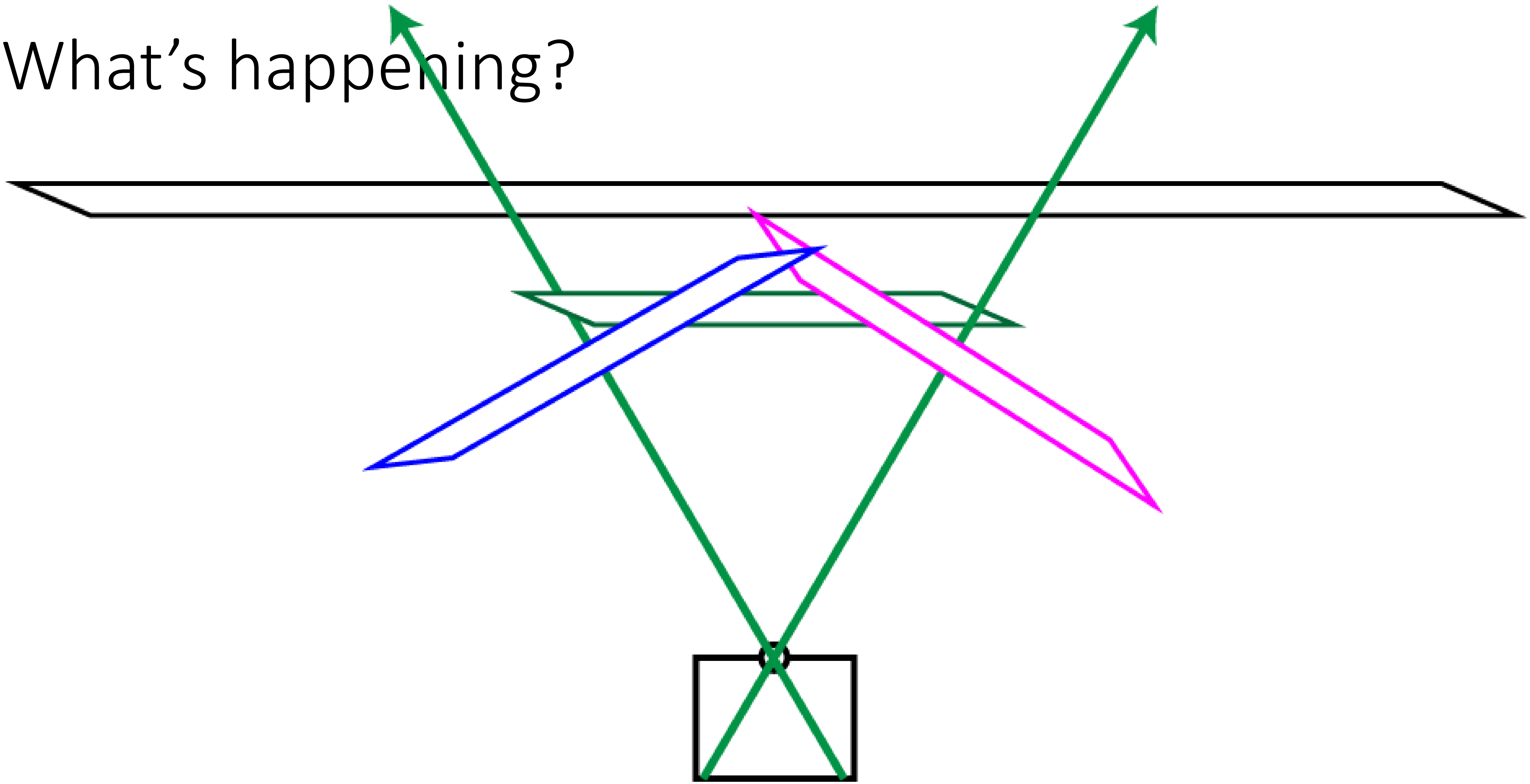
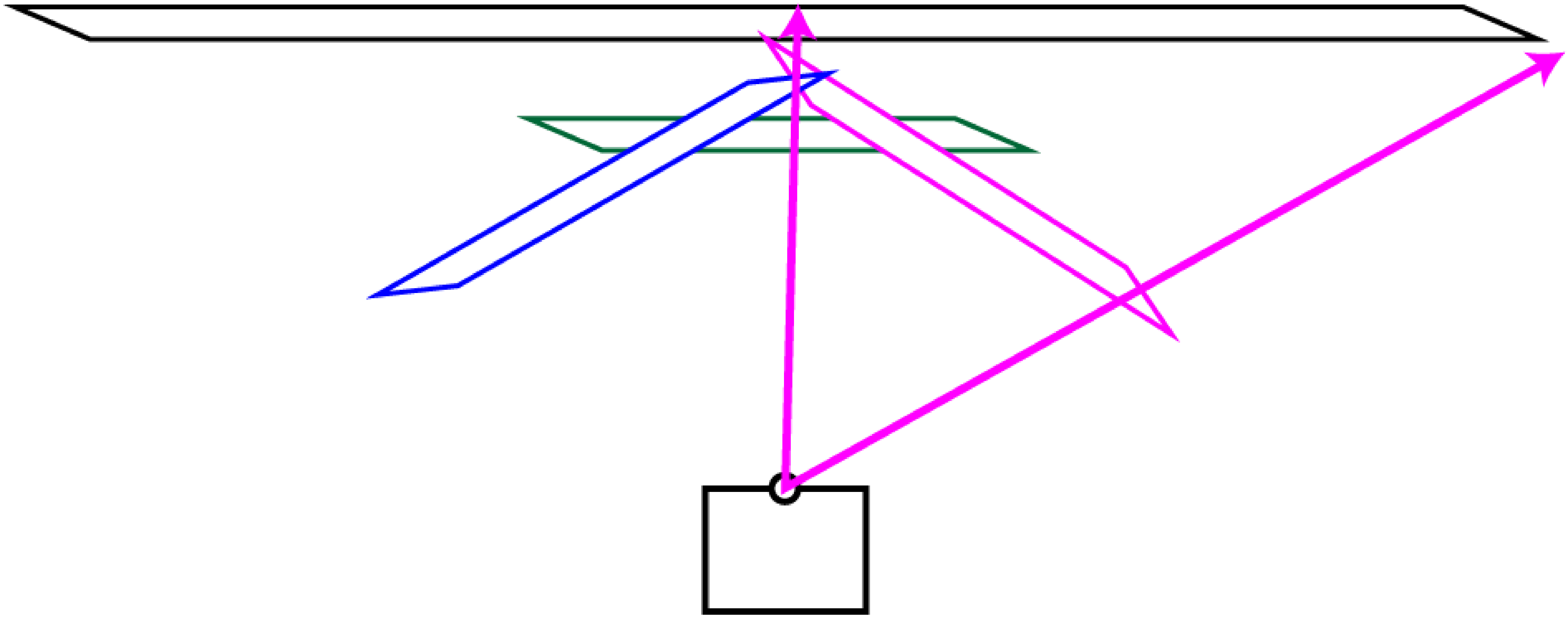
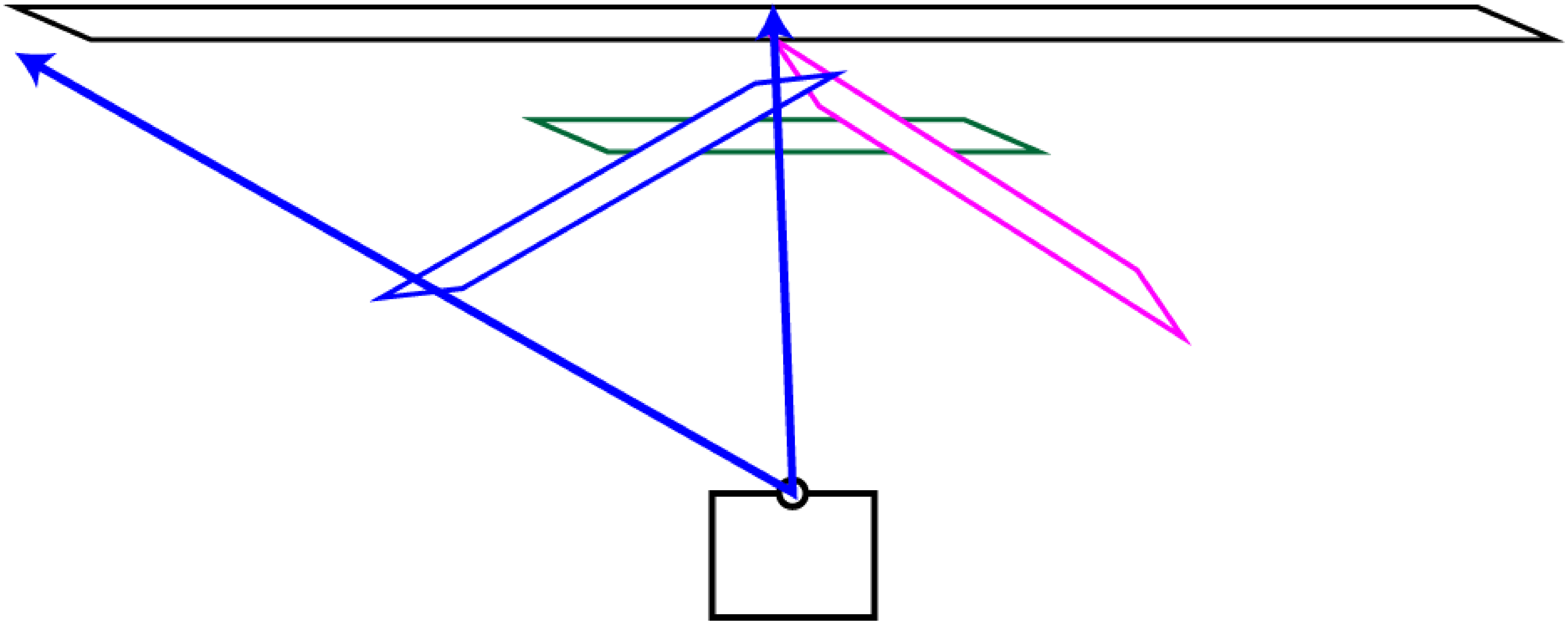


What’s happening?



What’s happening?



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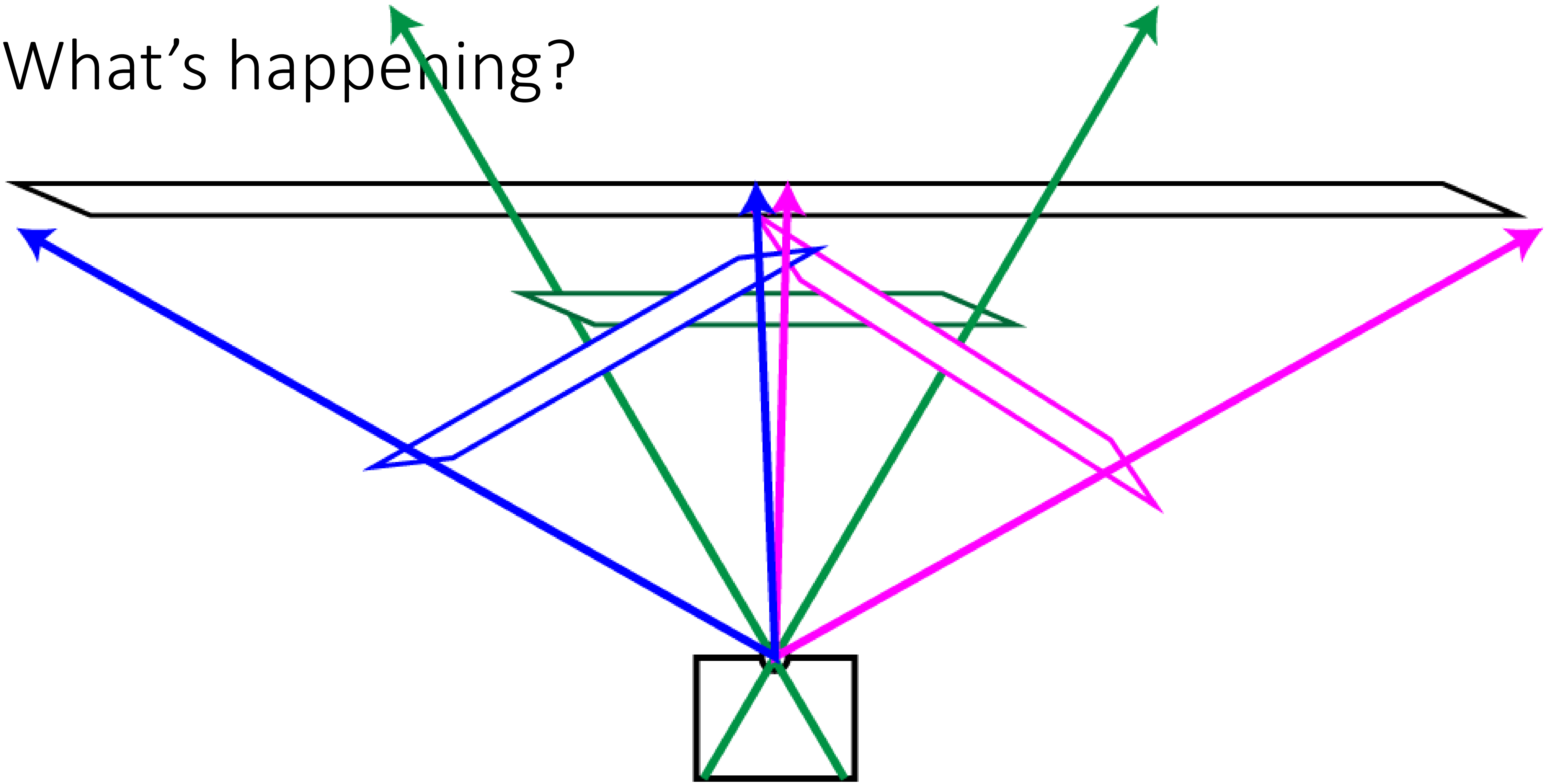

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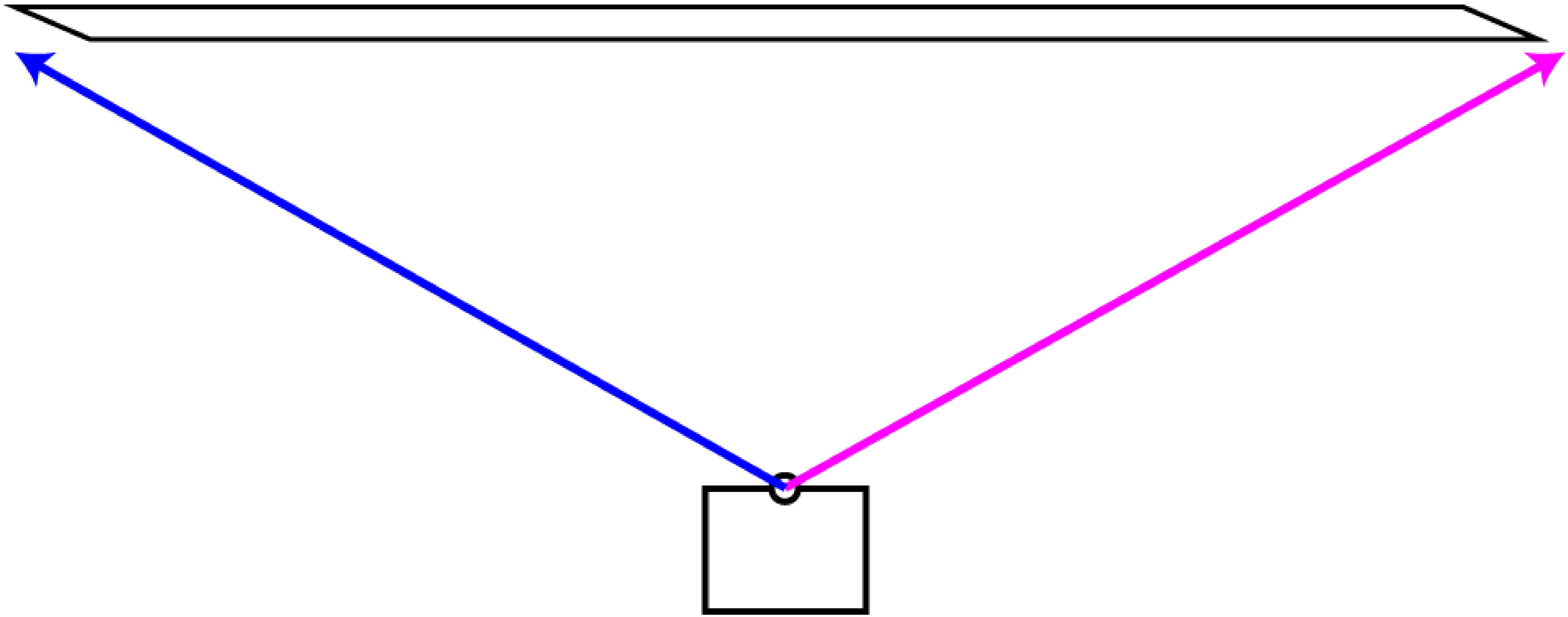

Very bad for big panoramas!

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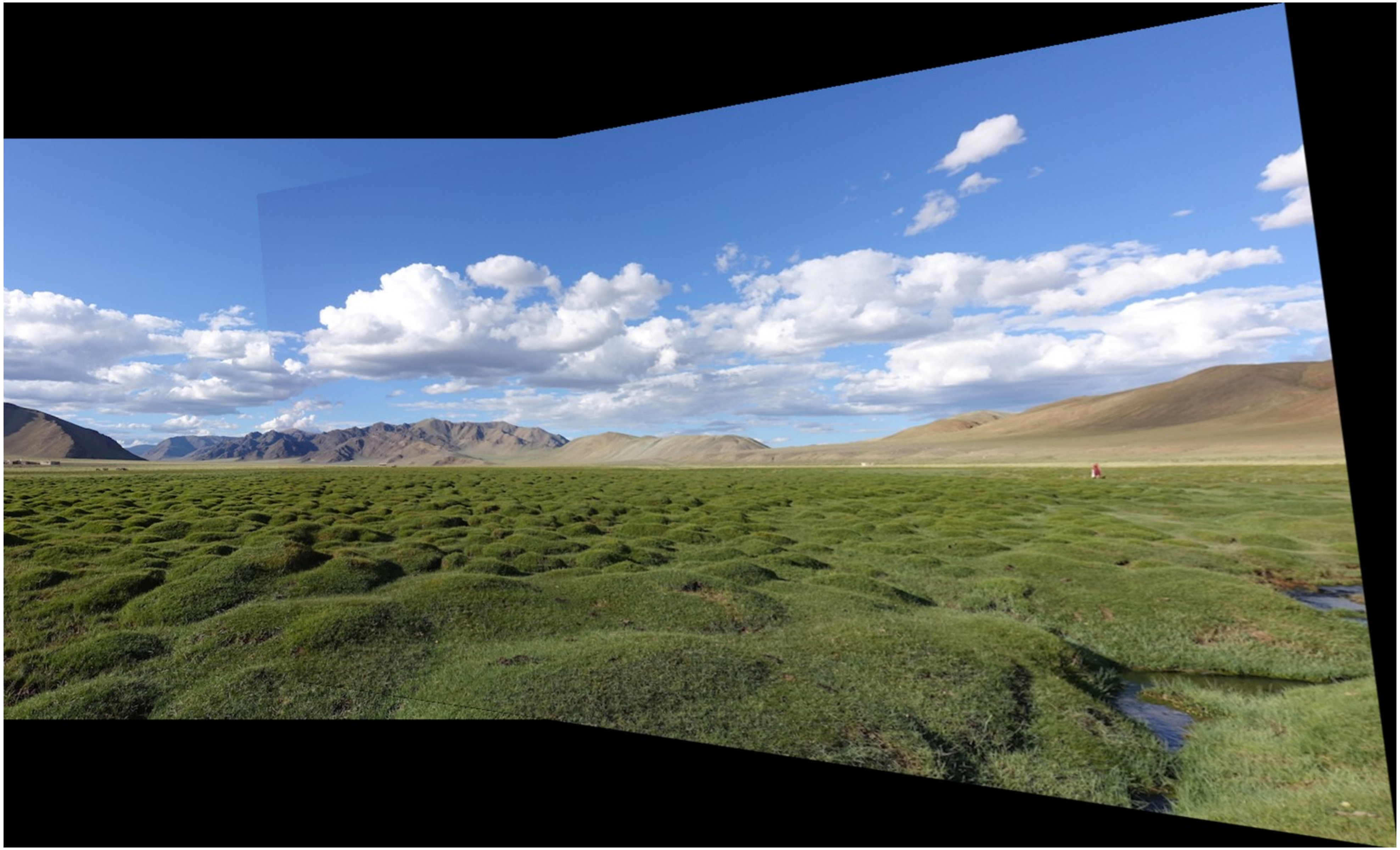

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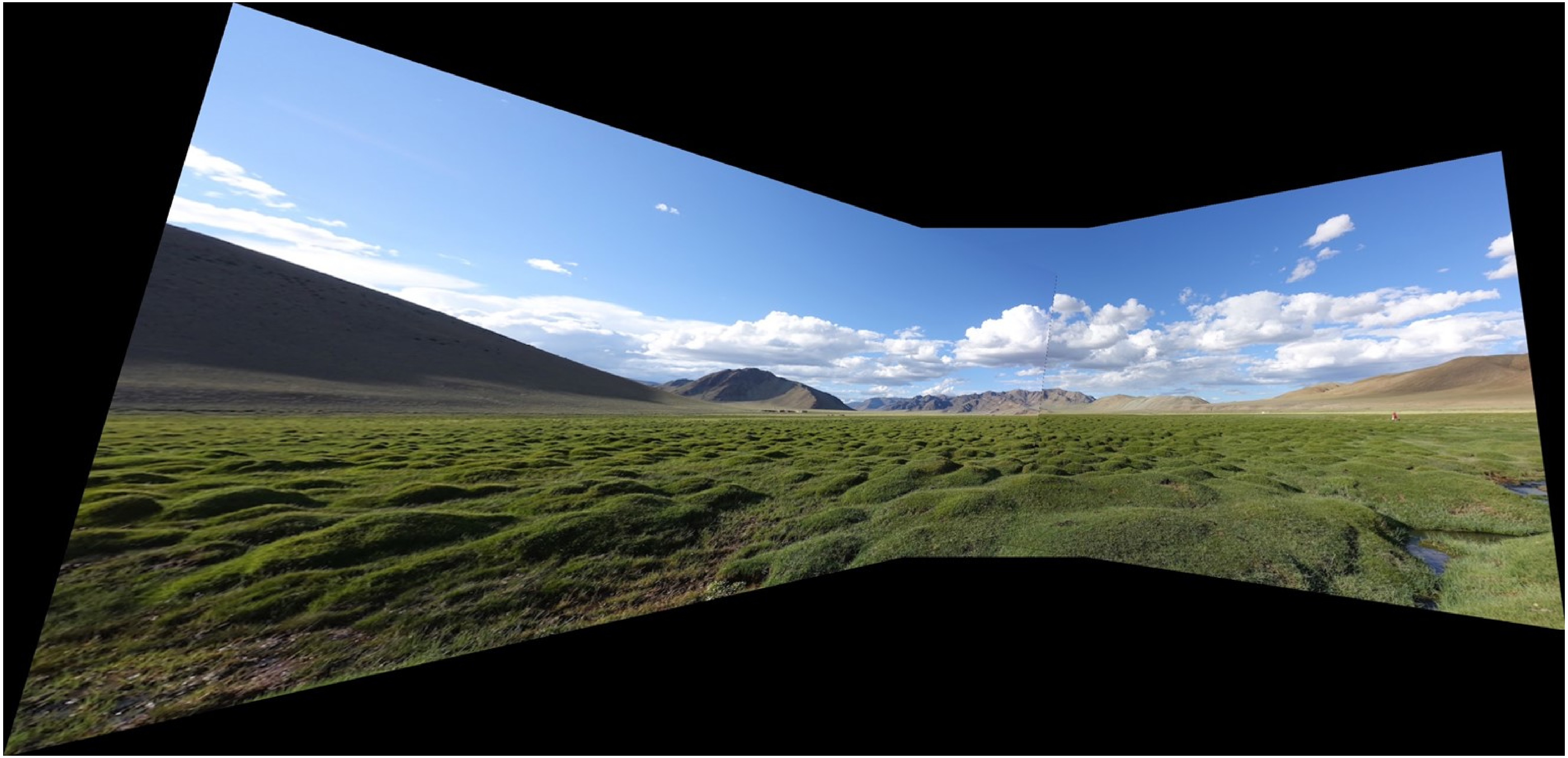

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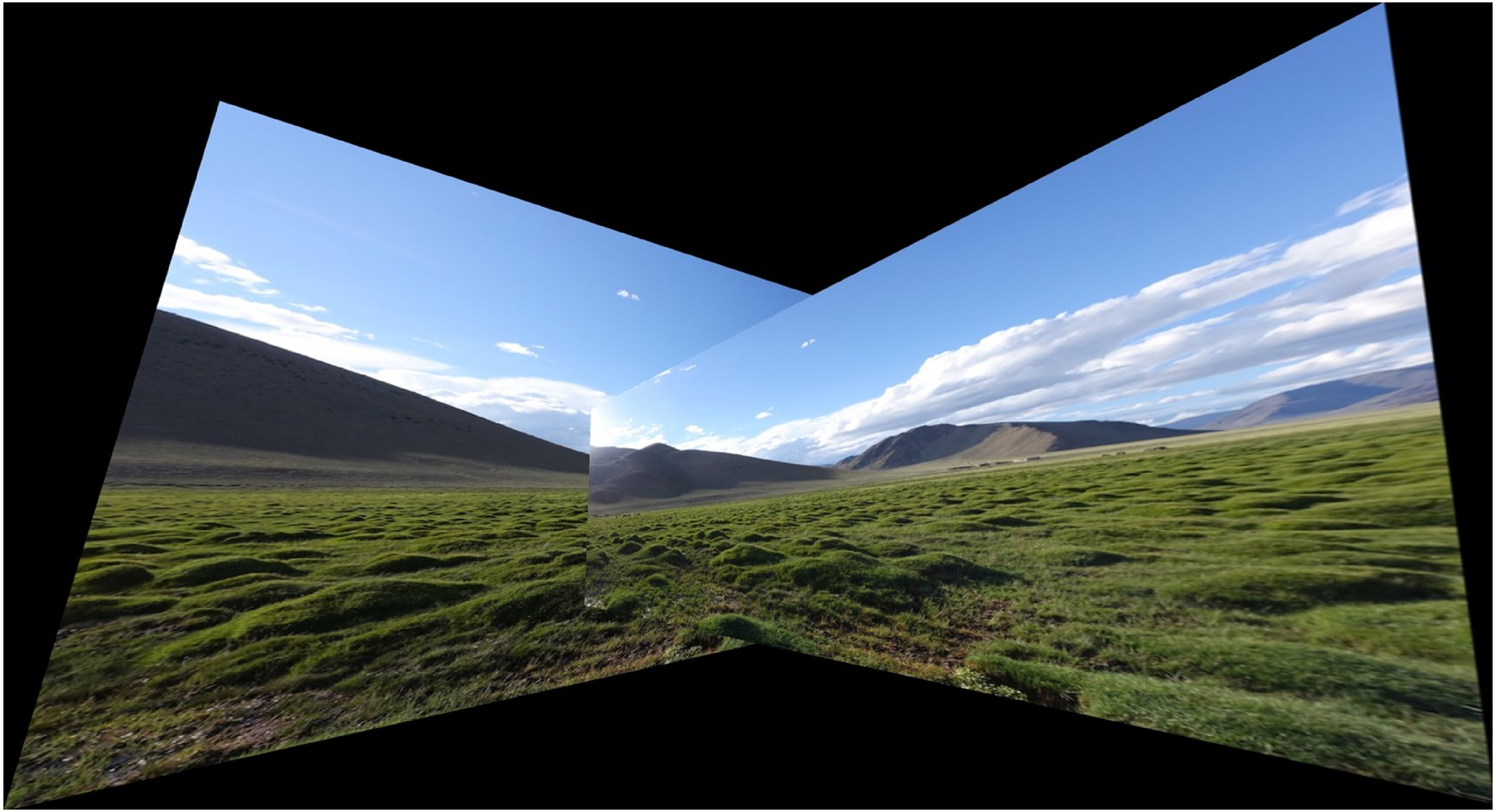
Fails :-(



How do we fix it? Cylinders!

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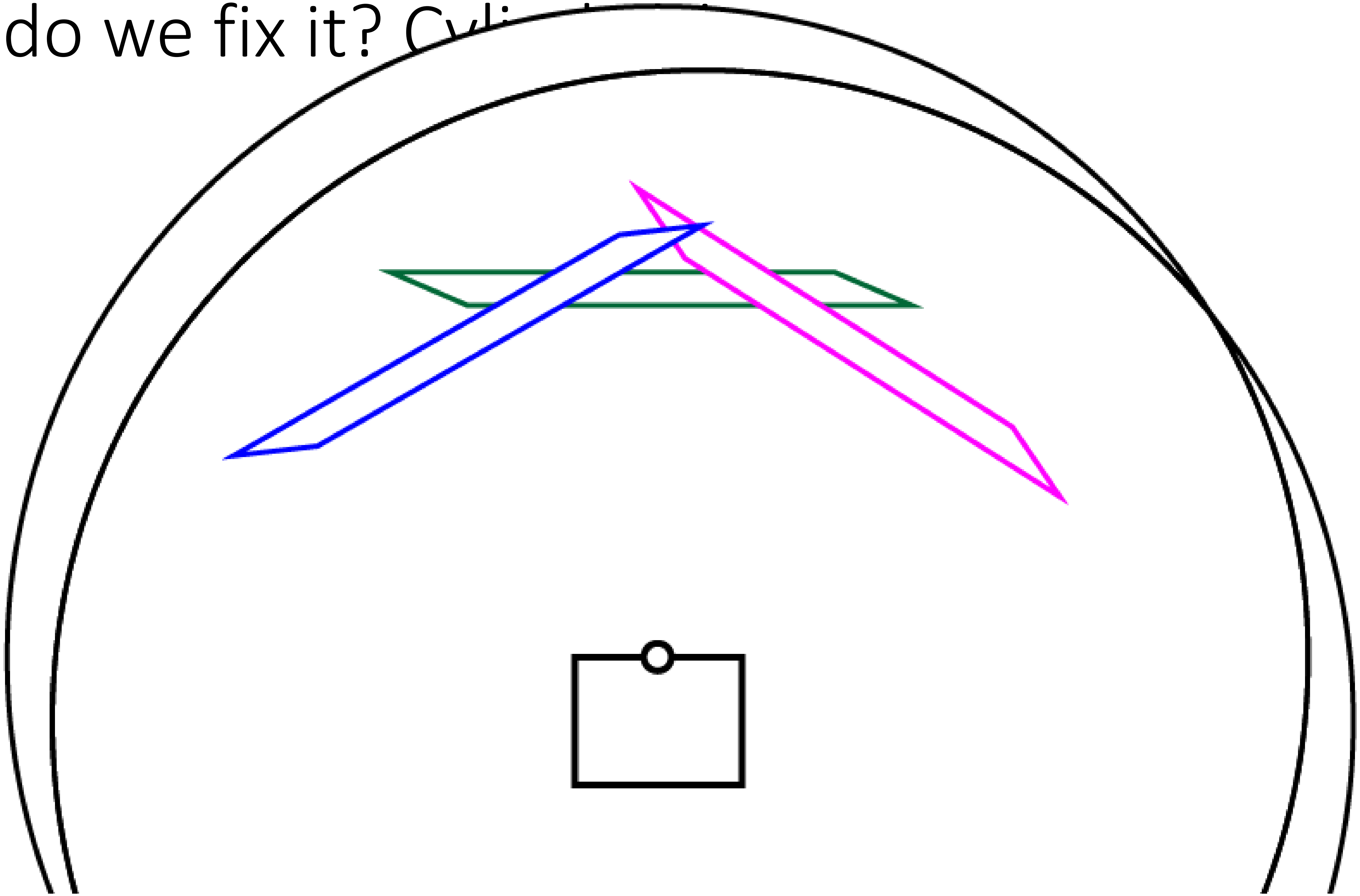
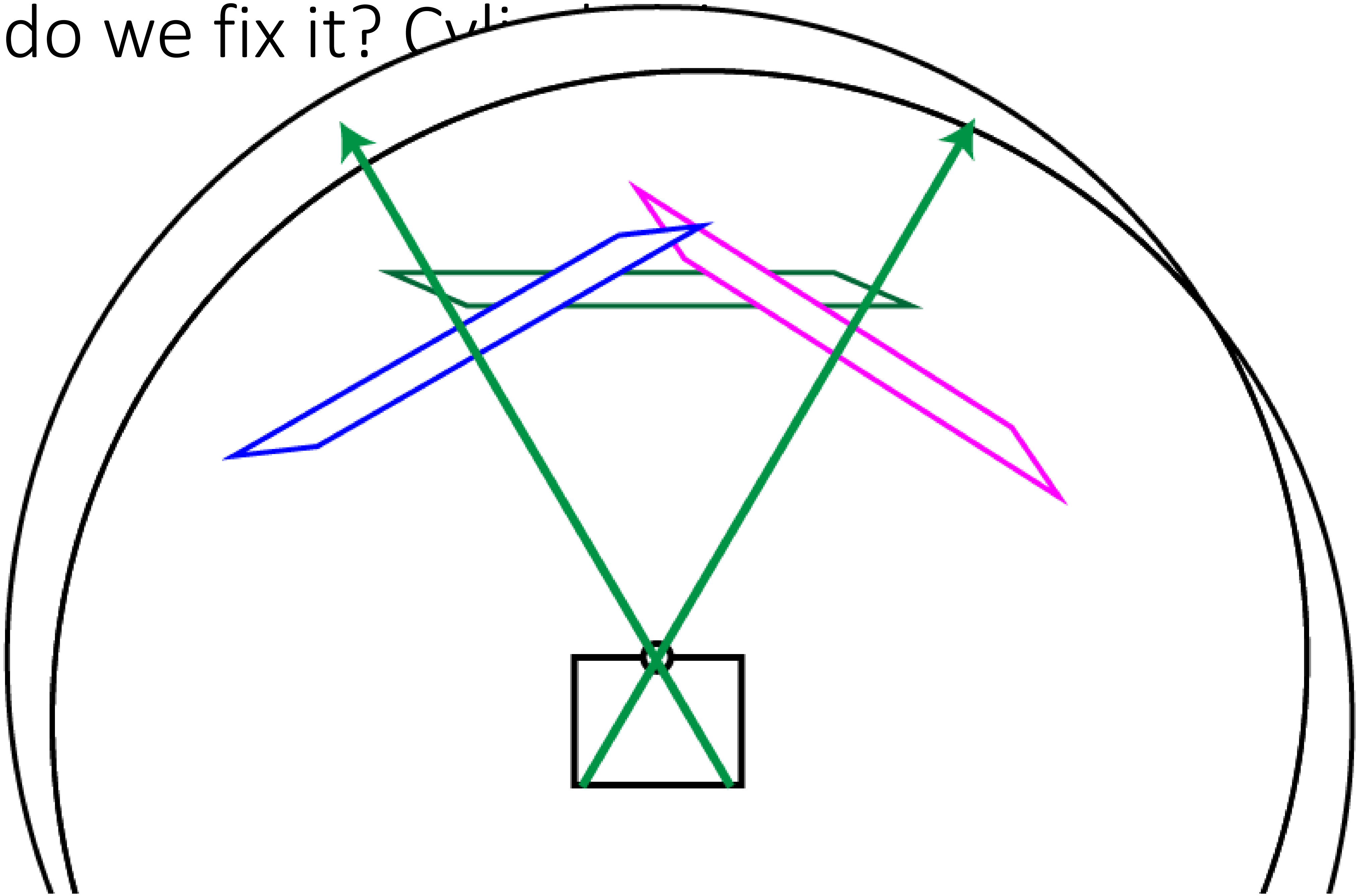
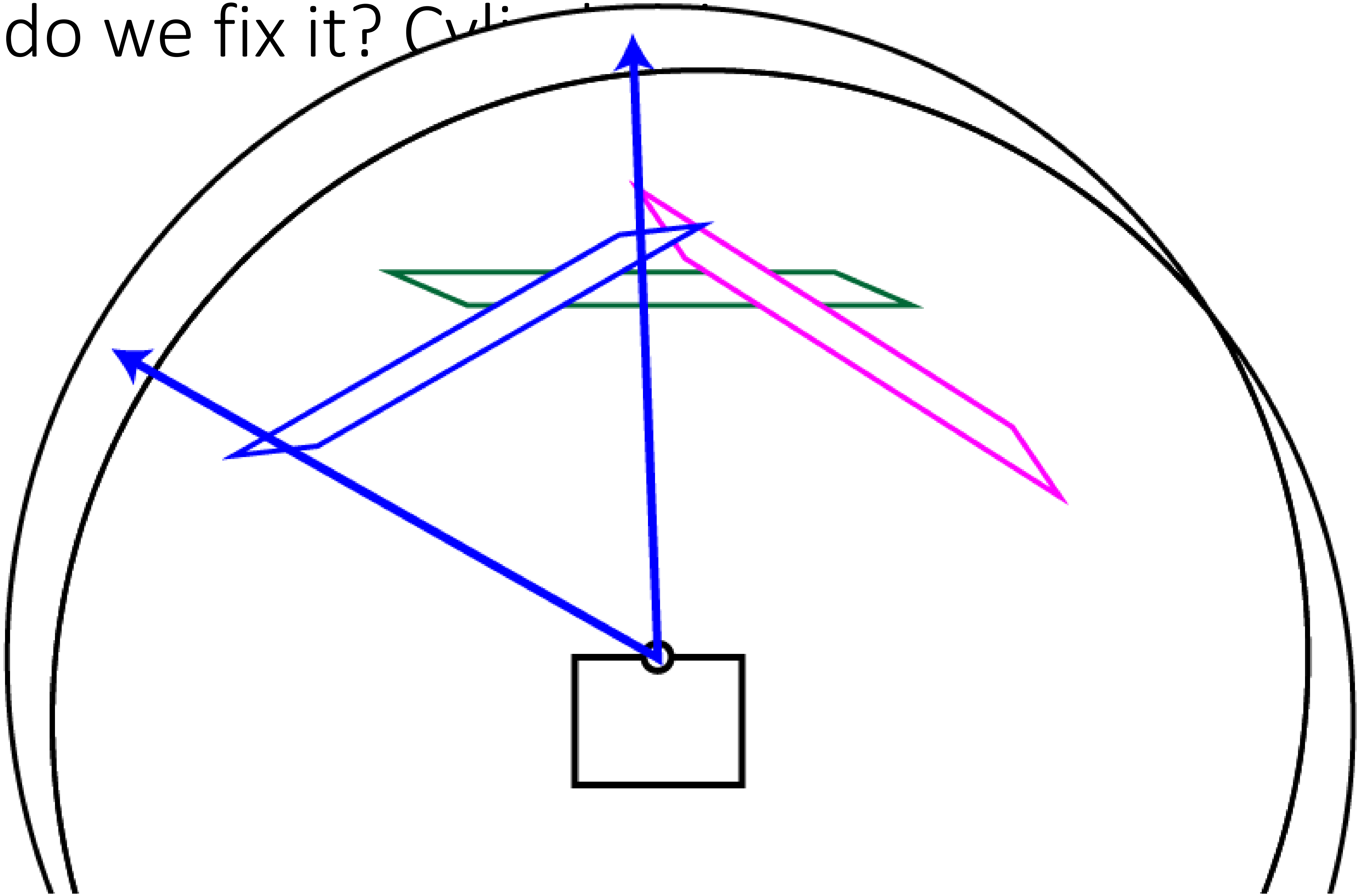
How do we fix it? Cyl



How do we fix it? Cyl



How do we fix it? Cyl



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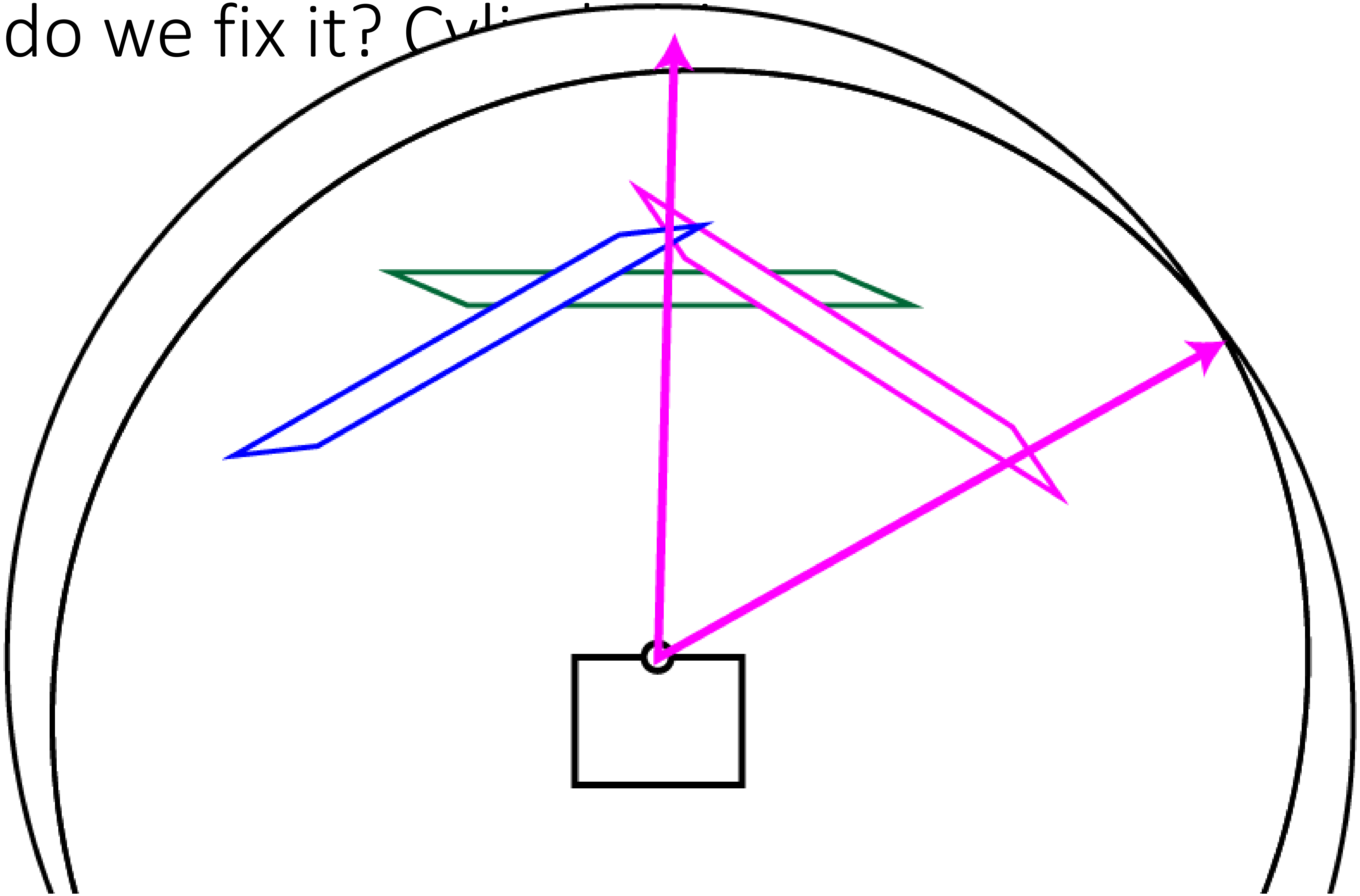

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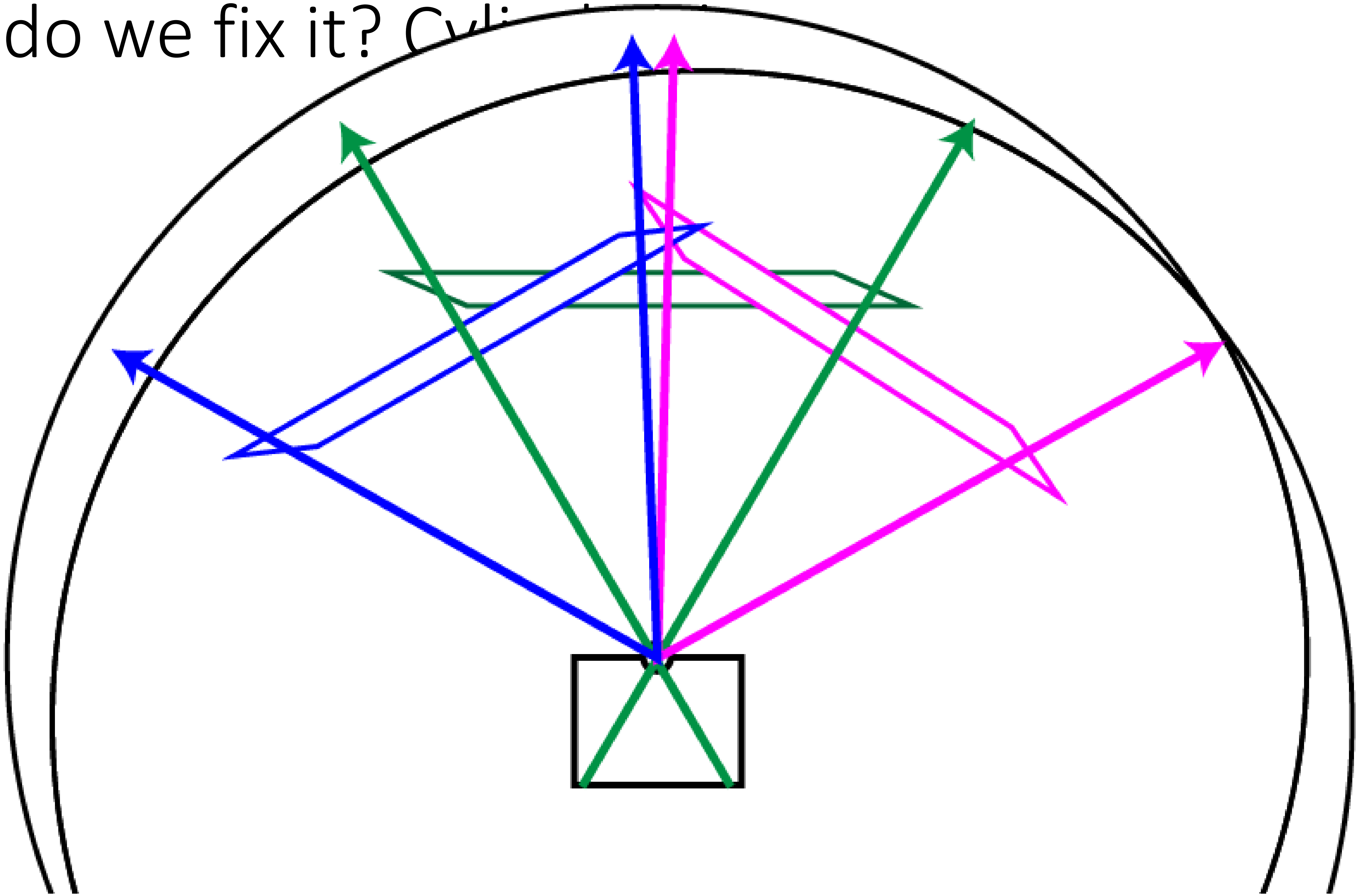
How do we fix it? Cyl



How do we fix it? Cylinders!

Calculate angle and height:

$\theta = (x - xc) / f$

$h = (y - yc) / f$

Find unit cylindrical coords:

$X' = \sin(\theta)$

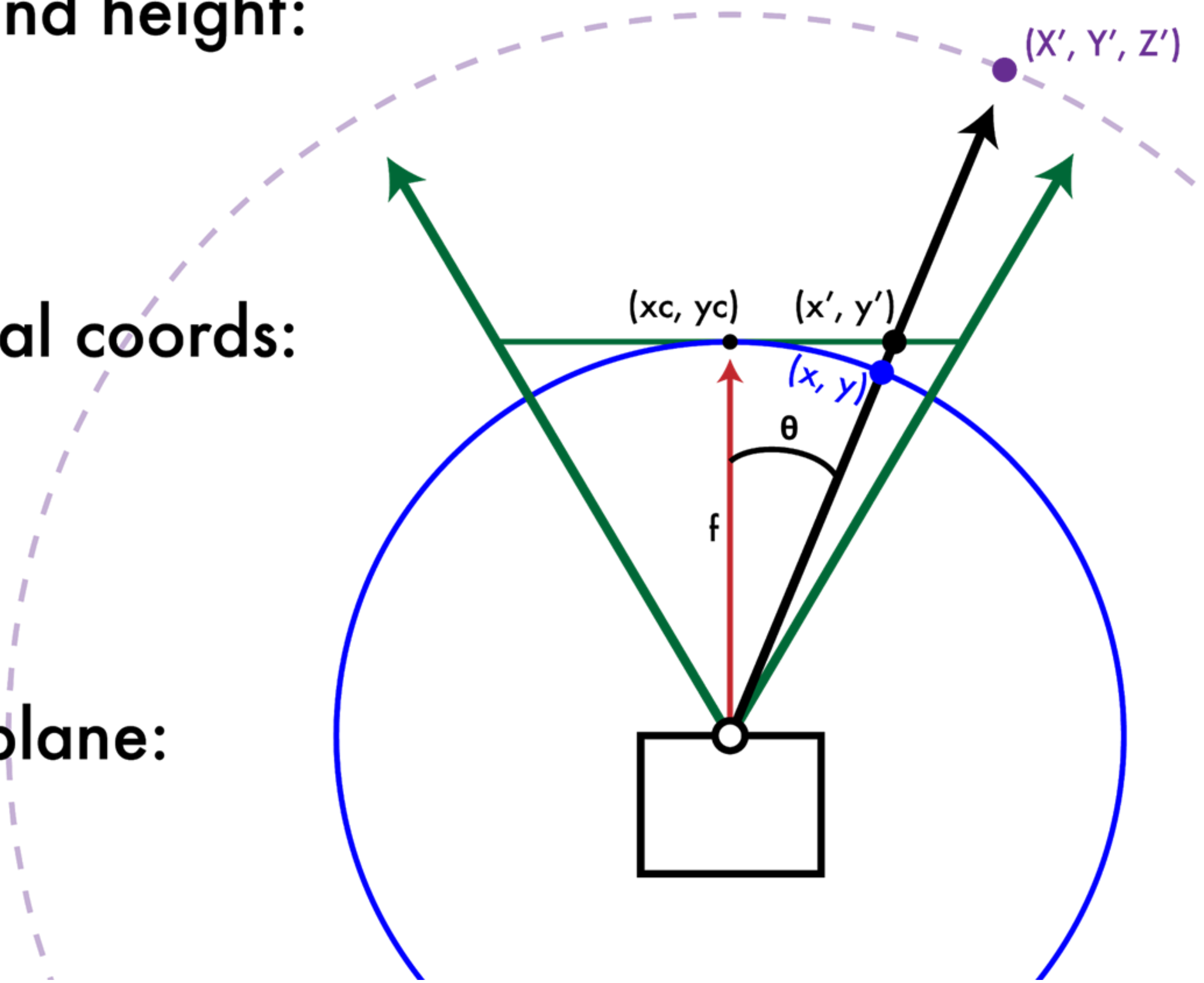
$Y' = h$

$Z' = \cos(\theta)$

Project to image plane:

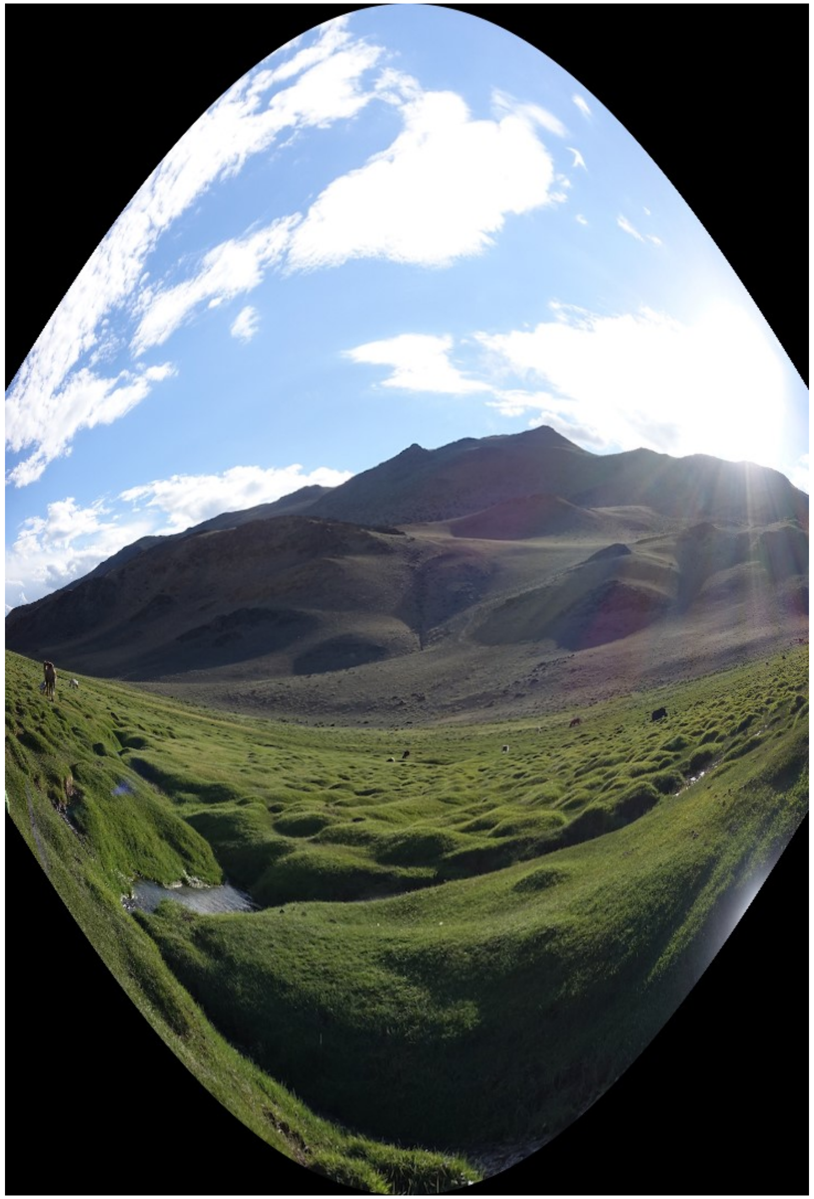
$x' = f X'/Z' + xc$

$y' = f Y'/Z' + yc$



Dependant on focal length!

f = 300



f = 500

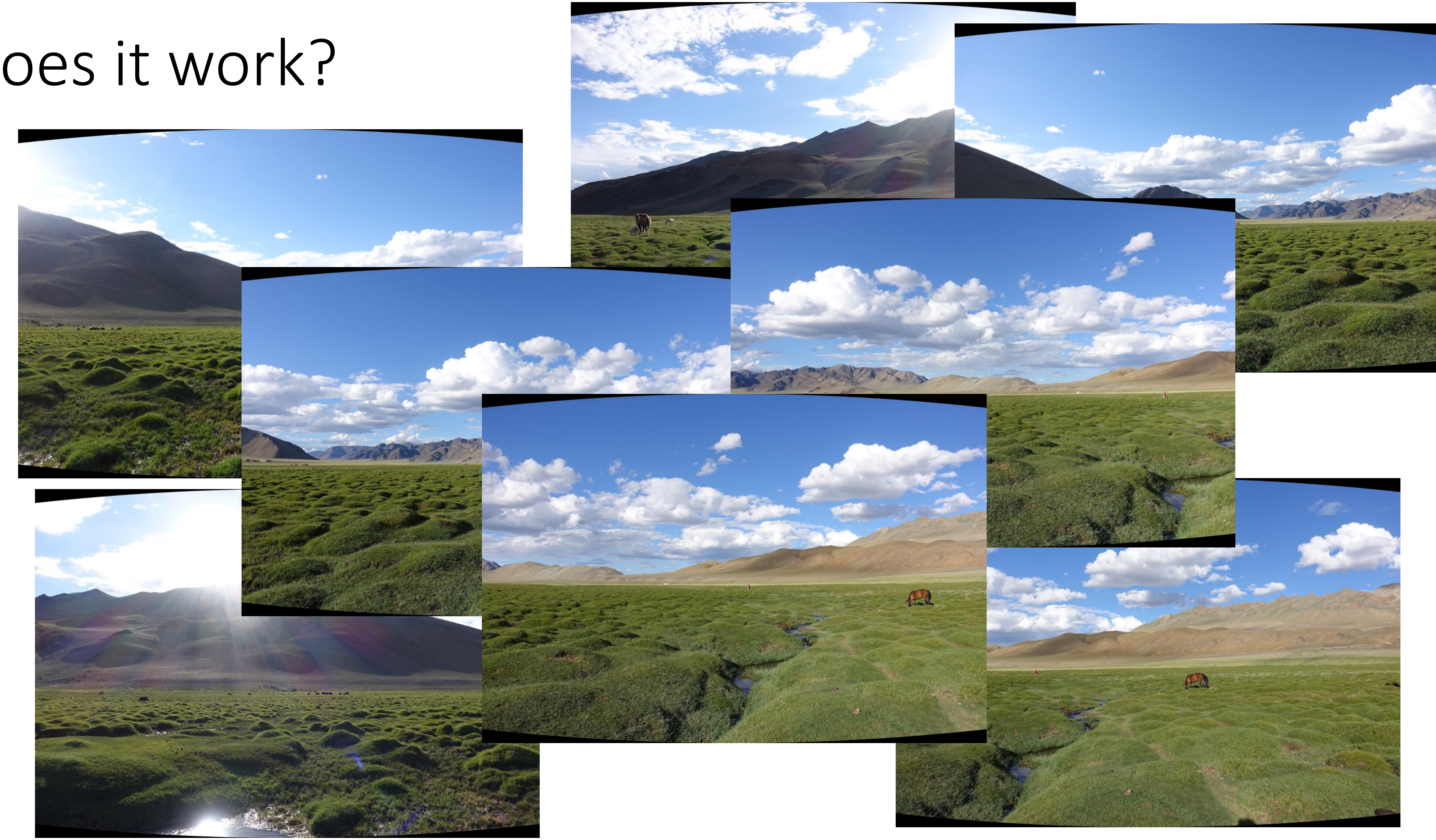
f = 1000

f = 1400

f = 10,000

$f = 10{,}000$

Does it work?



Does it work?

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Does it work?

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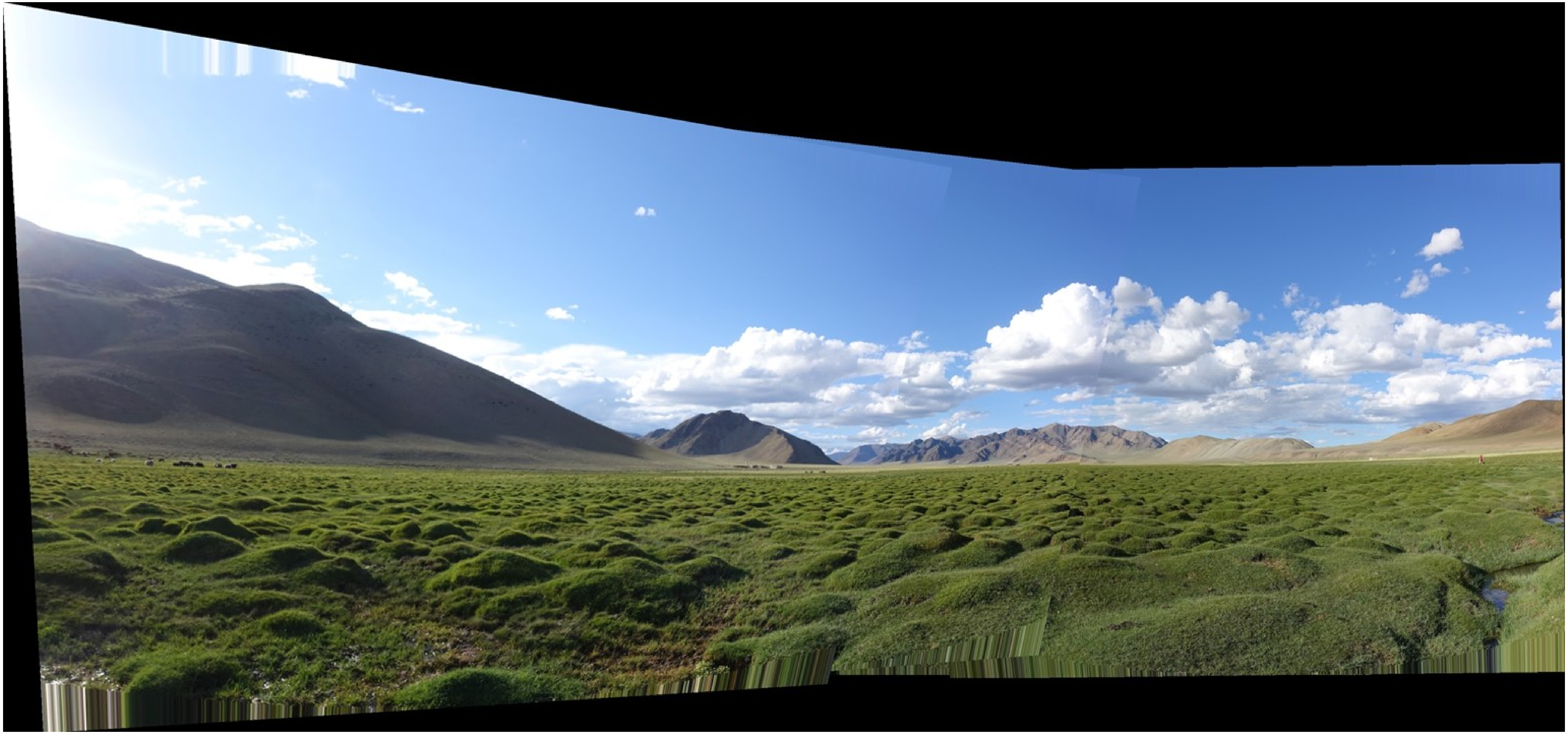
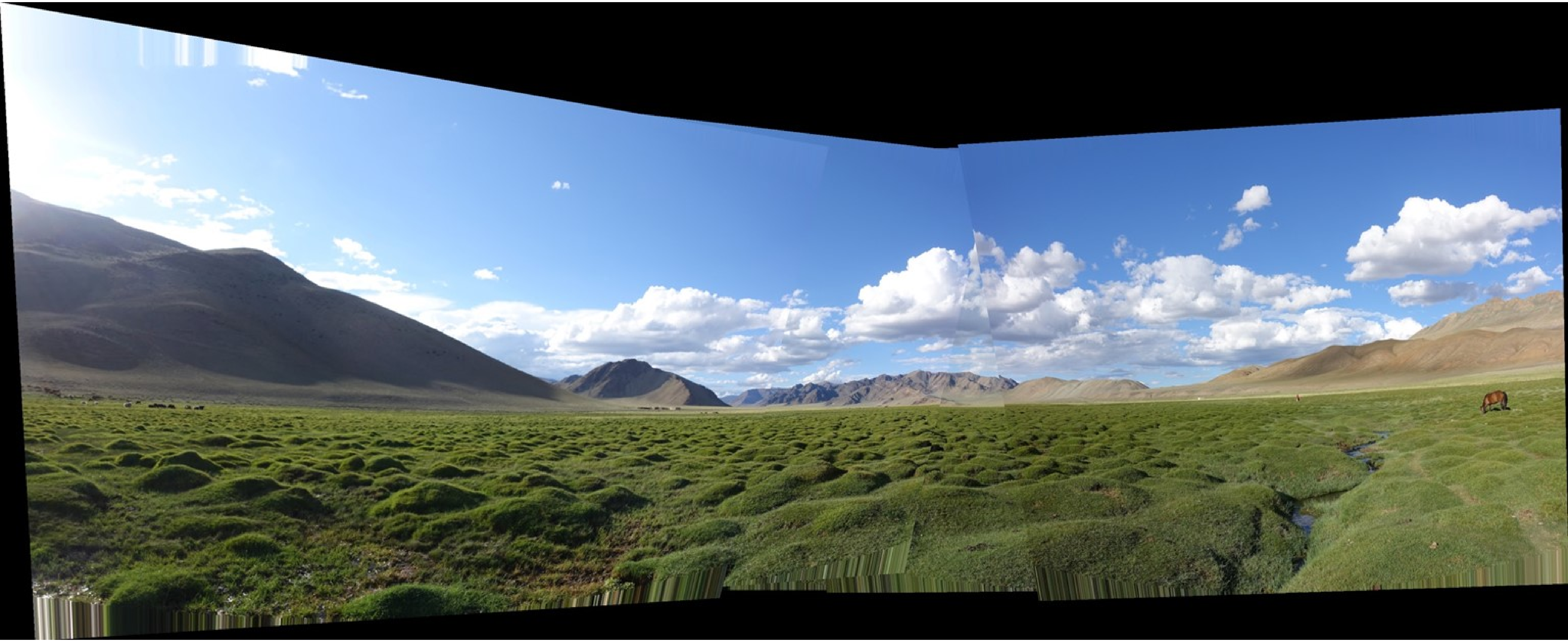


Does it work?



Yes! Assuming camera is level and rotating around its vertical axis

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Master programmes in Artificial Intelligence 4 Careers in Europe

Thank you.

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